Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.2-Cosine/83-4.2.10-c+d-x-^m-a+b-cos-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [189]. This is test number [83].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (189)	0.00 (0)
Mathematica	100.00 (189)	0.00 (0)
Maxima	74.07 (140)	25.93 (49)
Fricas	72.49 (137)	27.51 (52)
Maple	71.43 (135)	28.57 (54)
Giac	59.26 (112)	40.74 (77)
Mupad	39.15 (74)	60.85 (115)
Sympy	29.10 (55)	70.90 (134)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

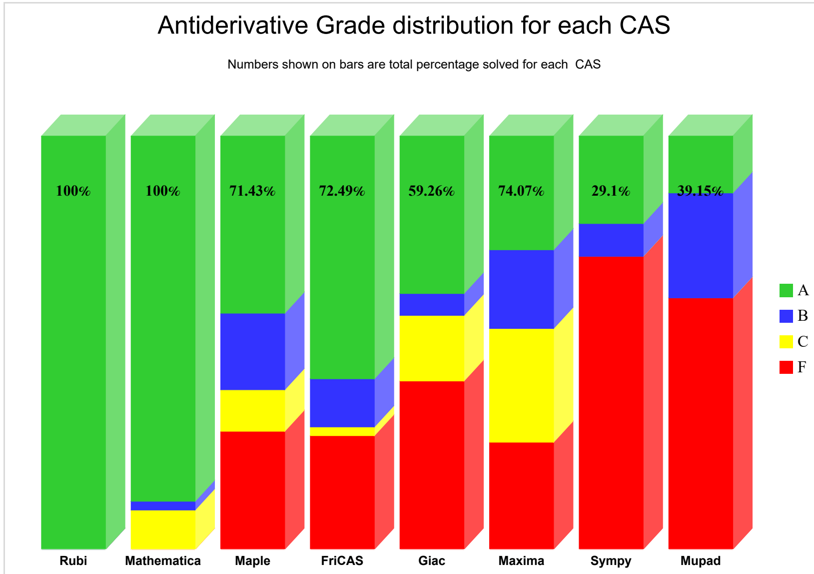
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

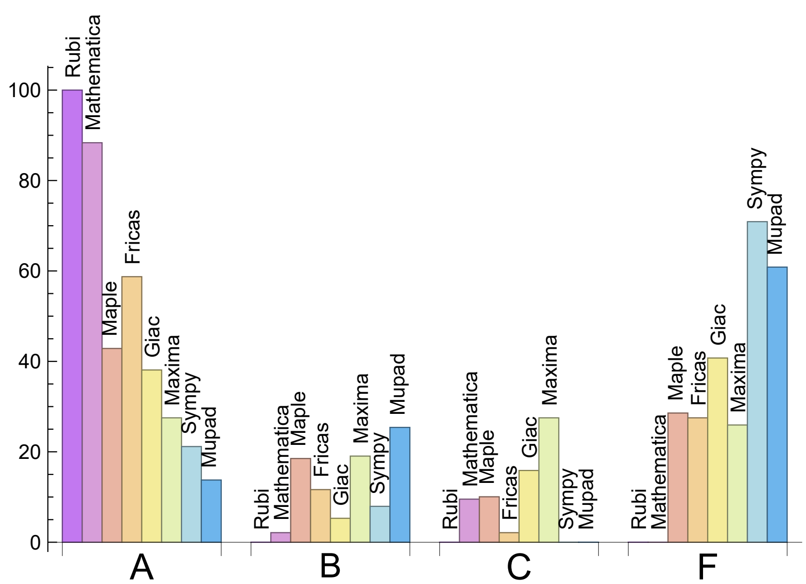
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	88.36	2.12	9.52	0.00
Fricas	58.73	11.64	2.12	27.51
Maple	42.86	18.52	10.05	28.57
Giac	38.10	5.29	15.87	40.74
Maxima	27.51	19.05	27.51	25.93
Sympy	21.16	7.94	0.00	70.90
Mupad	N/A	25.40	0.00	60.85

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	54	100.00 %	0.00 %	0.00 %
Fricas	52	17.31 %	0.00 %	82.69 %
Giac	77	100.00 %	0.00 %	0.00 %
Maxima	49	91.84 %	0.00 %	8.16 %
Sympy	134	97.01 %	1.49 %	1.49 %
Mupad	115	94.78 %	5.22 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

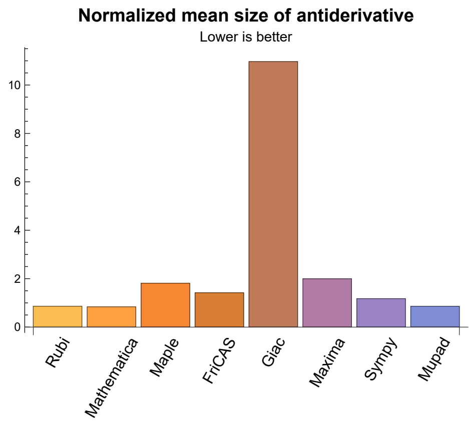
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

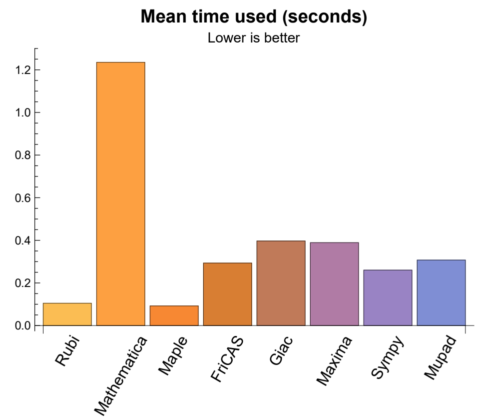
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	108.02	0.86	89.00	1.00
Mathematica	1.24	107.04	0.84	76.00	0.87
Maple	0.09	209.79	1.81	143.00	1.36
Maxima	0.39	269.16	2.00	138.00	1.39
Fricas	0.29	200.81	1.42	114.00	1.04
Sympy	0.26	127.25	1.17	46.00	1.19
Giac	0.40	1604.08	10.97	80.50	1.09
Mupad	0.31	71.41	0.86	34.50	0.85

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{32, 36, 40, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 131, 132, 136, 137, 141, 142, 174, 179, 183, 184, 188\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{139, 189\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

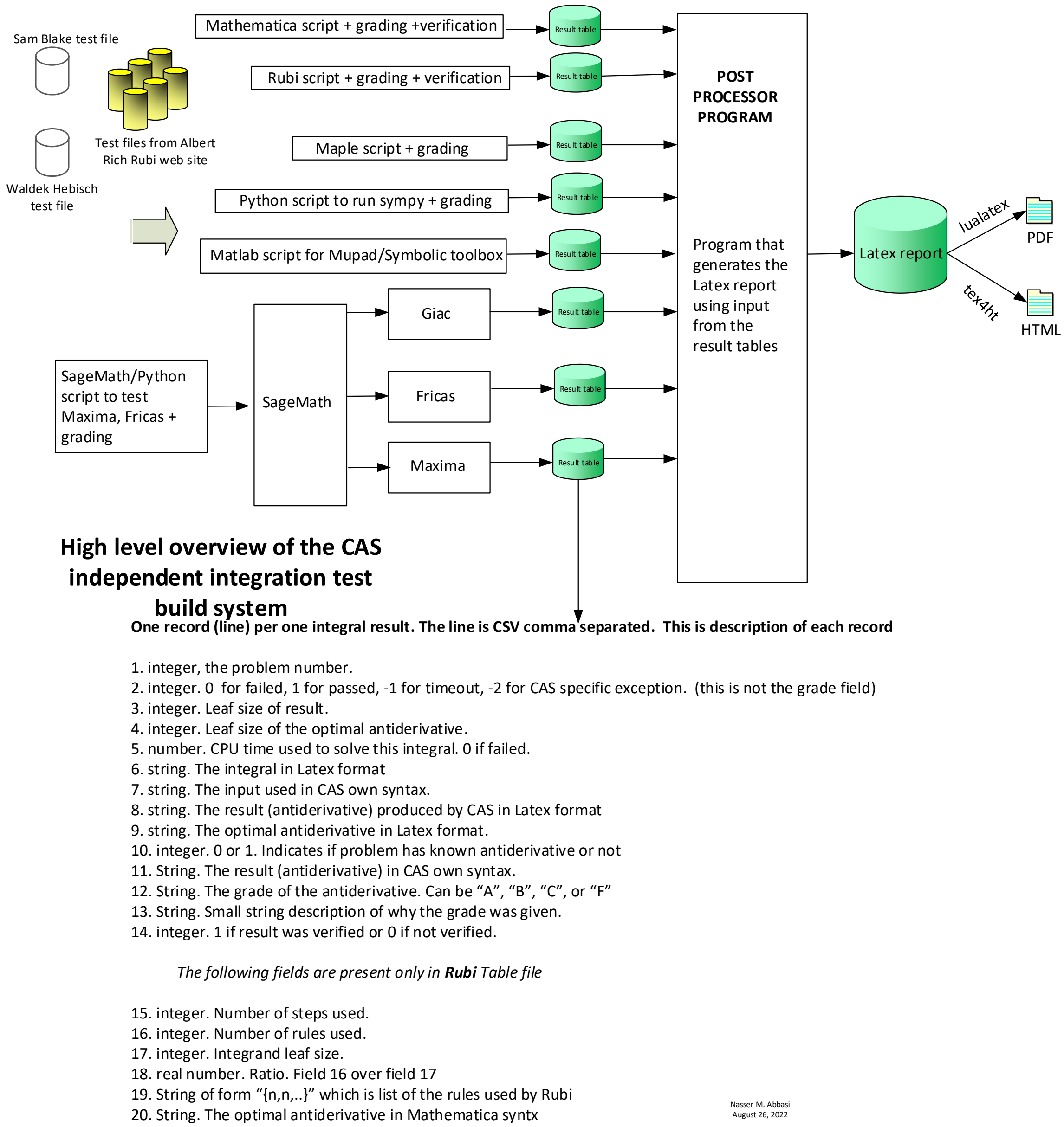
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 48, 49, 52, 54, 56, 57, 60, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188 }

B grade: { 39, 139, 187, 189 }

C grade: { 41, 42, 43, 44, 45, 46, 47, 50, 51, 53, 55, 58, 59, 61, 63, 64, 65, 66 }

F grade: { }

2.1.3 Maple

A grade: { 4, 5, 6, 7, 8, 13, 14, 15, 19, 20, 21, 22, 24, 25, 26, 27, 28, 31, 32, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 121, 122, 126, 127, 130, 131, 132, 135, 136, 137, 141, 142, 146, 153, 160, 174, 178, 179, 183, 184, 188 }

B grade: { 1, 2, 3, 9, 10, 11, 12, 16, 17, 18, 23, 29, 30, 33, 34, 37, 38, 39, 76, 79, 87, 118, 119, 120, 123, 124, 125, 128, 129, 133, 134, 138, 139, 187, 189 }

C grade: { 84, 104, 105, 106, 107, 108, 109, 110, 140, 143, 144, 145, 150, 151, 152, 157, 158, 159, 173 }

F grade: { 67, 68, 69, 70, 71, 72, 73, 74, 81, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 111, 112, 113, 114, 115, 116, 117, 147, 148, 149, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186 }

2.1.4 Maxima

A grade: { 4, 12, 19, 23, 24, 25, 32, 36, 40, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 125, 131, 132, 136, 137, 141, 142, 144, 145, 146, 150, 151, 152, 153, 160, 164, 165, 166, 174, 179, 183, 184, 188 }

B grade: { 1, 2, 3, 9, 10, 11, 16, 17, 18, 29, 30, 33, 34, 35, 37, 38, 118, 119, 120, 123, 124, 128, 129, 130, 133, 134, 135, 138, 139, 140, 143, 157, 158, 159, 173, 178 }

C grade: { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 26, 27, 28, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 121, 122, 126, 127, 147, 148, 149, 154, 155, 156, 167, 168, 169 }

F grade: { 31, 39, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 161, 162, 163, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186, 187, 189 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 32, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 135, 136, 137, 140, 141, 142, 146, 153, 160, 173, 174, 178, 179, 183, 188 }

B grade: { 7, 8, 22, 29, 30, 31, 33, 34, 37, 38, 39, 55, 128, 129, 133, 134, 138, 139, 185, 186, 187, 189 }

C grade: { 76, 79, 84, 87 }

F grade: { 75, 77, 78, 80, 81, 82, 83, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 97, 143, 144, 145, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 180, 181, 182, 184 }

2.1.6 Sympy

A grade: { 4, 19, 23, 24, 25, 26, 32, 36, 40, 63, 64, 65, 66, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 120, 125, 130, 131, 132, 135, 136, 137, 141, 142, 174, 179, 183, 184, 188 }

B grade: { 1, 2, 3, 9, 10, 11, 12, 16, 17, 18, 118, 119, 123, 124, 140 }

C grade: { }

F grade: { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 126, 127, 128, 129, 133, 134, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 9, 10, 11, 12, 16, 17, 18, 19, 23, 24, 25, 32, 36, 40, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 118, 119, 120, 123, 124, 125, 131, 132, 136, 137, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 174, 178, 179, 183, 184, 188 }

B grade: { 6, 14, 21, 35, 122, 127, 130, 135, 140, 173 }

C grade: { 5, 7, 8, 13, 15, 20, 22, 26, 27, 28, 41, 42, 43, 44, 48, 49, 50, 51, 56, 57, 58, 59, 63, 64, 65, 121, 126, 147, 148, 149 }

F grade: { 29, 30, 31, 33, 34, 37, 38, 39, 45, 46, 47, 52, 53, 54, 55, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 128, 129, 133, 134, 138, 139, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186, 187, 189 }

2.1.8 Mupad

A grade: { 32, 36, 40, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 131, 132, 136, 137, 141, 142, 174, 179, 183, 184, 188 }

B grade: { 1, 2, 3, 4, 9, 10, 11, 12, 16, 17, 18, 19, 23, 24, 25, 35, 64, 65, 76, 79, 84, 87, 89, 90, 91, 92, 118, 119, 120, 123, 124, 125, 130, 135, 140, 143, 144, 145, 146, 150, 151, 152, 153, 157, 158, 159, 160, 173 }

C grade: { }

F grade: { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 26, 27, 28, 29, 30, 31, 33, 34, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 72, 73, 74, 81, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 126, 127, 128, 129, 133, 134, 138, 139, 147, 148, 149, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	76	539	481	169	311	170	219
N.S.	1	1.00	0.84	5.92	5.29	1.86	3.42	1.87	2.41
time (sec)	N/A	0.063	0.174	0.145	0.340	0.379	0.356	0.455	0.418

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	302	278	109	202	110	147
N.S.	1	1.00	0.87	4.31	3.97	1.56	2.89	1.57	2.10
time (sec)	N/A	0.044	0.117	0.090	0.333	0.368	0.220	0.429	0.294

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	143	136	62	112	64	84
N.S.	1	1.00	0.90	2.92	2.78	1.27	2.29	1.31	1.71
time (sec)	N/A	0.028	0.096	0.072	0.331	0.375	0.146	0.520	0.120

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	51	50	28	46	30	34
N.S.	1	1.00	0.96	1.89	1.85	1.04	1.70	1.11	1.26
time (sec)	N/A	0.011	0.044	0.047	0.306	0.369	0.086	0.468	0.180

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	77	142	78	0	577	-1
N.S.	1	1.00	0.96	1.48	2.73	1.50	0.00	11.10	-0.02
time (sec)	N/A	0.065	0.057	0.079	0.354	0.394	0.000	0.459	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	114	164	123	0	523	-1
N.S.	1	1.00	0.89	1.56	2.25	1.68	0.00	7.16	-0.01
time (sec)	N/A	0.077	0.236	0.102	0.375	0.381	0.000	0.464	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	89	148	199	209	0	5518	-1
N.S.	1	1.00	0.86	1.42	1.91	2.01	0.00	53.06	-0.01
time (sec)	N/A	0.093	0.434	0.151	0.423	0.374	0.000	0.570	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	144	184	249	295	0	8378	-1
N.S.	1	1.00	1.13	1.45	1.96	2.32	0.00	65.97	-0.01
time (sec)	N/A	0.109	0.385	0.190	0.486	0.376	0.000	0.704	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	132	1027	717	287	660	222	349
N.S.	1	1.00	0.82	6.38	4.45	1.78	4.10	1.38	2.17
time (sec)	N/A	0.068	0.361	0.175	0.333	0.375	0.562	0.487	0.614

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	106	587	428	190	456	153	229
N.S.	1	1.00	0.79	4.38	3.19	1.42	3.40	1.14	1.71
time (sec)	N/A	0.049	0.280	0.109	0.335	0.369	0.369	0.437	0.458

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	77	289	222	113	264	94	179
N.S.	1	1.00	0.81	3.04	2.34	1.19	2.78	0.99	1.88
time (sec)	N/A	0.036	0.202	0.086	0.307	0.364	0.240	0.415	0.207

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	112	90	53	126	48	57
N.S.	1	1.00	0.91	2.04	1.64	0.96	2.29	0.87	1.04
time (sec)	N/A	0.017	0.123	0.059	0.288	0.387	0.123	0.550	0.096

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	114	163	88	0	610	-1
N.S.	1	1.00	0.83	1.46	2.09	1.13	0.00	7.82	-0.01
time (sec)	N/A	0.114	0.079	0.084	0.383	0.381	0.000	0.437	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	156	171	127	0	534	-1
N.S.	1	1.00	0.90	1.88	2.06	1.53	0.00	6.43	-0.01
time (sec)	N/A	0.095	0.413	0.123	0.381	0.379	0.000	0.503	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	102	193	204	218	0	5136	-1
N.S.	1	1.00	0.91	1.72	1.82	1.95	0.00	45.86	-0.01
time (sec)	N/A	0.128	0.637	0.202	0.423	0.373	0.000	0.647	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	385	1023	925	350	772	351	532
N.S.	1	1.00	1.71	4.55	4.11	1.56	3.43	1.56	2.36
time (sec)	N/A	0.164	0.589	0.182	0.336	0.394	0.782	0.500	1.143

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	121	560	535	227	495	231	364
N.S.	1	1.00	0.69	3.20	3.06	1.30	2.83	1.32	2.08
time (sec)	N/A	0.110	0.614	0.176	0.318	0.384	0.528	0.484	0.724

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	93	265	267	128	284	137	173
N.S.	1	1.00	0.76	2.15	2.17	1.04	2.31	1.11	1.41
time (sec)	N/A	0.064	0.392	0.136	0.309	0.379	0.337	0.463	0.592

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	95	103	60	126	69	77
N.S.	1	1.00	0.69	1.27	1.37	0.80	1.68	0.92	1.03
time (sec)	N/A	0.028	0.127	0.087	0.305	0.378	0.204	0.511	0.260

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	103	171	278	153	0	6075	-1
N.S.	1	1.00	0.85	1.41	2.30	1.26	0.00	50.21	-0.01
time (sec)	N/A	0.169	0.149	0.105	0.398	0.358	0.000	0.613	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	200	247	304	227	0	1000	-1
N.S.	1	1.00	1.38	1.70	2.10	1.57	0.00	6.90	-0.01
time (sec)	N/A	0.160	0.464	0.164	0.436	0.462	0.000	0.565	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	221	316	339	375	0	115446	-1
N.S.	1	1.00	1.20	1.72	1.84	2.04	0.00	627.42	-0.01
time (sec)	N/A	0.220	0.530	0.287	0.527	0.433	0.000	2.712	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	100	432	303	115	253	108	138
N.S.	1	1.00	0.58	2.51	1.76	0.67	1.47	0.63	0.80
time (sec)	N/A	0.103	0.282	0.122	0.305	0.389	0.640	0.463	0.796

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	92	237	188	88	209	84	104
N.S.	1	1.00	0.69	1.77	1.40	0.66	1.56	0.63	0.78
time (sec)	N/A	0.072	0.124	0.119	0.312	0.427	0.445	0.439	0.543

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	106	98	63	138	64	63
N.S.	1	1.00	0.66	1.32	1.22	0.79	1.72	0.80	0.79
time (sec)	N/A	0.029	0.086	0.093	0.301	0.398	0.266	0.414	0.339

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	52	91	61	60	428	-1
N.S.	1	1.00	0.88	0.88	1.54	1.03	1.02	7.25	-0.02
time (sec)	N/A	0.104	0.055	0.095	0.362	0.418	1.298	0.508	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	79	90	717	87	0	3220	-1
N.S.	1	1.00	1.20	1.36	10.86	1.32	0.00	48.79	-0.02
time (sec)	N/A	0.102	0.146	0.122	0.385	0.358	0.000	0.475	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	119	124	790	130	0	3920	-1
N.S.	1	1.00	1.32	1.38	8.78	1.44	0.00	43.56	-0.01
time (sec)	N/A	0.172	0.186	0.111	0.384	0.368	0.000	0.501	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	196	685	722	970	0	0	-1
N.S.	1	1.00	0.96	3.34	3.52	4.73	0.00	0.00	-0.00
time (sec)	N/A	0.095	0.122	0.191	0.630	0.446	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	130	392	402	598	0	0	-1
N.S.	1	1.00	0.95	2.86	2.93	4.36	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.068	0.100	0.585	0.439	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	87	128	0	306	0	0	-1
N.S.	1	1.00	1.16	1.71	0.00	4.08	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.007	0.040	0.000	0.406	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	3.042	0.046	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	109	316	1059	790	0	0	-1
N.S.	1	1.00	0.96	2.77	9.29	6.93	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.345	0.127	0.611	0.432	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	75	170	324	450	0	0	-1
N.S.	1	1.00	0.91	2.07	3.95	5.49	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.171	0.074	0.582	0.427	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	36	52	159	45	0	1459	55
N.S.	1	1.00	1.29	1.86	5.68	1.61	0.00	52.11	1.96
time (sec)	N/A	0.019	0.012	0.051	0.524	0.376	0.000	0.659	0.832

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	3.749	0.060	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	311	1127	3831	1315	0	0	-1
N.S.	1	1.00	0.92	3.34	11.37	3.90	0.00	0.00	-0.00
time (sec)	N/A	0.160	1.818	0.273	1.982	0.488	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	184	584	1891	795	0	0	-1
N.S.	1	1.00	0.95	3.03	9.80	4.12	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.620	0.160	0.865	0.443	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	389	267	0	435	0	0	-1
N.S.	1	1.00	3.32	2.28	0.00	3.72	0.00	0.00	-0.01
time (sec)	N/A	0.047	2.424	0.092	0.000	0.410	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.994	0.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	124	232	263	190	0	1239	-1
N.S.	1	1.00	0.64	1.20	1.36	0.98	0.00	6.39	-0.01
time (sec)	N/A	0.289	0.038	0.061	0.336	0.394	0.000	0.609	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	122	189	242	156	0	773	-1
N.S.	1	1.00	0.72	1.12	1.43	0.92	0.00	4.57	-0.01
time (sec)	N/A	0.170	0.061	0.041	0.324	0.369	0.000	0.495	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	124	144	196	126	0	422	-1
N.S.	1	1.00	0.87	1.01	1.38	0.89	0.00	2.97	-0.01
time (sec)	N/A	0.116	0.061	0.040	0.321	0.382	0.000	0.443	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	124	100	159	108	0	166	-1
N.S.	1	1.00	1.05	0.85	1.35	0.92	0.00	1.41	-0.01
time (sec)	N/A	0.092	0.037	0.043	0.325	0.376	0.000	0.451	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	147	140	129	144	0	0	-1
N.S.	1	1.00	1.06	1.01	0.93	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.218	0.040	0.622	0.389	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	190	180	129	208	0	0	-1
N.S.	1	1.00	1.13	1.07	0.77	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.211	0.041	0.627	0.404	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	228	220	129	296	0	0	-1
N.S.	1	1.00	1.18	1.14	0.67	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.239	0.040	0.606	0.422	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	194	242	295	258	0	1325	-1
N.S.	1	1.00	0.84	1.05	1.28	1.12	0.00	5.74	-0.00
time (sec)	N/A	0.299	1.366	0.067	0.541	0.389	0.000	0.643	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	175	197	274	195	0	817	-1
N.S.	1	1.00	0.86	0.97	1.35	0.96	0.00	4.02	-0.00
time (sec)	N/A	0.237	1.061	0.060	0.539	0.432	0.000	0.615	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	146	150	229	148	0	434	-1
N.S.	1	1.00	0.92	0.95	1.45	0.94	0.00	2.75	-0.01
time (sec)	N/A	0.184	0.349	0.059	0.530	0.442	0.000	0.521	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	145	108	187	114	0	165	-1
N.S.	1	1.00	1.12	0.83	1.44	0.88	0.00	1.27	-0.01
time (sec)	N/A	0.151	0.153	0.061	0.524	0.389	0.000	0.438	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	146	135	136	0	0	-1
N.S.	1	1.00	0.99	1.08	1.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.409	0.087	0.629	0.384	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	181	189	136	206	0	0	-1
N.S.	1	1.00	1.06	1.11	0.80	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.927	0.088	0.636	0.411	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	244	230	136	323	0	0	-1
N.S.	1	1.00	1.13	1.06	0.63	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.829	0.089	0.645	0.443	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	237	273	136	417	0	0	-1
N.S.	1	1.00	0.96	1.11	0.55	1.69	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.544	0.091	0.660	0.453	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	542	474	547	368	0	2469	-1
N.S.	1	1.00	1.32	1.16	1.33	0.90	0.00	6.02	-0.00
time (sec)	N/A	0.755	1.989	0.066	0.557	0.422	0.000	0.868	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	390	386	497	299	0	1541	-1
N.S.	1	1.00	1.10	1.09	1.40	0.84	0.00	4.35	-0.00
time (sec)	N/A	0.639	1.030	0.060	0.582	0.409	0.000	0.723	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	254	294	424	245	0	844	-1
N.S.	1	1.00	0.84	0.97	1.39	0.81	0.00	2.78	-0.00
time (sec)	N/A	0.313	0.282	0.059	0.561	0.425	0.000	0.567	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	236	212	377	213	0	330	-1
N.S.	1	1.00	0.92	0.82	1.47	0.83	0.00	1.28	-0.00
time (sec)	N/A	0.260	0.234	0.061	0.524	0.391	0.000	0.465	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	299	286	253	265	0	0	-1
N.S.	1	1.00	1.10	1.06	0.93	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.341	1.008	0.061	0.705	0.385	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	268	368	253	367	0	0	-1
N.S.	1	1.00	0.92	1.26	0.87	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.487	1.313	0.060	0.703	0.449	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	465	450	254	528	0	0	-1
N.S.	1	1.00	1.31	1.26	0.71	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.518	4.470	0.058	0.712	0.443	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	55	34	74	35	83	69	-1
N.S.	1	1.00	1.12	0.69	1.51	0.71	1.69	1.41	-0.02
time (sec)	N/A	0.037	0.009	0.042	0.523	0.378	3.167	0.443	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	48	27	67	26	61	53	26
N.S.	1	1.00	1.33	0.75	1.86	0.72	1.69	1.47	0.72
time (sec)	N/A	0.023	0.005	0.031	0.516	0.395	0.511	0.564	0.028

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	51	19	60	18	37	35	18
N.S.	1	1.00	2.12	0.79	2.50	0.75	1.54	1.46	0.75
time (sec)	N/A	0.014	0.007	0.030	0.521	0.385	0.463	0.409	0.033

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	63	28	21	31	61	0	-1
N.S.	1	1.00	1.80	0.80	0.60	0.89	1.74	0.00	-0.03
time (sec)	N/A	0.023	0.029	0.035	0.563	0.407	1.005	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	122	0	235	132	0	0	-1
N.S.	1	1.00	0.67	0.00	1.28	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.073	0.038	0.361	0.126	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	124	0	186	102	0	0	-1
N.S.	1	1.00	0.82	0.00	1.22	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.074	0.037	0.358	0.102	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	124	0	186	102	0	0	-1
N.S.	1	1.00	0.82	0.00	1.22	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.065	0.036	0.351	0.109	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	0	137	86	0	0	-1
N.S.	1	1.00	0.92	0.00	1.01	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.039	0.036	0.357	0.118	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	0	138	86	0	0	-1
N.S.	1	1.00	0.92	0.00	1.02	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.040	0.036	0.340	0.095	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	121	0	138	117	0	0	-1
N.S.	1	1.00	0.80	0.00	0.91	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.034	0.036	0.369	0.111	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	121	0	138	117	0	0	-1
N.S.	1	1.00	0.79	0.00	0.90	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.036	0.036	0.366	0.131	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	125	0	137	185	0	0	-1
N.S.	1	1.00	0.69	0.00	0.75	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.035	0.036	0.368	0.104	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.012	107.445	0.021	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	133	0	57	0	0	15
N.S.	1	1.00	1.00	8.31	0.00	3.56	0.00	0.00	0.94
time (sec)	N/A	0.006	0.013	0.046	0.000	0.100	0.000	0.000	0.183

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.018	0.380	0.019	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	1.197	0.022	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	179	0	68	0	0	35
N.S.	1	1.00	0.86	4.26	0.00	1.62	0.00	0.00	0.83
time (sec)	N/A	0.013	0.023	0.000	0.000	0.119	0.000	0.000	0.188

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.019	9.493	0.020	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.193	0.161	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	3.119	0.018	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.012	0.203	0.002	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	0	51	0	0	15
N.S.	1	1.00	1.00	1.12	0.00	3.19	0.00	0.00	0.94
time (sec)	N/A	0.007	0.013	0.000	0.000	0.087	0.000	0.000	0.200

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.020	0.138	0.019	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	1.610	0.019	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	182	0	93	0	0	42
N.S.	1	1.00	1.00	4.79	0.00	2.45	0.00	0.00	1.11
time (sec)	N/A	0.013	0.036	0.000	0.000	0.121	0.000	0.000	0.437

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.020	8.428	0.021	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	0	0	0	0	0	51
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	1.34
time (sec)	N/A	0.040	0.215	0.143	0.000	0.000	0.000	0.000	0.657

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	15
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.75
time (sec)	N/A	0.030	0.063	0.204	0.000	0.000	0.000	0.000	0.332

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	15	0	0	16
N.S.	1	1.00	0.71	0.00	0.00	0.62	0.00	0.00	0.67
time (sec)	N/A	0.032	0.043	0.211	0.000	0.352	0.000	0.000	0.139

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0	31
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.66
time (sec)	N/A	0.044	0.086	0.308	0.000	0.000	0.000	0.000	0.527

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.060	0.091	0.205	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	0.053	0.110	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	0.085	0.114	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.078	0.135	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.058	0.110	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	0.501	0.032	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	253	0	0	188	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.214	0.118	0.132	0.000	0.113	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	150	0	0	136	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.132	0.092	0.000	0.117	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	122	0	0	96	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.030	0.034	0.000	0.097	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.013	5.271	0.053	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.526	0.024	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	455	0	54	0	0	-1
N.S.	1	1.00	1.00	6.07	0.00	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.014	0.086	0.000	0.104	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	354	0	54	0	0	-1
N.S.	1	1.00	1.00	4.48	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.012	0.075	0.000	0.107	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	291	0	54	0	0	-1
N.S.	1	1.00	1.00	3.88	0.00	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.011	0.080	0.000	0.094	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	379	0	50	0	0	-1
N.S.	1	1.00	1.00	4.80	0.00	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.010	0.082	0.000	0.099	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	427	0	50	0	0	-1
N.S.	1	1.00	0.95	6.57	0.00	0.77	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.016	0.081	0.000	0.110	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	530	0	54	0	0	-1
N.S.	1	1.00	1.00	7.07	0.00	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.012	0.083	0.000	0.116	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	600	0	54	0	0	-1
N.S.	1	1.00	1.00	8.00	0.00	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.012	0.089	0.000	0.092	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	0	0	77	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.072	0.061	0.000	0.132	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	96	0	0	77	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.071	0.057	0.000	0.115	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	0	0	77	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.067	0.078	0.000	0.111	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	90	0	0	69	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.078	0.063	0.000	0.102	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	64	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.040	0.076	0.000	0.100	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	0	0	77	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.071	0.061	0.000	0.115	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	0	0	77	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.063	0.062	0.000	0.108	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	122	476	492	170	264	156	189
N.S.	1	1.00	1.37	5.35	5.53	1.91	2.97	1.75	2.12
time (sec)	N/A	0.077	0.310	0.112	0.339	0.401	0.251	0.416	0.258

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	80	236	256	104	151	94	112
N.S.	1	1.00	1.19	3.52	3.82	1.55	2.25	1.40	1.67
time (sec)	N/A	0.054	0.226	0.077	0.318	0.402	0.143	0.454	0.287

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	52	89	101	53	68	46	52
N.S.	1	1.00	1.18	2.02	2.30	1.20	1.55	1.05	1.18
time (sec)	N/A	0.027	0.139	0.054	0.305	0.382	0.101	0.420	0.092

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	102	182	95	0	692	-1
N.S.	1	1.00	0.85	1.57	2.80	1.46	0.00	10.65	-0.02
time (sec)	N/A	0.103	0.080	0.082	0.374	0.386	0.000	0.419	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	143	208	137	0	578	-1
N.S.	1	1.00	0.88	1.61	2.34	1.54	0.00	6.49	-0.01
time (sec)	N/A	0.114	0.219	0.118	0.415	0.396	0.000	0.434	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	217	1129	1021	373	779	339	452
N.S.	1	1.00	0.92	4.76	4.31	1.57	3.29	1.43	1.91
time (sec)	N/A	0.172	0.882	0.197	0.347	0.387	0.436	0.466	0.912

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	193	564	535	216	456	207	255
N.S.	1	1.00	1.15	3.36	3.18	1.29	2.71	1.23	1.52
time (sec)	N/A	0.124	0.407	0.128	0.317	0.403	0.297	0.390	0.592

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	218	216	102	219	107	117
N.S.	1	1.00	0.68	1.85	1.83	0.86	1.86	0.91	0.99
time (sec)	N/A	0.064	0.305	0.089	0.284	0.391	0.150	0.451	0.203

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	114	197	361	191	0	6933	-1
N.S.	1	1.00	0.79	1.36	2.49	1.32	0.00	47.81	-0.01
time (sec)	N/A	0.231	0.133	0.105	0.371	0.386	0.000	0.576	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	206	276	396	289	0	1133	-1
N.S.	1	1.00	1.30	1.74	2.49	1.82	0.00	7.13	-0.01
time (sec)	N/A	0.206	0.330	0.153	0.409	0.424	0.000	0.610	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	151	364	1006	442	0	0	-1
N.S.	1	1.00	1.13	2.72	7.51	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.220	0.142	0.397	0.387	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	125	197	300	240	0	0	-1
N.S.	1	1.00	1.24	1.95	2.97	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.225	0.094	0.408	0.388	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	60	176	62	70	234	65
N.S.	1	1.00	1.43	1.22	3.59	1.27	1.43	4.78	1.33
time (sec)	N/A	0.044	0.057	0.078	0.333	0.383	0.313	0.474	0.661

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	1.755	0.049	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	1.749	0.049	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	250	678	3441	800	0	0	-1
N.S.	1	1.00	0.92	2.50	12.70	2.95	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.687	0.493	0.920	0.407	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	212	358	812	411	0	0	-1
N.S.	1	1.00	1.00	1.69	3.83	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.171	0.717	0.403	0.592	0.409	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	113	109	834	126	146	757	175
N.S.	1	1.00	0.92	0.89	6.78	1.02	1.19	6.15	1.42
time (sec)	N/A	0.063	0.289	0.217	0.315	0.381	0.483	0.606	4.287

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	7.646	0.124	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	8.039	0.127	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	198	468	1042	493	0	0	-1
N.S.	1	1.00	1.49	3.52	7.83	3.71	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.771	0.143	0.388	0.411	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	292	247	330	307	0	0	-1
N.S.	1	1.00	2.86	2.42	3.24	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.135	3.598	0.107	0.371	0.419	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	57	72	176	63	90	229	65
N.S.	1	1.00	1.14	1.44	3.52	1.26	1.80	4.58	1.30
time (sec)	N/A	0.043	0.148	0.088	0.281	0.400	0.393	0.504	0.522

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	1.506	0.050	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	1.499	0.049	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	53	132	206	0	0	98	83
N.S.	1	1.00	0.48	1.20	1.87	0.00	0.00	0.89	0.75
time (sec)	N/A	0.092	0.148	0.080	0.594	0.000	0.000	0.432	0.546

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	44	105	122	0	0	77	63
N.S.	1	1.00	0.50	1.19	1.39	0.00	0.00	0.88	0.72
time (sec)	N/A	0.074	0.109	0.057	0.546	0.000	0.000	0.448	0.431

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	34	80	61	0	0	57	46
N.S.	1	1.00	0.64	1.51	1.15	0.00	0.00	1.08	0.87
time (sec)	N/A	0.039	0.080	0.053	0.541	0.000	0.000	0.490	0.205

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	20	32	0	30	33
N.S.	1	1.00	1.12	1.65	0.77	1.23	0.00	1.15	1.27
time (sec)	N/A	0.009	0.024	0.073	0.553	0.354	0.000	0.433	0.332

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	55	0	61	0	0	166	-1
N.S.	1	1.00	0.65	0.00	0.73	0.00	0.00	1.98	-0.01
time (sec)	N/A	0.081	0.058	0.040	0.552	0.000	0.000	0.458	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	75	0	193	0	0	560	-1
N.S.	1	1.00	0.68	0.00	1.75	0.00	0.00	5.09	-0.01
time (sec)	N/A	0.091	0.101	0.027	0.554	0.000	0.000	0.539	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	98	0	227	0	0	662	-1
N.S.	1	1.00	0.65	0.00	1.50	0.00	0.00	4.38	-0.01
time (sec)	N/A	0.108	0.171	0.029	0.544	0.000	0.000	0.537	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	33	87	48	0	0	55	91
N.S.	1	1.00	0.49	1.28	0.71	0.00	0.00	0.81	1.34
time (sec)	N/A	0.073	0.039	0.072	0.505	0.000	0.000	0.397	0.426

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	29	70	36	0	0	43	70
N.S.	1	1.00	0.55	1.32	0.68	0.00	0.00	0.81	1.32
time (sec)	N/A	0.061	0.033	0.058	0.517	0.000	0.000	0.426	0.340

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	55	24	0	0	31	50
N.S.	1	1.00	0.69	1.72	0.75	0.00	0.00	0.97	1.56
time (sec)	N/A	0.034	0.016	0.057	0.514	0.000	0.000	0.425	0.307

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	25	12	18	0	17	34
N.S.	1	1.00	1.20	1.67	0.80	1.20	0.00	1.13	2.27
time (sec)	N/A	0.008	0.008	0.073	0.529	0.347	0.000	0.443	0.288

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	17	0	0	16	-1
N.S.	1	1.00	1.00	0.00	0.74	0.00	0.00	0.70	-0.04
time (sec)	N/A	0.058	0.006	0.037	0.545	0.000	0.000	0.434	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	33	0	23	0	0	34	-1
N.S.	1	1.00	0.79	0.00	0.55	0.00	0.00	0.81	-0.02
time (sec)	N/A	0.061	0.040	0.033	0.522	0.000	0.000	0.435	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	0	19	0	0	48	-1
N.S.	1	1.00	0.66	0.00	0.28	0.00	0.00	0.72	-0.01
time (sec)	N/A	0.073	0.053	0.033	0.544	0.000	0.000	0.448	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	34	86	129	0	0	55	92
N.S.	1	1.00	0.47	1.19	1.79	0.00	0.00	0.76	1.28
time (sec)	N/A	0.078	0.037	0.060	0.528	0.000	0.000	0.417	0.434

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	30	69	100	0	0	51	71
N.S.	1	1.00	0.54	1.23	1.79	0.00	0.00	0.91	1.27
time (sec)	N/A	0.066	0.031	0.052	0.523	0.000	0.000	0.443	0.343

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	23	54	72	0	0	31	48
N.S.	1	1.00	0.68	1.59	2.12	0.00	0.00	0.91	1.41
time (sec)	N/A	0.036	0.018	0.049	0.503	0.000	0.000	0.458	0.316

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	19	25	23	19	0	26	34
N.S.	1	1.00	1.19	1.56	1.44	1.19	0.00	1.62	2.12
time (sec)	N/A	0.008	0.007	0.077	0.506	0.366	0.000	0.420	0.292

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	0	16	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.67	-0.04
time (sec)	N/A	0.061	0.016	0.030	0.000	0.000	0.000	0.427	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	0	0	0	0	34	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.77	-0.02
time (sec)	N/A	0.068	0.019	0.025	0.000	0.000	0.000	0.601	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	45	0	0	0	0	48	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.69	-0.01
time (sec)	N/A	0.078	0.040	0.025	0.000	0.000	0.000	0.479	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	67	0	98	0	0	113	-1
N.S.	1	1.00	0.36	0.00	0.53	0.00	0.00	0.61	-0.01
time (sec)	N/A	0.130	0.199	0.033	0.501	0.000	0.000	0.427	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	54	0	72	0	0	85	-1
N.S.	1	1.00	0.37	0.00	0.50	0.00	0.00	0.59	-0.01
time (sec)	N/A	0.100	0.154	0.032	0.505	0.000	0.000	0.416	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	45	0	48	0	0	59	-1
N.S.	1	1.00	0.51	0.00	0.54	0.00	0.00	0.66	-0.01
time (sec)	N/A	0.053	0.049	0.033	0.519	0.000	0.000	0.437	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	0	29	0	0	32	-1
N.S.	1	1.00	0.65	0.00	0.53	0.00	0.00	0.58	-0.02
time (sec)	N/A	0.090	0.014	0.033	0.542	0.000	0.000	0.440	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	53	0	37	0	0	62	-1
N.S.	1	1.00	0.67	0.00	0.47	0.00	0.00	0.78	-0.01
time (sec)	N/A	0.088	0.059	0.032	0.595	0.000	0.000	0.478	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	66	0	33	0	0	92	-1
N.S.	1	1.00	0.61	0.00	0.30	0.00	0.00	0.84	-0.01
time (sec)	N/A	0.119	0.041	0.032	0.532	0.000	0.000	0.431	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	199	0	0	0	0	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.086	0.030	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	146	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.120	0.050	0.031	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	89	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.032	0.030	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	54	90	126	0	93	45
N.S.	1	1.00	0.87	1.17	1.96	2.74	0.00	2.02	0.98
time (sec)	N/A	0.017	0.012	0.114	0.547	0.382	0.000	0.464	0.332

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.047	0.589	0.031	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	170	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.121	0.071	0.029	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	117	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.036	0.030	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.025	0.030	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	25	81	87	0	20	-1
N.S.	1	1.00	0.97	0.68	2.19	2.35	0.00	0.54	-0.03
time (sec)	N/A	0.015	0.013	0.076	0.564	0.376	0.000	0.458	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.051	2.031	0.030	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	257	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.290	0.033	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	185	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.081	0.033	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.125	0.031	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.052	6.787	0.033	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.040	0.984	0.026	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	290	0	0	1030	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	2.69	0.00	0.00	-0.00
time (sec)	N/A	0.370	0.557	0.027	0.000	0.481	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	379	0	0	1263	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	3.84	0.00	0.00	-0.00
time (sec)	N/A	0.449	0.462	0.043	0.000	0.532	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	756	414	0	915	0	0	-1
N.S.	1	1.00	3.53	1.93	0.00	4.28	0.00	0.00	-0.00
time (sec)	N/A	0.263	0.564	0.067	0.000	0.503	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.032	0.516	0.023	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	933	674	0	1491	0	0	-1
N.S.	1	1.00	3.15	2.28	0.00	5.04	0.00	0.00	-0.00
time (sec)	N/A	0.354	8.418	0.904	0.000	0.574	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [81] had the largest ratio of [28]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.00	14	0.143
2	A	4	2	1.00	14	0.143
3	A	3	2	1.00	14	0.143
4	A	2	2	1.00	12	0.167
5	A	3	3	1.00	14	0.214
6	A	4	4	1.00	14	0.286
7	A	5	4	1.00	14	0.286
8	A	6	4	1.00	14	0.286
9	A	6	4	1.00	16	0.250
10	A	4	3	1.00	16	0.188
11	A	4	4	1.00	16	0.250
12	A	2	1	1.00	14	0.071
13	A	5	4	1.00	16	0.250
14	A	5	5	1.00	16	0.312
15	A	7	6	1.00	16	0.375
16	A	12	4	1.00	16	0.250
17	A	8	4	1.00	16	0.250
18	A	6	4	1.00	16	0.250
19	A	3	3	1.00	14	0.214
20	A	8	4	1.00	16	0.250
21	A	8	4	1.00	16	0.250
22	A	12	5	1.00	16	0.312
23	A	8	3	1.00	12	0.250
24	A	8	4	1.00	12	0.333
25	A	3	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	8	4	1.00	12	0.333
27	A	8	4	1.00	12	0.333
28	A	14	5	1.00	12	0.417
29	A	9	5	1.00	14	0.357
30	A	7	4	1.00	14	0.286
31	A	5	3	1.00	12	0.250
32	A	0	0	0.00	0	0.000
33	A	6	6	1.00	16	0.375
34	A	5	5	1.00	16	0.312
35	A	2	2	1.00	14	0.143
36	A	0	0	0.00	0	0.000
37	A	15	8	1.00	16	0.500
38	A	9	6	1.00	16	0.375
39	A	6	4	1.00	14	0.286
40	A	0	0	0.00	0	0.000
41	A	8	6	1.00	16	0.375
42	A	7	6	1.00	16	0.375
43	A	6	6	1.00	16	0.375
44	A	5	5	1.00	16	0.312
45	A	6	6	1.00	16	0.375
46	A	7	6	1.00	16	0.375
47	A	8	6	1.00	16	0.375
48	A	10	9	1.00	18	0.500
49	A	9	8	1.00	18	0.444
50	A	8	7	1.00	18	0.389
51	A	7	6	1.00	18	0.333
52	A	7	7	1.00	18	0.389
53	A	9	8	1.00	18	0.444
54	A	9	9	1.00	18	0.500
55	A	11	8	1.00	18	0.444
56	A	23	8	1.00	18	0.444
57	A	20	8	1.00	18	0.444
58	A	14	7	1.00	18	0.389
59	A	12	6	1.00	18	0.333
60	A	12	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	18	7	1.00	18	0.389
62	A	19	8	1.00	18	0.444
63	A	4	3	1.00	8	0.375
64	A	3	3	1.00	8	0.375
65	A	2	2	1.00	8	0.250
66	A	3	3	1.00	8	0.375
67	A	5	3	1.00	16	0.188
68	A	4	3	1.00	16	0.188
69	A	4	3	1.00	16	0.188
70	A	3	2	1.00	16	0.125
71	A	3	2	1.00	16	0.125
72	A	4	3	1.00	16	0.188
73	A	4	3	1.00	16	0.188
74	A	5	3	1.00	16	0.188
75	A	0	0	0.00	0	0.000
76	A	1	1	1.00	10	0.100
77	A	0	0	0.00	0	0.000
78	A	0	0	0.00	0	0.000
79	A	2	2	1.00	10	0.200
80	A	0	0	0.00	0	0.000
81	A	2	1	1.00	28	0.036
82	A	0	0	0.00	0	0.000
83	A	0	0	0.00	0	0.000
84	A	1	1	1.00	10	0.100
85	A	0	0	0.00	0	0.000
86	A	0	0	0.00	0	0.000
87	A	2	2	1.00	10	0.200
88	A	0	0	0.00	0	0.000
89	A	2	1	1.00	25	0.040
90	A	2	1	1.00	17	0.059
91	A	2	1	1.00	20	0.050
92	A	3	1	1.00	20	0.050
93	A	3	2	1.00	21	0.095
94	A	4	2	1.00	20	0.100
95	A	4	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	2	1.00	20	0.100
97	A	7	5	1.00	24	0.208
98	A	0	0	0.00	0	0.000
99	A	8	3	1.00	16	0.188
100	A	5	3	1.00	16	0.188
101	A	3	2	1.00	14	0.143
102	A	0	0	0.00	0	0.000
103	A	0	0	0.00	0	0.000
104	A	3	2	1.00	12	0.167
105	A	3	2	1.00	12	0.167
106	A	3	2	1.00	12	0.167
107	A	3	2	1.00	10	0.200
108	A	3	2	1.00	12	0.167
109	A	3	2	1.00	12	0.167
110	A	3	2	1.00	12	0.167
111	A	5	3	1.00	14	0.214
112	A	5	3	1.00	14	0.214
113	A	5	3	1.00	14	0.214
114	A	5	3	1.00	12	0.250
115	A	5	3	1.00	14	0.214
116	A	5	3	1.00	14	0.214
117	A	5	3	1.00	14	0.214
118	A	6	3	1.00	18	0.167
119	A	5	3	1.00	18	0.167
120	A	4	3	1.00	16	0.188
121	A	5	4	1.00	18	0.222
122	A	6	5	1.00	18	0.278
123	A	10	6	1.00	20	0.300
124	A	9	7	1.00	20	0.350
125	A	6	4	1.00	18	0.222
126	A	9	5	1.00	20	0.250
127	A	9	5	1.00	20	0.250
128	A	7	7	1.00	20	0.350
129	A	6	6	1.00	20	0.300
130	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	0	0	0.00	0	0.000
132	A	0	0	0.00	0	0.000
133	A	10	9	1.00	20	0.450
134	A	9	9	1.00	20	0.450
135	A	4	4	1.00	18	0.222
136	A	0	0	0.00	0	0.000
137	A	0	0	0.00	0	0.000
138	A	7	7	1.00	21	0.333
139	A	6	6	1.00	21	0.286
140	A	3	3	1.00	19	0.158
141	A	0	0	0.00	0	0.000
142	A	0	0	0.00	0	0.000
143	A	5	3	1.00	18	0.167
144	A	4	3	1.00	18	0.167
145	A	3	3	1.00	16	0.188
146	A	1	1	1.00	14	0.071
147	A	4	4	1.00	18	0.222
148	A	5	5	1.00	18	0.278
149	A	6	5	1.00	18	0.278
150	A	5	3	1.00	14	0.214
151	A	4	3	1.00	14	0.214
152	A	3	3	1.00	12	0.250
153	A	1	1	1.00	10	0.100
154	A	2	2	1.00	14	0.143
155	A	3	3	1.00	14	0.214
156	A	4	3	1.00	14	0.214
157	A	5	3	1.00	15	0.200
158	A	4	3	1.00	15	0.200
159	A	3	3	1.00	13	0.231
160	A	1	1	1.00	11	0.091
161	A	2	2	1.00	15	0.133
162	A	3	3	1.00	15	0.200
163	A	4	3	1.00	15	0.200
164	A	9	5	1.00	14	0.357
165	A	7	5	1.00	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	4	1.00	12	0.333
167	A	5	3	1.00	14	0.214
168	A	5	3	1.00	14	0.214
169	A	7	4	1.00	14	0.286
170	A	10	6	1.00	18	0.333
171	A	8	5	1.00	18	0.278
172	A	6	4	1.00	16	0.250
173	A	2	2	1.00	14	0.143
174	A	0	0	0.00	0	0.000
175	A	10	6	1.00	15	0.400
176	A	8	5	1.00	15	0.333
177	A	6	4	1.00	13	0.308
178	A	2	2	1.00	11	0.182
179	A	0	0	0.00	0	0.000
180	A	16	9	1.00	14	0.643
181	A	10	7	1.00	14	0.500
182	A	7	5	1.00	12	0.417
183	A	0	0	0.00	0	0.000
184	A	0	0	0.00	0	0.000
185	A	12	7	1.00	12	0.583
186	A	10	6	1.00	16	0.375
187	A	8	5	1.00	14	0.357
188	A	0	0	0.00	0	0.000
189	A	11	8	1.00	18	0.444

Chapter 3

Listing of integrals

Local contents

3.1 $\int (c+dx)^4 \cos(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{24d^3(c+dx)\cos(a+bx)}{b^4}+\frac{4d(c+dx)^3\cos(a+bx)}{b^2}+\frac{24d^4\sin(a+bx)}{b^5}-\frac{12d^2(c+dx)^2\sin(a+bx)}{b^3}+\frac{(c+d}{}$$

```
[Out] -24*d^3*(d*x+c)*cos(b*x+a)/b^4+4*d*(d*x+c)^3*cos(b*x+a)/b^2+24*d^4*sin(b*x+
a)/b^5-12*d^2*(d*x+c)^2*sin(b*x+a)/b^3+(d*x+c)^4*sin(b*x+a)/b
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2717}

$$\frac{24d^4\sin(a+bx)}{b^5}-\frac{24d^3(c+dx)\cos(a+bx)}{b^4}-\frac{12d^2(c+dx)^2\sin(a+bx)}{b^3}+\frac{4d(c+dx)^3\cos(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x],x]
```

```
[Out] (-24*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 +
 (24*d^4*Sin[a + b*x])/b^5 - (12*d^2*(c + d*x)^2*Sin[a + b*x])/b^3 + ((c +
d*x)^4*Sin[a + b*x])/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4\cos(a+bx)\,dx &= \frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(4d)\int (c+dx)^3\sin(a+bx)\,dx}{b}\\
&= \frac{4d(c+dx)^3\cos(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(12d^2)\int (c+dx)^2\cos(a+}{b^2}\\
&= \frac{4d(c+dx)^3\cos(a+bx)}{b^2}-\frac{12d^2(c+dx)^2\sin(a+bx)}{b^3}+\frac{(c+dx)^4\sin(a+bx)}{b}\\
&= -\frac{24d^3(c+dx)\cos(a+bx)}{b^4}+\frac{4d(c+dx)^3\cos(a+bx)}{b^2}-\frac{12d^2(c+dx)^2\sin(a-}{b^3}\\
&= -\frac{24d^3(c+dx)\cos(a+bx)}{b^4}+\frac{4d(c+dx)^3\cos(a+bx)}{b^2}+\frac{24d^4\sin(a+bx)}{b^5}-1
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 76, normalized size = 0.84

$$\frac{4bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)+\left(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4\right)\sin(a+bx)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x],x]
```

```
[Out] (4*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (24*d^4 - 12*b^2
*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Sin[a + b*x])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(91) = 182.
time = 0.14, size = 539, normalized size = 5.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^4*a^4*d^4*sin(b*x+a)-4/b^3*a^3*c*d^3*sin(b*x+a)-4/b^4*a^3*d^4*(cos
(b*x+a)+(b*x+a)*sin(b*x+a))+6/b^2*a^2*c^2*d^2*sin(b*x+a)+12/b^3*a^2*c*d^3*(
cos(b*x+a)+(b*x+a)*sin(b*x+a))+6/b^4*a^2*d^4*((b*x+a)^2*sin(b*x+a)-2*sin(b*
x+a)+2*(b*x+a)*cos(b*x+a))-4/b*a*c^3*d*sin(b*x+a)-12/b^2*a*c^2*d^2*(cos(b*x
+a)+(b*x+a)*sin(b*x+a))-12/b^3*a*c*d^3*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2
*(b*x+a)*cos(b*x+a))-4/b^4*a*d^4*((b*x+a)^3*sin(b*x+a)+3*(b*x+a)^2*cos(b*x+
a)-6*cos(b*x+a)-6*(b*x+a)*sin(b*x+a))+c^4*sin(b*x+a)+4/b*c^3*d*(cos(b*x+a)+
(b*x+a)*sin(b*x+a))+6/b^2*c^2*d^2*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x
+a)*cos(b*x+a))+4/b^3*c*d^3*((b*x+a)^3*sin(b*x+a)+3*(b*x+a)^2*cos(b*x+a)-6*
cos(b*x+a)-6*(b*x+a)*sin(b*x+a))+1/b^4*d^4*((b*x+a)^4*sin(b*x+a)+4*(b*x+a)^
3*cos(b*x+a)-12*(b*x+a)^2*sin(b*x+a)+24*sin(b*x+a)-24*(b*x+a)*cos(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(91) = 182.
time = 0.34, size = 481, normalized size = 5.29

$$\frac{c^4\sin(bx+a)-\frac{4ac^3d\sin(bx+a)}{b}+\frac{6a^2c^2d^2\sin(bx+a)}{b^2}-\frac{4a^3cd^3\sin(bx+a)}{b^3}+\frac{a^4d^4\sin(bx+a)}{b^4}+\frac{4((bx+a)\sin(bx+a)+\cos(bx+a))c^3d}{b}-\frac{12((bx+a)\sin(bx+a)+\cos(bx+a))ac^2d^2}{b^2}+\frac{12((bx+a)\sin(bx+a)+\cos(bx+a))a^2cd^3}{b^3}-\frac{4((bx+a)\sin(bx+a)+\cos(bx+a))a^3d^4}{b^4}+\frac{6\left(2(bx+a)\cos(bx+a)+\left((bx+a)^2-2\right)\sin(bx+a)\right)c^2d^2}{b^2}-\frac{12\left(2(bx+a)\cos(bx+a)+\left((bx+a)^2-2\right)\sin(bx+a)\right)acd^3}{b^3}+\frac{6\left(2(bx+a)\cos(bx+a)+\left((bx+a)^2-2\right)\sin(bx+a)\right)a^2d^4}{b^4}+\frac{4\left(3\left((bx+a)^2-2\right)\cos(bx+a)+\left((bx+a)^3-6bx-6a\right)\sin(bx+a)\right)cd^3}{b^3}-\frac{4\left(3\left((bx+a)^2-2\right)\cos(bx+a)+\left((bx+a)^3-6bx-6a\right)\sin(bx+a)\right)ad^4}{b^4}+\frac{\left(4\left((bx+a)^3-6bx-6a\right)\cos(bx+a)+\left((bx+a)^4-12(bx+a)^2+24\right)\sin(bx+a)\right)d^4}{b^4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^4*sin(b*x + a) - 4*a*c^3*d*sin(b*x + a)/b + 6*a^2*c^2*d^2*sin(b*x + a)/b
^2 - 4*a^3*c*d^3*sin(b*x + a)/b^3 + a^4*d^4*sin(b*x + a)/b^4 + 4*((b*x + a)
*sin(b*x + a) + cos(b*x + a))*c^3*d/b - 12*((b*x + a)*sin(b*x + a) + cos(b*
x + a))*a*c^2*d^2/b^2 + 12*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a^2*c*d^
3/b^3 - 4*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a^3*d^4/b^4 + 6*(2*(b*x +
 a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 12*(2*(b*x
```

```
 + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 + 6*(2*(b*
x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 + 4*(3*((
b*x + a)^2 - 2)*cos(b*x + a) + ((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*
d^3/b^3 - 4*(3*((b*x + a)^2 - 2)*cos(b*x + a) + ((b*x + a)^3 - 6*b*x - 6*a)
*sin(b*x + a))*a*d^4/b^4 + (4*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) + ((
b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [**A**]
time = 0.38, size = 169, normalized size = 1.86

$$\frac{4\,(b^3d^4x^3 + 3\,b^3cd^3x^2 + b^3c^3d - 6\,bcd^3 + 3\,(b^3c^2d^2 - 2\,bd^4)x)\cos{(bx + a)} + (b^4d^4x^4 + 4\,b^4cd^3x^3 + b^4c^4 - 12\,b^2c^2d^2 + 24\,d^4 + 6\,(b^4c^2d^2 - 2\,b^2d^4)x^2 + 4\,(b^4c^3d - 6\,b^2cd^3)x)\sin{(bx + a)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2
- 2*b*d^4)*x)*cos(b*x + a) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*
b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b
^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.
time = 0.36, size = 311, normalized size = 3.42

$$\begin{cases} \frac{c^4\sin{(a+bx)}}{b} + \frac{4c^3dx\sin{(a+bx)}}{b} + \frac{6c^2d^2x^2\sin{(a+bx)}}{b} + \frac{4cd^3x^3\sin{(a+bx)}}{b} + \frac{d^4x^4\sin{(a+bx)}}{b} + \frac{4c^3d\cos{(a+bx)}}{b^2} + \frac{12c^2d^2x\cos{(a+bx)}}{b^2} + \frac{12cd^3x^2\cos{(a+bx)}}{b^2} + \frac{4d^4x^3\cos{(a+bx)}}{b^2} - \frac{12c^2d^2\sin{(a+bx)}}{b^3} - \frac{24cd^3x\sin{(a+bx)}}{b^3} - \frac{12d^4x^2\sin{(a+bx)}}{b^3} - \frac{24cd^3\cos{(a+bx)}}{b^4} - \frac{24d^4x\cos{(a+bx)}}{b^4} + \frac{24d^4\sin{(a+bx)}}{b^5} & \text{for}\: b \neq 0 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\cos{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a),x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)/b + 4*c**3*d*x*sin(a + b*x)/b + 6*c**2*d**2*x*
*2*sin(a + b*x)/b + 4*c*d**3*x**3*sin(a + b*x)/b + d**4*x**4*sin(a + b*x)/b
 + 4*c**3*d*cos(a + b*x)/b**2 + 12*c**2*d**2*x*cos(a + b*x)/b**2 + 12*c*d**
3*x**2*cos(a + b*x)/b**2 + 4*d**4*x**3*cos(a + b*x)/b**2 - 12*c**2*d**2*sin
(a + b*x)/b**3 - 24*c*d**3*x*sin(a + b*x)/b**3 - 12*d**4*x**2*sin(a + b*x)/
b**3 - 24*c*d**3*cos(a + b*x)/b**4 - 24*d**4*x*cos(a + b*x)/b**4 + 24*d**4*
sin(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 +
 c*d**3*x**4 + d**4*x**5/5)*cos(a), True))
```

Giac [**A**]
time = 0.45, size = 170, normalized size = 1.87

$$\frac{4\,(b^3d^4x^3 + 3\,b^3cd^3x^2 + 3\,b^3c^2d^2x + b^3c^3d - 6\,bd^4x - 6\,bcd^3)\cos{(bx + a)}}{b^5} + \frac{(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^4c^3dx + b^4c^4 - 12\,b^2d^4x^2 - 24\,b^2cd^3x - 12\,b^2c^2d^2 + 24\,d^4)\sin{(bx + a)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x
- 6*b*c*d^3)*cos(b*x + a)/b^5 + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*
d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^
2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

Mupad [B]
time = 0.42, size = 219, normalized size = 2.41

$$\frac{\sin(a+bx)\,(b^4c^4-12b^2c^2d^2+24d^4)}{b^5}-\frac{4\cos(a+bx)\,(6cd^3-b^2c^3d)}{b^4}+\frac{4d^4x^3\cos(a+bx)}{b^2}-\frac{12x\cos(a+bx)\,(2d^4-b^2c^2d^2)}{b^4}+\frac{d^4x^4\sin(a+bx)}{b}-\frac{4x\sin(a+bx)\,(6cd^3-b^2c^3d)}{b^3}-\frac{6x^2\sin(a+bx)\,(2d^4-b^2c^2d^2)}{b^3}+\frac{12cd^3x^2\cos(a+bx)}{b^2}+\frac{4cd^3x^3\sin(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (sin(a + b*x)*(24*d^4 + b^4*c^4 - 12*b^2*c^2*d^2))/b^5 - (4*cos(a + b*x)*(6
*c*d^3 - b^2*c^3*d))/b^4 + (4*d^4*x^3*cos(a + b*x))/b^2 - (12*x*cos(a + b*x
)*(2*d^4 - b^2*c^2*d^2))/b^4 + (d^4*x^4*sin(a + b*x))/b - (4*x*sin(a + b*x)
*(6*c*d^3 - b^2*c^3*d))/b^3 - (6*x^2*sin(a + b*x)*(2*d^4 - b^2*c^2*d^2))/b^
3 + (12*c*d^3*x^2*cos(a + b*x))/b^2 + (4*c*d^3*x^3*sin(a + b*x))/b
```

3.2 $\int (c+dx)^3 \cos(a+bx)\, dx$

Optimal. Leaf size=70

$$-\frac{6d^3\cos(a+bx)}{b^4}+\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{(c+dx)^3\sin(a+bx)}{b}$$

```
[Out] -6*d^3*cos(b*x+a)/b^4+3*d*(d*x+c)^2*cos(b*x+a)/b^2-6*d^2*(d*x+c)*sin(b*x+a)
/b^3+(d*x+c)^3*sin(b*x+a)/b
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2718}

$$-\frac{6d^3\cos(a+bx)}{b^4}-\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{b^2}+\frac{(c+dx)^3\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x],x]
```

```
[Out] (-6*d^3*Cos[a + b*x])/b^4 + (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - (6*d^2*(c
+ d*x)*Sin[a + b*x])/b^3 + ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\cos(a+bx)\,dx &= \frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\sin(a+bx)\,dx}{b}\\
&= \frac{3d(c+dx)^2\cos(a+bx)}{b^2}+\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(6d^2)\int (c+dx)\cos(a+bx}{b^2}\\
&= \frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{(c+dx)^3\sin(a+bx)}{b}+\\
&= -\frac{6d^3\cos(a+bx)}{b^4}+\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{(c-}{}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 61, normalized size = 0.87

$$\frac{3d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\sin(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x],x]
```

```
[Out] (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + b*(c + d*x)*(-6*d^2 + b^2*(c
 + d*x)^2)*Sin[a + b*x])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(70) = 140$.
time = 0.09, size = 302, normalized size = 4.31

method	result
risch	$\frac{3d\left(d^2x^2b^2+2b^2cdx+b^2c^2-2d^2\right)\cos(bx+a)}{b^4}+\frac{\left(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2\right)\sin(bx+a)}{b^3}$
norman	$\frac{\frac{\left(-6b^2c^2d+12d^3\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^4}+\frac{3d^3x^2}{b^2}+\frac{6c\,d^2x}{b^2}+\frac{2c\left(b^2c^2-6d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{2d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{3d^3x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\cdots}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
derivativedivides	$-\frac{a^3d^3\sin(bx+a)}{b^3}+\frac{3a^2c\,d^2\sin(bx+a)}{b^2}+\frac{3a^2d^3(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^3}-\frac{3a\,c^2d\sin(bx+a)}{b}-\frac{6ac\,d^2(\cos(bx+a)+(bx+a)\sin(bx+\cdots}{b^2}$
default	$-\frac{a^3d^3\sin(bx+a)}{b^3}+\frac{3a^2c\,d^2\sin(bx+a)}{b^2}+\frac{3a^2d^3(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^3}-\frac{3a\,c^2d\sin(bx+a)}{b}-\frac{6ac\,d^2(\cos(bx+a)+(bx+a)\sin(bx+\cdots}{b^2}$
meijerg	$\frac{8d^3\cos(a)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2b^2}{2}+3\right)\cos(bx)}{4\sqrt{\pi}}-\frac{xb\left(-\frac{x^2b^2}{2}+3\right)\sin(bx)}{4\sqrt{\pi}}\right)}{b^4}-\frac{8d^3\sin(a)\sqrt{\pi}\left(\frac{xb\left(-\frac{5x^2b^2}{2}+15\right)\cos(bx)}{20\sqrt{\pi}}\cdots\right.}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^3*a^3*d^3*sin(b*x+a)+3/b^2*a^2*c*d^2*sin(b*x+a)+3/b^3*a^2*d^3*(co
s(b*x+a)+(b*x+a)*sin(b*x+a))-3/b*a*c^2*d*sin(b*x+a)-6/b^2*a*c*d^2*(cos(b*x+
a)+(b*x+a)*sin(b*x+a))-3/b^3*a*d^3*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*
x+a)*cos(b*x+a))+c^3*sin(b*x+a)+3/b*c^2*d*(cos(b*x+a)+(b*x+a)*sin(b*x+a))+3
/b^2*c*d^2*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x+a)*cos(b*x+a))+1/b^3*d
^3*((b*x+a)^3*sin(b*x+a)+3*(b*x+a)^2*cos(b*x+a)-6*cos(b*x+a)-6*(b*x+a)*sin(
b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(70) = 140$.
time = 0.33, size = 278, normalized size = 3.97

$$\frac{c^3\sin(bx+a)-\frac{3ac^2d\sin(bx+a)}{b}+\frac{3a^2cd^2\sin(bx+a)}{b^2}-\frac{a^3d^3\sin(bx+a)}{b^3}+\frac{3((bx+a)\sin(bx+a)+\cos(bx+a))c^2d}{b}-\frac{6((bx+a)\sin(bx+a)+\cos(bx+a))acd^2}{b^2}+\frac{3((bx+a)\sin(bx+a)+\cos(bx+a))a^2d^3}{b^3}+\frac{3\left(2(bx+a)\cos(bx+a)+\left((bx+a)^2-2\right)\sin(bx+a)\right)cd^2}{b^2}-\frac{3\left(2(bx+a)\cos(bx+a)+\left((bx+a)^2-2\right)\sin(bx+a)\right)ad^3}{b^3}+\frac{\left(3\left((bx+a)^2-2\right)\cos(bx+a)+\left((bx+a)^3-6bx-6a\right)\sin(bx+a)\right)d^3}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^3*sin(b*x + a) - 3*a*c^2*d*sin(b*x + a)/b + 3*a^2*c*d^2*sin(b*x + a)/b^2
 - a^3*d^3*sin(b*x + a)/b^3 + 3*((b*x + a)*sin(b*x + a) + cos(b*x + a))*c^2
*d/b - 6*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a*c*d^2/b^2 + 3*((b*x + a)
*sin(b*x + a) + cos(b*x + a))*a^2*d^3/b^3 + 3*(2*(b*x + a)*cos(b*x + a) + (
(b*x + a)^2 - 2)*sin(b*x + a))*c*d^2/b^2 - 3*(2*(b*x + a)*cos(b*x + a) + ((
b*x + a)^2 - 2)*sin(b*x + a))*a*d^3/b^3 + (3*((b*x + a)^2 - 2)*cos(b*x + a)
 + ((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A]
time = 0.37, size = 109, normalized size = 1.56

$$\frac{3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3)\cos{(bx+a)}+(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3-6\,bcd^2+3\,(b^3c^2d-2\,bd^3)x)\sin{(bx+a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*
sin(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(70) = 140.
time = 0.22, size = 202, normalized size = 2.89

$$\begin{cases} \frac{c^3\sin{(a+bx)}}{b}+\frac{3c^2dx\sin{(a+bx)}}{b}+\frac{3cd^2x^2\sin{(a+bx)}}{b}+\frac{d^3x^3\sin{(a+bx)}}{b}+\frac{3c^2d\cos{(a+bx)}}{b^2}+\frac{6cd^2x\cos{(a+bx)}}{b^2}+\frac{3d^3x^2\cos{(a+bx)}}{b^2}-\frac{6cd^2\sin{(a+bx)}}{b^3}-\frac{6d^3x\sin{(a+bx)}}{b^3}-\frac{6d^3\cos{(a+bx)}}{b^4} & \text{for}\ b\neq 0 \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\cos{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a),x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)/b + 3*c**2*d*x*sin(a + b*x)/b + 3*c*d**2*x**2*
sin(a + b*x)/b + d**3*x**3*sin(a + b*x)/b + 3*c**2*d*cos(a + b*x)/b**2 + 6*
c*d**2*x*cos(a + b*x)/b**2 + 3*d**3*x**2*cos(a + b*x)/b**2 - 6*c*d**2*sin(a
 + b*x)/b**3 - 6*d**3*x*sin(a + b*x)/b**3 - 6*d**3*cos(a + b*x)/b**4, Ne(b,
 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a), True)
)
```

Giac [A]
time = 0.43, size = 110, normalized size = 1.57

$$\frac{3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3)\cos{(bx+a)}}{b^4}+\frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2)\sin{(bx+a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)/b^4 + (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^
2)*sin(b*x + a)/b^4
```

Mupad [B]
time = 0.29, size = 147, normalized size = 2.10

$$\frac{3\,d^3\,x^2\,\cos\left(a+b\,x\right)}{b^2}-\frac{\sin\left(a+b\,x\right)\,\left(6\,c\,d^2-b^2\,c^3\right)}{b^3}-\frac{3\,\cos\left(a+b\,x\right)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^4}+\frac{d^3\,x^3\,\sin\left(a+b\,x\right)}{b}-\frac{3\,x\,\sin\left(a+b\,x\right)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^3}+\frac{6\,c\,d^2\,x\,\cos\left(a+b\,x\right)}{b^2}+\frac{3\,c\,d^2\,x^2\,\sin\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (3*d^3*x^2*cos(a + b*x))/b^2 - (sin(a + b*x)*(6*c*d^2 - b^2*c^3))/b^3 - (3*
cos(a + b*x)*(2*d^3 - b^2*c^2*d))/b^4 + (d^3*x^3*sin(a + b*x))/b - (3*x*sin
(a + b*x)*(2*d^3 - b^2*c^2*d))/b^3 + (6*c*d^2*x*cos(a + b*x))/b^2 + (3*c*d^
2*x^2*sin(a + b*x))/b
```

3.3 $\int (c+dx)^2 \cos(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{2d^2\sin(a+bx)}{b^3}+\frac{(c+dx)^2\sin(a+bx)}{b}$$

```
[Out] 2*d*(d*x+c)*cos(b*x+a)/b^2-2*d^2*sin(b*x+a)/b^3+(d*x+c)^2*sin(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2717}

$$-\frac{2d^2\sin(a+bx)}{b^3}+\frac{2d(c+dx)\cos(a+bx)}{b^2}+\frac{(c+dx)^2\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x],x]
```

```
[Out] (2*d*(c + d*x)*Cos[a + b*x])/b^2 - (2*d^2*Sin[a + b*x])/b^3 + ((c + d*x)^2*
Sin[a + b*x])/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cos(a+bx)\,dx &= \frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(2d)\int (c+dx)\sin(a+bx)\,dx}{b}\\ &= \frac{2d(c+dx)\cos(a+bx)}{b^2}+\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(2d^2)\int \cos(a+bx)\,dx}{b^2}\\ &= \frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{2d^2\sin(a+bx)}{b^3}+\frac{(c+dx)^2\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 44, normalized size = 0.90

$$\frac{2bd(c+dx)\cos(a+bx)+(-2d^2+b^2(c+dx)^2)\sin(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x],x]
```

```
[Out] (2*b*d*(c + d*x)*Cos[a + b*x] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a + b*x])/b^
3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(49) = 98.
time = 0.07, size = 143, normalized size = 2.92

method	result
risch	$\frac{2d(dx+c)\cos(bx+a)}{b^2}+\frac{(d^2x^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{b^3}$
norman	$\frac{\frac{4cd}{b^2}+\frac{2d^2x}{b^2}+\frac{2\left(b^2c^2-2d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{2d^2x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2d^2x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{4cdx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
derivativedivides	$\frac{\frac{a^2d^2\sin(bx+a)}{b^2}-\frac{2acd\sin(bx+a)}{b}-\frac{2a\,d^2(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^2}+c^2\sin(bx+a)+\frac{2cd(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}+\frac{d^2((b\ldots}{}}{b}$
default	$\frac{\frac{a^2d^2\sin(bx+a)}{b^2}-\frac{2acd\sin(bx+a)}{b}-\frac{2a\,d^2(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^2}+c^2\sin(bx+a)+\frac{2cd(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}+\frac{d^2((b\ldots}{}}{b}$
meijerg	$\frac{4d^2\cos(a)\sqrt{\pi}\left(\frac{x\left(b^2\right)^{\frac{3}{2}}\cos(bx)}{2\sqrt{\pi}\,b^2}-\frac{\left(b^2\right)^{\frac{3}{2}}\left(-\frac{3x^2b^2}{2}+3\right)\sin(bx)}{6\sqrt{\pi}\,b^3}\right)}{b^2\sqrt{b^2}}-\frac{4d^2\sin(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2b^2}{2}+1\right)\cos(bx)}{2\sqrt{\pi}}+\ldots\right)}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^2*a^2*d^2*sin(b*x+a)-2/b*a*c*d*sin(b*x+a)-2/b^2*a*d^2*(cos(b*x+a)+
(b*x+a)*sin(b*x+a))+c^2*sin(b*x+a)+2/b*c*d*(cos(b*x+a)+(b*x+a)*sin(b*x+a))+
1/b^2*d^2*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x+a)*cos(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(49) = 98.
time = 0.33, size = 136, normalized size = 2.78

$$\frac{c^2\sin(bx+a)-\frac{2\,acd\sin(bx+a)}{b}+\frac{a^2d^2\sin(bx+a)}{b^2}+\frac{2\,((bx+a)\sin(bx+a)+\cos(bx+a))cd}{b}-\frac{2\,((bx+a)\sin(bx+a)+\cos(bx+a))ad^2}{b^2}+\frac{\left(2\,(bx+a)\cos(bx+a)+\left((bx+a)^2-2\right)\sin(bx+a)\right)d^2}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^2*sin(b*x + a) - 2*a*c*d*sin(b*x + a)/b + a^2*d^2*sin(b*x + a)/b^2 + 2*(
(b*x + a)*sin(b*x + a) + cos(b*x + a))*c*d/b - 2*((b*x + a)*sin(b*x + a) +
cos(b*x + a))*a*d^2/b^2 + (2*(b*x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin
(b*x + a))*d^2/b^2)/b
```

Fricas [A]
time = 0.38, size = 62, normalized size = 1.27

$$\frac{2\left(bd^2x+bcd\right)\cos\left(bx+a\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2\right)\sin\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*(b*d^2*x + b*c*d)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 -
2*d^2)*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.
time = 0.15, size = 112, normalized size = 2.29

$$\begin{cases} \frac{c^2\sin\left(a+bx\right)}{b}+\frac{2cdx\sin\left(a+bx\right)}{b}+\frac{d^2x^2\sin\left(a+bx\right)}{b}+\frac{2cd\cos\left(a+bx\right)}{b^2}+\frac{2d^2x\cos\left(a+bx\right)}{b^2}-\frac{2d^2\sin\left(a+bx\right)}{b^3} & \text{for}\; b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a),x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)/b + 2*c*d*x*sin(a + b*x)/b + d**2*x**2*sin(a +
 b*x)/b + 2*c*d*cos(a + b*x)/b**2 + 2*d**2*x*cos(a + b*x)/b**2 - 2*d**2*sin
(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cos(a), True)
)
```

Giac [A]
time = 0.52, size = 64, normalized size = 1.31

$$\frac{2\left(bd^2x+bcd\right)\cos\left(bx+a\right)}{b^3}+\frac{\left(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2\right)\sin\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - 2*d^2)*sin(b*x + a)/b^3
```

Mupad [B]
time = 0.12, size = 84, normalized size = 1.71

$$\frac{d^2\,x^2\,\sin\left(a+b\,x\right)}{b}-\frac{\sin\left(a+b\,x\right)\,\left(2\,d^2-b^2\,c^2\right)}{b^3}+\frac{2\,c\,d\,\cos\left(a+b\,x\right)}{b^2}+\frac{2\,d^2\,x\,\cos\left(a+b\,x\right)}{b^2}+\frac{2\,c\,d\,x\,\sin\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (d^2*x^2*sin(a + b*x))/b - (sin(a + b*x)*(2*d^2 - b^2*c^2))/b^3 + (2*c*d*co
s(a + b*x))/b^2 + (2*d^2*x*cos(a + b*x))/b^2 + (2*c*d*x*sin(a + b*x))/b
```

3.4 $\int (c+dx)\cos(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}$$

```
[Out] d*cos(b*x+a)/b^2+(d*x+c)*sin(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3377, 2718}

$$\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x],x]
```

```
[Out] (d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos(a+bx)\,dx &= \frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)\,dx}{b}\\ &= \frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 0.96

$$\frac{d\cos(a+bx)+b(c+dx)\sin(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x],x]
```

```
[Out] (d*Cos[a + b*x] + b*(c + d*x)*Sin[a + b*x])/b^2
```

Maple [A]
time = 0.05, size = 51, normalized size = 1.89

method	result
risch	$\frac{d\cos(bx+a)}{b^2}+\frac{(dx+c)\sin(bx+a)}{b}$
derivativedivides	$\frac{-\frac{da\sin(bx+a)}{b}+c\sin(bx+a)+\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}}{b}$
default	$\frac{-\frac{da\sin(bx+a)}{b}+c\sin(bx+a)+\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}}{b}$
norman	$\frac{\frac{2d}{b^2}+\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
meijerg	$\frac{2d\cos(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(bx)}{2\sqrt{\pi}}\right)}{b^2}-\frac{2d\sin(a)\sqrt{\pi}\left(-\frac{xb\cos(bx)}{2\sqrt{\pi}}+\frac{\sin(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{c\cos(a)\sin(bx)}{b}-$ c si

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*sin(b*x+a)+c*sin(b*x+a)+1/b*d*(cos(b*x+a)+(b*x+a)*sin(b*x+a))
)
```

Maxima [A]
time = 0.31, size = 50, normalized size = 1.85

$$\frac{c\sin(bx+a)-\frac{ad\sin(bx+a)}{b}+\frac{((bx+a)\sin(bx+a)+\cos(bx+a))d}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c*sin(b*x + a) - a*d*sin(b*x + a)/b + ((b*x + a)*sin(b*x + a) + cos(b*x +
a))*d/b)/b
```

Fricas [A]
time = 0.37, size = 28, normalized size = 1.04

$$\frac{d\cos(bx+a)+(bdx+bc)\sin(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a) + (b*d*x + b*c)*sin(b*x + a))/b^2
```

Sympy [A]
time = 0.09, size = 46, normalized size = 1.70

$$\begin{cases} \frac{c\sin{(a+bx)}}{b} + \frac{dx\sin{(a+bx)}}{b} + \frac{d\cos{(a+bx)}}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\cos{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a),x)
```

```
[Out] Piecewise((c*sin(a + b*x)/b + d*x*sin(a + b*x)/b + d*cos(a + b*x)/b**2, Ne(
b, 0)), ((c*x + d*x**2/2)*cos(a), True))
```

Giac [A]
time = 0.47, size = 30, normalized size = 1.11

$$\frac{d\cos{(bx+a)}}{b^2} + \frac{(bdx+bc)\sin{(bx+a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] d*cos(b*x + a)/b^2 + (b*d*x + b*c)*sin(b*x + a)/b^2
```

Mupad [B]
time = 0.18, size = 34, normalized size = 1.26

$$\frac{c\,\sin{(a+b\,x)} + d\,x\,\sin{(a+b\,x)}}{b} + \frac{d\,\cos{(a+b\,x)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x),x)
```

```
[Out] (c*sin(a + b*x) + d*x*sin(a + b*x))/b + (d*cos(a + b*x))/b^2
```

3.5 $\int \frac{\cos(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=52

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] Ci(b*c/d+b*x)*cos(a-b*c/d)/d-Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A]
time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3384, 3380, 3383}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d - (Sin[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{c+dx}\,dx &= \cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\\ &= \frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.96

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)-\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x] - Sin[a - (b*c)/d]*SinIntegral
[(b*c)/d + b*x])/d
```

Maple [A]
time = 0.08, size = 77, normalized size = 1.48

method	result	size
derivativedivides	$-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+bc}{d}\right)\sin\left(\frac{-da+bc}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(bx+a+\frac{-da+bc}{d}\right)\cos\left(\frac{-da+bc}{d}\right)}{d}$	77
default	$-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+bc}{d}\right)\sin\left(\frac{-da+bc}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(bx+a+\frac{-da+bc}{d}\right)\cos\left(\frac{-da+bc}{d}\right)}{d}$	77
risch	$-\frac{e^{-\frac{i(da-bc)}{d}}\text{expIntegral}\left(1,ibx+ia-\frac{i(da-bc)}{d}\right)}{2d}-\frac{e^{\frac{i(da-bc)}{d}}\text{expIntegral}\left(1,-ibx-ia-\frac{-iad+ibc}{d}\right)}{2d}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a
*d+b*c)/d)/d
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 142, normalized size = 2.73

$$-\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integra
l_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(I*exp_
integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 0.39, size = 78, normalized size = 1.50

$$\frac{\left(\text{Ci}\left(\frac{bdx+bc}{d}\right)+\text{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-2\,\sin\left(-\frac{bc-ad}{d}\right)\text{Si}\left(\frac{bdx+bc}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos(-
(b*c - a*d)/d) - 2*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx)}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 577, normalized size = 11.10

$$\frac{\Re(\operatorname{Ci}(bx+\frac{bc}{d}))\tan(\frac{1}{2}a)^2\tan(\frac{bc}{2d})^2+\Re(\operatorname{Ci}(-bx-\frac{bc}{d}))\tan(\frac{1}{2}a)^2\tan(\frac{bc}{2d})^2-2\Im(\operatorname{Ci}(bx+\frac{bc}{d}))\tan(\frac{1}{2}a)^2\tan(\frac{bc}{2d})+2\Im(\operatorname{Ci}(-bx-\frac{bc}{d}))\tan(\frac{1}{2}a)^2\tan(\frac{bc}{2d})-4\operatorname{Si}(\frac{bdx+bc}{d})\tan(\frac{1}{2}a)^2\tan(\frac{bc}{2d})+2\Im(\operatorname{Ci}(bx+\frac{bc}{d}))\tan(\frac{1}{2}a)\tan(\frac{bc}{2d})^2-2\Im(\operatorname{Ci}(-bx-\frac{bc}{d}))\tan(\frac{1}{2}a)\tan(\frac{bc}{2d})^2+4\operatorname{Si}(\frac{bdx+bc}{d})\tan(\frac{1}{2}a)\tan(\frac{bc}{2d})^2-\Re(\operatorname{Ci}(bx+\frac{bc}{d}))\tan(\frac{1}{2}a)^2-\Re(\operatorname{Ci}(-bx-\frac{bc}{d}))\tan(\frac{1}{2}a)^2+4\Re(\operatorname{Ci}(bx+\frac{bc}{d}))\tan(\frac{1}{2}a)\tan(\frac{bc}{2d})+4\Re(\operatorname{Ci}(-bx-\frac{bc}{d}))\tan(\frac{1}{2}a)\tan(\frac{bc}{2d})-\Re(\operatorname{Ci}(bx+\frac{bc}{d}))\tan(\frac{bc}{2d})^2-\Re(\operatorname{Ci}(-bx-\frac{bc}{d}))\tan(\frac{bc}{2d})^2-2\Im(\operatorname{Ci}(bx+\frac{bc}{d}))\tan(\frac{1}{2}a)+2\Im(\operatorname{Ci}(-bx-\frac{bc}{d}))\tan(\frac{1}{2}a)-4\operatorname{Si}(\frac{bdx+bc}{d})\tan(\frac{1}{2}a)+2\Im(\operatorname{Ci}(bx+\frac{bc}{d}))\tan(\frac{bc}{2d})-2\Im(\operatorname{Ci}(-bx-\frac{bc}{d}))\tan(\frac{bc}{2d})+4\operatorname{Si}(\frac{bdx+bc}{d})\tan(\frac{bc}{2d})+\Re(\operatorname{Ci}(bx+\frac{bc}{d}))+\Re(\operatorname{Ci}(-bx-\frac{bc}{d}))}{2\left(d\tan(\frac{1}{2}a)^2\tan(\frac{bc}{2d})^2+d\tan(\frac{1}{2}a)^2+d\tan(\frac{bc}{2d})^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + r
eal_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*imag
_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*imag_part(
cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*sin_integral((b
*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(b*x +
 b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x - b*c/
d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
)*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 - re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 + 4*real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 4*real_part(cos_integral(-b*x -
b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - real_part(cos_integral(b*x + b*c/d))*ta
n(1/2*b*c/d)^2 - real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*a) - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a) + 2
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*imag_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(1/
2*b*c/d) + real_part(cos_integral(b*x + b*c/d)) + real_part(cos_integral(-b
*x - b*c/d)))/(d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2 + d*tan(1/2
*b*c/d)^2 + d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x), x)
```

3.6 $\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos(a+bx)}{d(c+dx)}-\frac{b\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

```
[Out] -cos(b*x+a)/d/(d*x+c)-b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2-b*Ci(b*c/d+b*x)*sin(
a-b*c/d)/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3378, 3384, 3380, 3383}

$$-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\cos(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(Cos[a + b*x]/(d*(c + d*x))) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)
/d])/d^2 - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx &= -\frac{\cos(a+bx)}{d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d} \\ &= -\frac{\cos(a+bx)}{d(c+dx)} - \frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} - \frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} \\ &= -\frac{\cos(a+bx)}{d(c+dx)} - \frac{b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2} - \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 65, normalized size = 0.89

$$-\frac{\frac{d\cos(a+bx)}{c+dx} + b\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right) + b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(((d*Cos[a + b*x])/(c + d*x) + b*CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d]
 + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])/d^2)
```

Maple [A]
time = 0.10, size = 114, normalized size = 1.56

method	result
derivativedivides	$b\left(-\frac{\cos(bx+a)}{(-da+bc+d(bx+a))d} - \frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+bc}{d}\right)\cos\left(\frac{-da+bc}{d}\right)}{d} - \frac{\text{cosineIntegral}\left(bx+a+\frac{-da+bc}{d}\right)\sin\left(\frac{-da+bc}{d}\right.}{d}}{d}\right.$
default	$b\left(-\frac{\cos(bx+a)}{(-da+bc+d(bx+a))d} - \frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+bc}{d}\right)\cos\left(\frac{-da+bc}{d}\right)}{d} - \frac{\text{cosineIntegral}\left(bx+a+\frac{-da+bc}{d}\right)\sin\left(\frac{-da+bc}{d}\right.}{d}}{d}\right.$
risch	$\frac{ib\,\mathrm{e}^{-\frac{i(da-bc)}{d}}\text{expIntegral}\left(1,ibx+ia-\frac{i(da-bc)}{d}\right)}{2d^2} - \frac{ib\,\mathrm{e}^{\frac{i(da-bc)}{d}}\text{expIntegral}\left(1,-ibx-ia-\frac{-iad+ibc}{d}\right)}{2d^2} - \frac{(-2dxb-2bc)}{2d(dx+c)(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-cos(b*x+a)/(-d*a+b*c+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b
*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 164, normalized size = 2.25

$$-\frac{b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(-i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2*(-I
*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(2,
 -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b*c*d + (b*x +
 a)*d^2 - a*d^2)*b)
```

Fricas [A]
time = 0.38, size = 123, normalized size = 1.68

$$-\frac{2\,(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+2\,d\cos\left(bx+a\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2
*d*cos(b*x + a) + ((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b
*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(73) = 146.
time = 0.46, size = 523, normalized size = 7.16

$$-\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\,\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+b^3c\,\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)-ab^2d\,\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)-(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)-b^3c\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)+ab^2d\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)+b^2d\cos\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)\right)d^2}{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) +
b^3*c*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a
*d)/d)*sin(-(b*c - a*d)/d) - a*b^2*d*cos_integral(((d*x + c)*(b - b*c/(d*x
+ c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) - (d*x + c)*(b -
b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)/d)*sin_integral(-((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - b^3*c*cos(-(b*c
- a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*
c - a*d)/d) + a*b^2*d*cos(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c
/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + b^2*d*cos(-(d*x + c)*(b - b*c
/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^2, x)
```

3.7 $\int \frac{\cos(a+bx)}{(c+dx)^3} \, dx$

Optimal. Leaf size=104

$$-\frac{\cos(a+bx)}{2d(c+dx)^2}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b\sin(a+bx)}{2d^2(c+dx)}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

```
[Out] -1/2*b^2*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^3-1/2*cos(b*x+a)/d/(d*x+c)^2+1/2*b^2*
Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+1/2*b*sin(b*x+a)/d^2/(d*x+c)
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3380, 3383}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b\sin(a+bx)}{2d^2(c+dx)}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -1/2*Cos[a + b*x]/(d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)
/d + b*x])/(2*d^3) + (b*Sin[a + b*x])/(2*d^2*(c + d*x)) + (b^2*Sin[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos(a+bx)}{2d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{\cos(a+bx)}{2d(c+dx)^2} + \frac{b\sin(a+bx)}{2d^2(c+dx)} - \frac{b^2\int \frac{\cos(a+bx)}{c+dx}\,dx}{2d^2}$$
$$= -\frac{\cos(a+bx)}{2d(c+dx)^2} + \frac{b\sin(a+bx)}{2d^2(c+dx)} - \frac{\left(b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2} + \frac{\left(b^2\sin\left(a-\frac{bc}{d}\right)\right)\int}{2d^2}$$
$$= -\frac{\cos(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{2d^3} + \frac{b\sin(a+bx)}{2d^2(c+dx)} + \frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+b\right.}{2d^3}$$

Mathematica [A]
time = 0.43, size = 89, normalized size = 0.86

$$\frac{-b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d(-d\cos(a+bx)+b(c+dx)\sin(a+bx))}{(c+dx)^2}+b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^3,x]
[Out] (-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + (d*(-(d*Cos[a + b*x]) +
 b*(c + d*x)*Sin[a + b*x]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[
b*(c/d + x)])/(2*d^3)
```

Maple [A]
time = 0.15, size = 148, normalized size = 1.42

method	result
derivativedivides	$b^2\left(-\frac{\cos(bx+a)}{2(-da+bc+d(bx+a))^2 d} - \frac{-\frac{\sin(bx+a)}{(-da+bc+d(bx+a))d}+\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+bc}{d}\right)\sin\left(\frac{-da+bc}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(b\right.}{}}{d}}{2d}\right.$
default	$b^2\left(-\frac{\cos(bx+a)}{2(-da+bc+d(bx+a))^2 d} - \frac{-\frac{\sin(bx+a)}{(-da+bc+d(bx+a))d}+\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+bc}{d}\right)\sin\left(\frac{-da+bc}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(b\right.}{}}{d}}{2d}\right.$

risch	$\frac{b^2 e^{-\frac{i(da-bc)}{d}} \operatorname{expIntegral}\left(1, ibx+ia-\frac{i(da-bc)}{d}\right)}{4d^3} + \frac{b^2 e^{\frac{i(da-bc)}{d}} \operatorname{expIntegral}\left(1, -ibx-ia-\frac{-iad+ibc}{d}\right)}{4d^3} + \frac{(-2b^2d^3x^2-4b}{4d^2(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*cos(b*x+a)/(-d*a+b*c+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-d*a+b*c+d*
(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c
)/d)*cos((-a*d+b*c)/d)/d)/d)/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.42, size = 199, normalized size = 1.91

$$-\frac{b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(-i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3*(-I
*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(3,
 -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^2*c^2*d - 2*
a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(96) = 192.
time = 0.37, size = 209, normalized size = 2.01

$$-\frac{2\,d^2\cos(bx+a)-2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)+\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-2\left(bd^2x+bcd\right)\sin(bx+a)}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*
c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) - 2*(b*d^2*x + b*c*d
)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(d*x+c)**3,x)

[Out] Integral(cos(a + b*x)/(c + d*x)**3, x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 5518, normalized size = 53.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="giac")

```
[Out] -1/4*(b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(co
s_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2
*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*
tan(1/2*b*c/d) - 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part
(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4
*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^
2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-b*x -
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d^2*x^2*rea
l_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b^2*d^2*
x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*
b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2
*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*imag_part(cos_i
ntegral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 8*b^2*c
*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/
d) - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)
^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 8*b^2*c*d*x*si
n_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b^
2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*
b*c/d)^2 + b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_i
```

```
ntegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*d^2*x^2*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 4*b^2*d^2*x^2*sin_in
tegral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c*d*x*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*d*x*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*d^2*x^2*
imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b^2*
d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)
 + 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d
) + 8*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2
*a)*tan(1/2*b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*b^2*d^2*x^2*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^2*c^2*imag_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*c
^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/
2*b*c/d) - 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d) - 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b^2*d^2*x^2*sin_integr
al((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*b^2*c^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*c^
2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 + 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)
*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2 - 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*
x)^2*tan(1/2*a) + 4*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2
*b*x)^2*tan(1/2*a) - 8*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)
^2*tan(1/2*a) - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)
^2 - b^2*d^2*x^2*real_part(cos_integral(-b*x - ...
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^3,x)
[Out] int(cos(a + b*x)/(c + d*x)^3, x)
```

3.8 $\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=127

$$-\frac{\cos(a+bx)}{3d(c+dx)^3}+\frac{b^2\cos(a+bx)}{6d^3(c+dx)}+\frac{b^3\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{6d^4}+\frac{b\sin(a+bx)}{6d^2(c+dx)^2}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+\right.}{6d^4}$$

```
[Out] -1/3*cos(b*x+a)/d/(d*x+c)^3+1/6*b^2*cos(b*x+a)/d^3/(d*x+c)+1/6*b^3*cos(a-b*
c/d)*Si(b*c/d+b*x)/d^4+1/6*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^4+1/6*b*sin(b*x
+a)/d^2/(d*x+c)^2
```

Rubi [A]
time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,
Rules used = {3378, 3384, 3380, 3383}

$$\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{6d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{6d^4}+\frac{b^2\cos(a+bx)}{6d^3(c+dx)}+\frac{b\sin(a+bx)}{6d^2(c+dx)^2}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^4,x]
```

```
[Out] -1/3*Cos[a + b*x]/(d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(6*d^3*(c + d*x)) +
(b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(6*d^4) + (b*Sin[a + b*x]
)/(6*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(
6*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx &= -\frac{\cos(a+bx)}{3d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{3d} \\
&= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} - \frac{b^2\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{6d^2} \\
&= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{b^3\int \frac{\sin(a+bx)}{c+dx}\,dx}{6d^3} \\
&= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{\left(b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{6d^3} + \frac{(b^3 \ldots}{} \\
&= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b^3\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{6d^4} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{b^3\cos\left(a \ldots\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 144, normalized size = 1.13

$$\frac{d\cos(bx)\left((-2d^2+b^2(c+dx)^2)\cos(a)+bd(c+dx)\sin(a)\right)+d\left(bd(c+dx)\cos(a)-(-2d^2+b^2(c+dx)^2)\sin(a)\right)\sin(bx)+b^3(c+dx)^3\left(\mathrm{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)}{6d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^4,x]
```

```
[Out] (d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d*
(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3*(
c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*Si
nIntegral[b*(c/d + x)]))/(6*d^4*(c + d*x)^3)
```

Maple [A]
time = 0.19, size = 184, normalized size = 1.45

method	result

derivativedivides	$b^3\left(-\frac{\cos(bx+a)}{3(-da+bc+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-da+bc+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-da+bc+d(bx+a))d}-\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+bc}{d}\right)\cos(}{d}}{}}{2d}}{3d}\right.$
default	$b^3\left(-\frac{\cos(bx+a)}{3(-da+bc+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-da+bc+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-da+bc+d(bx+a))d}-\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+bc}{d}\right)\cos(}{d}}{}}{2d}}{3d}\right.$
risch	$-\frac{ib^3e^{-\frac{i(da-bc)}{d}}\text{expIntegral}\left(1,ibx+ia-\frac{i(da-bc)}{d}\right)}{12d^4}+\frac{ib^3e^{\frac{i(da-bc)}{d}}\text{expIntegral}\left(1,-ibx-ia-\frac{-iad+ibc}{d}\right)}{12d^4}-\frac{(-2b^5d^5}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*cos(b*x+a)/(-d*a+b*c+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a)/(-d*a+b*
c+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-d*a+b*c+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+
b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/
d)/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 249, normalized size = 1.96

$$-\frac{b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(-i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^4*(-I
*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(4,
 -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^3*c^3*d - 3*
a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*
d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(117) = 234$.
time = 0.38, size = 295, normalized size = 2.32

$$\frac{2\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)+2\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)+2\left(bd^3x+bcd^2\right)\sin\left(bx+a\right)+\left(\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{12\left(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="fricas")

[Out] 1/12*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + 2*(b*d^3*x + b*c*d^2)*sin(b*x + a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(d*x+c)**4,x)

[Out] Integral(cos(a + b*x)/(c + d*x)**4, x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 8378, normalized size = 65.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="giac")

[Out] 1/12*(b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/

```
2*a)*tan(1/2*b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^3*d^3*x^3*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 6*b^3*c*d^2*x^2*real_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
+ 6*b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d) - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(-b*x -
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*sin_integral((b*d*
x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos
_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^3*
c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(b*x +
b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c^2*d*x*imag_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a) + 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*x)^2*tan(1/2*a) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^3*d^3*x^3*real_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 12*b^3*c*d^
2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b
*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^3*c*d^2*x^2*sin_integral((b*d*x + b*
c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 2*b^3*d^3*x^3*real_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^3*d^3*x^3*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^3*c^2*d*x
*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*
tan(1/2*a)^2*tan(1/2*b*c/d) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x +
b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*sin_
integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*r
eal_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*d^3
*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*
b^3*c^2*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*
tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_par
t(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c*d^2*x
```

```
^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^3*
x^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^3*c^3*imag_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*c^3
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
b*c/d)^2 + 2*b^3*c^3*sin_integral((b*d*x + b*c)...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b\,x)}{{(c+d\,x)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^4,x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^4, x)
```

3.9 $\int (c+dx)^4 \cos^2(a+bx)\,dx$

Optimal. Leaf size=161

$$\frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{3d^4\cos(a+bx)\sin(a+}{4b^5}$$

```
[Out] 3/4*d^4*x/b^4-1/2*d*(d*x+c)^3/b^2+1/10*(d*x+c)^5/d-3/2*d^3*(d*x+c)*cos(b*x+
a)^2/b^4+d*(d*x+c)^3*cos(b*x+a)^2/b^2+3/4*d^4*cos(b*x+a)*sin(b*x+a)/b^5-3/2
*d^2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3+1/2*(d*x+c)^4*cos(b*x+a)*sin(b*x+a
)/b
```

Rubi [A]
time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$\frac{3d^4\sin(a+bx)\cos(a+bx)}{4b^5}-\frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{3d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b^3}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2,x]
```

```
[Out] (3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^3*(
c + d*x)*Cos[a + b*x]^2)/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*
d^4*Cos[a + b*x]*Sin[a + b*x])/(4*b^5) - (3*d^2*(c + d*x)^2*Cos[a + b*x]*Si
n[a + b*x])/(2*b^3) + ((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos^2(a+bx)\,dx &= \frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^4\,d\\ &= \frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}-\frac{3d^2(c+dx}{}\\ &= -\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{d(c+dx)^3\cos^2(a+b}{b^2}\\ &= \frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{d(c+dx)^3\cos^2(}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 132, normalized size = 0.82

$$\frac{8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)+20bd(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+10(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\sin(2(a+bx))}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2,x]
```

```
[Out] (8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) + 20
*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 10*(3*d^4 -
6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(80*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(147) = 294$.
time = 0.18, size = 1027, normalized size = 6.38

method	result
risch	$\frac{d^4x^5}{10}+\frac{c\,d^3x^4}{2}+d^2c^2x^3+d\,c^3x^2+\frac{c^4x}{2}+\frac{c^5}{10d}+\frac{d\left(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3c\,d^2\right)\cos(2}{4b^4}$
norman	$\frac{\frac{d^4x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{\left(2c^4b^4-6b^2c^2d^2+3d^4\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^5}+\frac{\left(2c^4b^4+6b^2c^2d^2-3d^4\right)x}{4b^4}+\frac{d^2\left(2b^2c^2+d^2\right)x^3}{2b^2}+\frac{2cd\left(2b^2c^2-3d^2\right)x\tan\left(\frac{bx}{2}\right.}{b^3}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^4*a^4*d^4*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-4/b^3*a^3*c*d^
3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-4/b^4*a^3*d^4*((b*x+a)*(1/2*cos
(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+6/b^2*a^2
*c^2*d^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+12/b^3*a^2*c*d^3*((b*x+a
)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)
+6/b^4*a^2*d^4*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*
x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)-4/
b*a*c^3*d*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-12/b^2*a*c^2*d^2*((b*x+
a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2
)-12/b^3*a*c*d^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(
b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)-
4/b^4*a*d^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a
)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/8*
(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+c^4*(1/2*cos(b*x+a)*sin(b*x+a)+1/
2*b*x+1/2*a)+4/b*c^3*d*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+6/b^2*c^2*d^2*((b*x+a)^2*(1/2*cos(b*x+a)*sin
(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1
/4*b*x-1/4*a-1/3*(b*x+a)^3)+4/b^3*c*d^3*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+
a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*si
n(b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+1/b^4
*d^4*((b*x+a)^4*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+(b*x+a)^3*cos(b*x
+a)^2-3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/2*(b*x+a)*cos
(b*x+a)^2+3/4*cos(b*x+a)*sin(b*x+a)+3/4*b*x+3/4*a+(b*x+a)^3-2/5*(b*x+a)^5))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(147) = 294$.
time = 0.33, size = 717, normalized size = 4.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/40*(10*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^4 - 40*(2*b*x + 2*a + sin(2*b*x
 + 2*a))*a*c^3*d/b + 60*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*c^2*d^2/b^2 -
40*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^3*c*d^3/b^3 + 10*(2*b*x + 2*a + sin(2
*b*x + 2*a))*a^4*d^4/b^4 + 20*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a)
 + cos(2*b*x + 2*a))*c^3*d/b - 60*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x +
2*a) + cos(2*b*x + 2*a))*a*c^2*d^2/b^2 + 60*(2*(b*x + a)^2 + 2*(b*x + a)*si
n(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^2*c*d^3/b^3 - 20*(2*(b*x + a)^2 + 2*(b
*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^3*d^4/b^4 + 10*(4*(b*x + a)^
3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*
c^2*d^2/b^2 - 20*(4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x
+ a)^2 - 1)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 10*(4*(b*x + a)^3 + 6*(b*x + a)
```

```
*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a^2*d^4/b^4 + 1
0*(2*(b*x + a)^4 + 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^
3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*c*d^3/b^3 - 10*(2*(b*x + a)^4 + 3*(2*(b*
x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x
+ 2*a))*a*d^4/b^4 + (4*(b*x + a)^5 + 10*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2
*b*x + 2*a) + 5*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*sin(2*b*x + 2*a))*d^4/b
^4)/b
```

Fricas [**A**]
time = 0.37, size = 287, normalized size = 1.78

$$\frac{2 b^{5} d^{4} x^{5} + 10 b^{5} c d^{3} x^{4} + 10 \left(2 b^{5} c^{2} d^{2} - b^{3} d^{4}\right) x^{3} + 10 \left(2 b^{5} c^{3} d - 3 b^{3} c d^{3}\right) x^{2} + 10 \left(2 b^{3} d^{4} x^{3} + 6 b^{3} c d^{3} x^{2} + 2 b^{3} c^{3} d - 3 b c d^{3} + 3 \left(2 b^{3} c^{2} d^{2} - b d^{4}\right) x\right) \cos\left(b x + a\right)^{2} + 5 \left(2 b^{4} d^{4} x^{4} + 8 b^{4} c d^{3} x^{3} + 2 b^{4} c^{4} - 6 b^{2} c^{2} d^{2} + 3 d^{4} + 6 \left(2 b^{4} c^{2} d^{2} - b^{2} d^{4}\right) x^{2} + 4 \left(2 b^{4} c^{3} d - 3 b^{2} c d^{3}\right) x\right) \cos\left(b x + a\right) \sin\left(b x + a\right) + 5 \left(2 b^{5} c^{4} - 6 b^{3} c^{2} d^{2} + 3 b d^{4}\right) x}{20 b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 10*(2*b^5*c^2*d^2 - b^3*d^4)*x^3 +
 10*(2*b^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 +
 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2
*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)
*sin(b*x + a) + 5*(2*b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.
time = 0.56, size = 660, normalized size = 4.10

$$\begin{cases} \frac{c^{4} x \sin^{2}{\left(a + b x \right)}}{2} + \frac{c^{4} x \cos^{2}{\left(a + b x \right)}}{2} + c^{3} d x^{2} \sin^{2}{\left(a + b x \right)} + c^{3} d x^{2} \cos^{2}{\left(a + b x \right)} + c^{2} d^{2} x^{3} \sin^{2}{\left(a + b x \right)} + c^{2} d^{2} x^{3} \cos^{2}{\left(a + b x \right)} + \frac{c d^{3} x^{4} \sin^{2}{\left(a + b x \right)}}{2} + \frac{c d^{3} x^{4} \cos^{2}{\left(a + b x \right)}}{2} + \frac{d^{4} x^{5} \sin^{2}{\left(a + b x \right)}}{10} + \frac{d^{4} x^{5} \cos^{2}{\left(a + b x \right)}}{10} + \frac{c^{4} \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{2 b} + \frac{2 c^{3} d x \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b} + \frac{3 c^{2} d^{2} x^{2} \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b} + \frac{2 c d^{3} x^{3} \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b} + \frac{d^{4} x^{4} \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{2 b} - \frac{c^{3} d \sin^{2}{\left(a + b x \right)}}{b^{2}} - \frac{3 c^{2} d^{2} x \sin^{2}{\left(a + b x \right)}}{2 b^{2}} + \frac{3 c^{2} d^{2} x \cos^{2}{\left(a + b x \right)}}{2 b^{2}} - \frac{3 c d^{3} x^{2} \sin^{2}{\left(a + b x \right)}}{2 b^{2}} + \frac{3 c d^{3} x^{2} \cos^{2}{\left(a + b x \right)}}{2 b^{2}} - \frac{d^{4} x^{3} \sin^{2}{\left(a + b x \right)}}{2 b^{2}} + \frac{d^{4} x^{3} \cos^{2}{\left(a + b x \right)}}{2 b^{2}} - \frac{3 c^{2} d^{2} \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{2 b^{3}} - \frac{3 c d^{3} x \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b^{3}} - \frac{3 d^{4} x^{2} \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{2 b^{3}} + \frac{3 c d^{3} \sin^{2}{\left(a + b x \right)}}{2 b^{4}} + \frac{3 d^{4} x \sin^{2}{\left(a + b x \right)}}{4 b^{4}} - \frac{3 d^{4} x \cos^{2}{\left(a + b x \right)}}{4 b^{4}} + \frac{3 d^{4} \sin{\left(a + b x \right)} \cos{\left(a + b x \right)}}{4 b^{5}} & \text{for}\: b \neq 0 \\\left(c^{4} x + 2 c^{3} d x^{2} + 2 c^{2} d^{2} x^{3} + c d^{3} x^{4} + \frac{d^{4} x^{5}}{5}\right) \cos^{2}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**2/2 + c**4*x*cos(a + b*x)**2/2 + c**3*d*x**
2*sin(a + b*x)**2 + c**3*d*x**2*cos(a + b*x)**2 + c**2*d**2*x**3*sin(a + b*
x)**2 + c**2*d**2*x**3*cos(a + b*x)**2 + c*d**3*x**4*sin(a + b*x)**2/2 + c*
d**3*x**4*cos(a + b*x)**2/2 + d**4*x**5*sin(a + b*x)**2/10 + d**4*x**5*cos(
a + b*x)**2/10 + c**4*sin(a + b*x)*cos(a + b*x)/(2*b) + 2*c**3*d*x*sin(a +
b*x)*cos(a + b*x)/b + 3*c**2*d**2*x**2*sin(a + b*x)*cos(a + b*x)/b + 2*c*d*
*3*x**3*sin(a + b*x)*cos(a + b*x)/b + d**4*x**4*sin(a + b*x)*cos(a + b*x)/(
2*b) - c**3*d*sin(a + b*x)**2/b**2 - 3*c**2*d**2*x*sin(a + b*x)**2/(2*b**2)
 + 3*c**2*d**2*x*cos(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*sin(a + b*x)**2/(
2*b**2) + 3*c*d**3*x**2*cos(a + b*x)**2/(2*b**2) - d**4*x**3*sin(a + b*x)**
2/(2*b**2) + d**4*x**3*cos(a + b*x)**2/(2*b**2) - 3*c**2*d**2*sin(a + b*x)*
cos(a + b*x)/(2*b**3) - 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)/b**3 - 3*d**4*
x**2*sin(a + b*x)*cos(a + b*x)/(2*b**3) + 3*c*d**3*sin(a + b*x)**2/(2*b**4)
 + 3*d**4*x*sin(a + b*x)**2/(4*b**4) - 3*d**4*x*cos(a + b*x)**2/(4*b**4) +
```

```
3*d**4*sin(a + b*x)*cos(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x
**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a)**2, True))
```

Giac [A]
time = 0.49, size = 222, normalized size = 1.38

$$\frac{1}{10}\,d^4x^5+\frac{1}{2}\,cd^3x^4+c^2d^2x^3+c^3dx^2+\frac{1}{2}\,c^4x+\frac{\left(2\,b^3d^4x^3+6\,b^3cd^3x^2+6\,b^3c^2d^2x+2\,b^3c^3d-3\,bd^4x-3\,bcd^3\right)\cos\left(2\,bx+2\,a\right)}{4\,b^5}+\frac{\left(2\,b^4d^4x^4+8\,b^4cd^3x^3+12\,b^4c^2d^2x^2+8\,b^4c^3dx+2\,b^4c^4-6\,b^2d^4x^2-12\,b^2cd^3x-6\,b^2c^2d^2+3\,d^4\right)\sin\left(2\,bx+2\,a\right)}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x + 1/4*(2
*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*x
- 3*b*c*d^3)*cos(2*b*x + 2*a)/b^5 + 1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 +
12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d
^3*x - 6*b^2*c^2*d^2 + 3*d^4)*sin(2*b*x + 2*a)/b^5
```

Mupad [B]
time = 0.61, size = 349, normalized size = 2.17

$$\frac{\frac{15\,d^4\,\sin\left(2\,a+2\,b\,x\right)}{2}+10\,b^5\,c^4\,x+5\,b^4\,c^4\,\sin\left(2\,a+2\,b\,x\right)+2\,b^5\,d^4\,x^5+10\,b^3\,c^3\,d\,\cos\left(2\,a+2\,b\,x\right)+20\,b^5\,c^3\,d\,x^2+10\,b^5\,c\,d^3\,x^4-15\,b^2\,c^2\,d^2\,\sin\left(2\,a+2\,b\,x\right)+10\,b^3\,d^4\,x^3\,\cos\left(2\,a+2\,b\,x\right)+20\,b^5\,c^2\,d^2\,x^3-15\,b^2\,d^4\,x^2\,\sin\left(2\,a+2\,b\,x\right)+5\,b^4\,d^4\,x^4\,\sin\left(2\,a+2\,b\,x\right)-15\,b\,c\,d^3\,\cos\left(2\,a+2\,b\,x\right)-15\,b\,d^4\,x\,\cos\left(2\,a+2\,b\,x\right)+30\,b^4\,c^2\,d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)-30\,b^2\,c\,d^3\,x\,\sin\left(2\,a+2\,b\,x\right)+20\,b^4\,c^3\,d\,x\,\sin\left(2\,a+2\,b\,x\right)+30\,b^3\,c^2\,d^2\,x\,\cos\left(2\,a+2\,b\,x\right)+30\,b^3\,c\,d^3\,x^2\,\cos\left(2\,a+2\,b\,x\right)+20\,b^4\,c\,d^3\,x^3\,\sin\left(2\,a+2\,b\,x\right)}{20\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] ((15*d^4*sin(2*a + 2*b*x))/2 + 10*b^5*c^4*x + 5*b^4*c^4*sin(2*a + 2*b*x) +
2*b^5*d^4*x^5 + 10*b^3*c^3*d*cos(2*a + 2*b*x) + 20*b^5*c^3*d*x^2 + 10*b^5*c
*d^3*x^4 - 15*b^2*c^2*d^2*sin(2*a + 2*b*x) + 10*b^3*d^4*x^3*cos(2*a + 2*b*x
) + 20*b^5*c^2*d^2*x^3 - 15*b^2*d^4*x^2*sin(2*a + 2*b*x) + 5*b^4*d^4*x^4*si
n(2*a + 2*b*x) - 15*b*c*d^3*cos(2*a + 2*b*x) - 15*b*d^4*x*cos(2*a + 2*b*x)
+ 30*b^4*c^2*d^2*x^2*sin(2*a + 2*b*x) - 30*b^2*c*d^3*x*sin(2*a + 2*b*x) + 2
0*b^4*c^3*d*x*sin(2*a + 2*b*x) + 30*b^3*c^2*d^2*x*cos(2*a + 2*b*x) + 30*b^3
*c*d^3*x^2*cos(2*a + 2*b*x) + 20*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/(20*b^5)
```

3.10 $\int (c+dx)^3 \cos^2(a+bx)\,dx$

Optimal. Leaf size=134

$$-\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}-\frac{3d^3\cos^2(a+bx)}{8b^4}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}-\frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3}$$

```
[Out] -3/4*c*d^2*x/b^2-3/8*d^3*x^2/b^2+1/8*(d*x+c)^4/d-3/8*d^3*cos(b*x+a)^2/b^4+3
/4*d*(d*x+c)^2*cos(b*x+a)^2/b^2-3/4*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^3+1
/2*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3392, 32, 3391}

$$-\frac{3d^3\cos^2(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}-\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2,x]
```

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) - (3*d^3*Cos
[a + b*x]^2)/(8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\,dx = \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b}+\frac{1}{2}\int (c+dx)$$
$$= \frac{(c+dx)^4}{8d}-\frac{3d^3\cos^2(a+bx)}{8b^4}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}-\frac{3d^2(c+dx)\cos($$
$$= -\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}-\frac{3d^3\cos^2(a+bx)}{8b^4}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}$$

Mathematica [**A**]
time = 0.28, size = 106, normalized size = 0.79

$$\frac{2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+3d(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))+2b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\sin(2(a+bx))}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2,x]
```

```
[Out] (2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c
 + d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Si
n[2*(a + b*x)])/(16*b^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(120) = 240$.
time = 0.11, size = 587, normalized size = 4.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^3*a^3*d^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/b^2*a^2*c*d
^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/b^3*a^2*d^3*((b*x+a)*(1/2*co
s(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-3/b*a*c^
2*d*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-6/b^2*a*c*d^2*((b*x+a)*(1/2*c
os(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-3/b^3*a
*d^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b
*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+c^3*(1/2*cos
(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/b*c^2*d*((b*x+a)*(1/2*cos(b*x+a)*sin(b*
x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+3/b^2*c*d^2*((b*x+a)^2*
(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(
b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+1/b^3*d^3*((b*x+a)^3*(1/2*co
s(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(
```

```
1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8
*(b*x+a)^4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(120) = 240$.
time = 0.34, size = 428, normalized size = 3.19

$$\frac{4\,(2\,bx+2\,a+\sin(2\,bx+2\,a))c^3-\frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ac^2d}{b}+\frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2cd^2}{b^2}-\frac{4\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^3d^3}{b^3}+\frac{6\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)c^2d}{b}-\frac{12\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)acd^2}{b^2}+\frac{6\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)a^2d^3}{b^3}+\frac{2\left(4\,(bx+a)^3+6\,(bx+a)\cos(2\,bx+2\,a)+3\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)\right)cd^2}{b^2}-\frac{2\left(4\,(bx+a)^3+6\,(bx+a)\cos(2\,bx+2\,a)+3\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)\right)ad^3}{b^3}+\frac{\left(2\,(bx+a)^4+3\left(2\,(bx+a)^2-1\right)\cos(2\,bx+2\,a)+2\left(2\,(bx+a)^3-3\,bx-3\,a\right)\sin(2\,bx+2\,a)\right)d^3}{b^3}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^3 - 12*(2*b*x + 2*a + sin(2*b*x
+ 2*a))*a*c^2*d/b + 12*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*c*d^2/b^2 - 4*(
2*b*x + 2*a + sin(2*b*x + 2*a))*a^3*d^3/b^3 + 6*(2*(b*x + a)^2 + 2*(b*x + a
)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)^2 + 2*(b*x
 + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)^2 +
 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^2*d^3/b^3 + 2*(4*(b*x +
 a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*
a))*c*d^2/b^2 - 2*(4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x
 + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(b*x + a)^4 + 3*(2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))
*d^3/b^3)/b
```

Fricas [A]
time = 0.37, size = 190, normalized size = 1.42

$$\frac{b^4d^3x^4+4\,b^4cd^2x^3+3\,(2\,b^4c^2d-b^2d^3)x^2+3\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\cos(bx+a)^2+2\,(2\,b^3d^3x^3+6\,b^3cd^2x^2+2\,b^3c^3-3\,bcd^2+3\,(2\,b^3c^2d-bd^3)x)\cos(bx+a)\sin(bx+a)+2\,(2\,b^4c^3-3\,b^2cd^2)x}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 3*(2*b^4*c^2*d - b^2*d^3)*x^2 + 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*
x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.
time = 0.37, size = 456, normalized size = 3.40

$$\begin{cases} \frac{c^3x\sin^2(a+bx)}{2}+\frac{c^3x\cos^2(a+bx)}{2}+\frac{3c^2dx^2\sin^2(a+bx)}{4}+\frac{3c^2dx^2\cos^2(a+bx)}{4}+\frac{cd^2x^3\sin^2(a+bx)}{2}+\frac{cd^2x^3\cos^2(a+bx)}{2}+\frac{d^3x^4\sin^2(a+bx)}{8}+\frac{d^3x^4\cos^2(a+bx)}{8}+\frac{c^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{3c^2dx\sin(a+bx)\cos(a+bx)}{2b}+\frac{3cd^2x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{d^3x^3\sin(a+bx)\cos(a+bx)}{2b}-\frac{3c^2d\sin^2(a+bx)}{4b^2}-\frac{3cd^2x\sin^2(a+bx)}{4b^2}+\frac{3cd^2x\cos^2(a+bx)}{4b^2}-\frac{3d^3x^2\sin^2(a+bx)}{8b^2}+\frac{3d^3x^2\cos^2(a+bx)}{8b^2}-\frac{3cd^2\sin(a+bx)\cos(a+bx)}{4b^3}-\frac{3d^3x\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{3d^3\sin^2(a+bx)}{8b^4} & \text{for}\ b\neq 0 \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**2/2 + c**3*x*cos(a + b*x)**2/2 + 3*c**2*d*x
**2*sin(a + b*x)**2/4 + 3*c**2*d*x**2*cos(a + b*x)**2/4 + c*d**2*x**3*sin(a
 + b*x)**2/2 + c*d**2*x**3*cos(a + b*x)**2/2 + d**3*x**4*sin(a + b*x)**2/8
+ d**3*x**4*cos(a + b*x)**2/8 + c**3*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c*
*2*d*x*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c*d**2*x**2*sin(a + b*x)*cos(a +
 b*x)/(2*b) + d**3*x**3*sin(a + b*x)*cos(a + b*x)/(2*b) - 3*c**2*d*sin(a +
b*x)**2/(4*b**2) - 3*c*d**2*x*sin(a + b*x)**2/(4*b**2) + 3*c*d**2*x*cos(a +
 b*x)**2/(4*b**2) - 3*d**3*x**2*sin(a + b*x)**2/(8*b**2) + 3*d**3*x**2*cos(
a + b*x)**2/(8*b**2) - 3*c*d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3) - 3*d**3
*x*sin(a + b*x)*cos(a + b*x)/(4*b**3) + 3*d**3*sin(a + b*x)**2/(8*b**4), Ne
(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a)**2,
 True))
```

Giac [A]
time = 0.44, size = 153, normalized size = 1.14

$$\frac{1}{8}\,d^{3}x^{4}+\frac{1}{2}\,cd^{2}x^{3}+\frac{3}{4}\,c^{2}dx^{2}+\frac{1}{2}\,c^{3}x+\frac{3\,(2\,b^{2}d^{3}x^{2}+4\,b^{2}cd^{2}x+2\,b^{2}c^{2}d-d^{3})\cos(2\,bx+2\,a)}{16\,b^{4}}+\frac{(2\,b^{3}d^{3}x^{3}+6\,b^{3}cd^{2}x^{2}+6\,b^{3}c^{2}dx+2\,b^{3}c^{3}-3\,bd^{3}x-3\,bcd^{2})\sin(2\,bx+2\,a)}{8\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x + 3/16*(2*b^2*d^3*x
^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(2*b*x + 2*a)/b^4 + 1/8*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^
2)*sin(2*b*x + 2*a)/b^4
```

Mupad [B]
time = 0.46, size = 229, normalized size = 1.71

$$\frac{4\,b^{4}\,c^{3}\,x-\frac{3\,d^{3}\,\cos(2\,a+2\,b\,x)}{2}+2\,b^{3}\,c^{3}\,\sin(2\,a+2\,b\,x)+b^{4}\,d^{3}\,x^{4}+3\,b^{2}\,c^{2}\,d\,\cos(2\,a+2\,b\,x)+6\,b^{4}\,c^{2}\,d\,x^{2}+4\,b^{4}\,c\,d^{2}\,x^{3}+3\,b^{2}\,d^{3}\,x^{2}\,\cos(2\,a+2\,b\,x)+2\,b^{3}\,d^{3}\,x^{3}\,\sin(2\,a+2\,b\,x)-3\,b\,c\,d^{2}\,\sin(2\,a+2\,b\,x)-3\,b\,d^{3}\,x\,\sin(2\,a+2\,b\,x)+6\,b^{2}\,c\,d^{2}\,x\,\cos(2\,a+2\,b\,x)+6\,b^{3}\,c^{2}\,d\,x\,\sin(2\,a+2\,b\,x)+6\,b^{3}\,c\,d^{2}\,x^{2}\,\sin(2\,a+2\,b\,x)}{8\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] (4*b^4*c^3*x - (3*d^3*cos(2*a + 2*b*x))/2 + 2*b^3*c^3*sin(2*a + 2*b*x) + b^
4*d^3*x^4 + 3*b^2*c^2*d*cos(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d^2*x^
3 + 3*b^2*d^3*x^2*cos(2*a + 2*b*x) + 2*b^3*d^3*x^3*sin(2*a + 2*b*x) - 3*b*c
*d^2*sin(2*a + 2*b*x) - 3*b*d^3*x*sin(2*a + 2*b*x) + 6*b^2*c*d^2*x*cos(2*a
+ 2*b*x) + 6*b^3*c^2*d*x*sin(2*a + 2*b*x) + 6*b^3*c*d^2*x^2*sin(2*a + 2*b*x
))/(8*b^4)
```

3.11 $\int (c+dx)^2 \cos^2(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] -1/4*d^2*x/b^2+1/6*(d*x+c)^3/d+1/2*d*(d*x+c)*cos(b*x+a)^2/b^2-1/4*d^2*cos(b
*x+a)*sin(b*x+a)/b^3+1/2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$-\frac{d^2\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2,x]
```

```
[Out] -1/4*(d^2*x)/b^2 + (c + d*x)^3/(6*d) + (d*(c + d*x)*Cos[a + b*x]^2)/(2*b^2)
 - (d^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) + ((c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\,dx = \frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^2 \ldots$$

$$= \frac{(c+dx)^3}{6d}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{(c+dx)^2 \ldots}{}$$

$$= -\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3}+ \ldots$$

Mathematica [A]
time = 0.20, size = 77, normalized size = 0.81

$$\frac{4b^3x(3c^2+3cdx+d^2x^2)+6bd(c+dx)\cos(2(a+bx))+3(-d^2+2b^2(c+dx)^2)\sin(2(a+bx))}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2,x]
```

```
[Out] (4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 6*b*d*(c + d*x)*Cos[2*(a + b*x)] + 3
*(-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(24*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(85) = 170$.
time = 0.09, size = 289, normalized size = 3.04

method	result
risch	$\frac{d^2x^3}{6}+\frac{cd\,x^2}{2}+\frac{c^2x}{2}+\frac{c^3}{6d}+\frac{d(dx+c)\cos(2bx+2a)}{4b^2}+\frac{(2d^2x^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{\frac{a^2d^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-(b \ldots\right.}{b^2}}{\ldots}$
default	$\frac{\frac{a^2d^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-(b \ldots\right.}{b^2}}{\ldots}$
norman	$\frac{cd\,x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{d^2x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{d^2x^3}{6}+\frac{cd\,x^2}{2}+\frac{d^2x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{d^2x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6}+\frac{(2b^2c^2-d^2)\tan\left(\frac{bx}{2} \ldots\right.}{2b^3}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^2*a^2*d^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2/b*a*c*d*(1/2
*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2/b^2*a*d^2*((b*x+a)*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+c^2*(1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a)+2/b*c*d*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/
2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+1/b^2*d^2*((b*x+a)^2*(1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*si
n(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(85) = 170$.
time = 0.31, size = 222, normalized size = 2.34

$$\frac{6\,(2\,bx+2\,a+\sin{(2\,bx+2\,a)})c^2-\frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))acd}{b}+\frac{6\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2d^2}{b^2}+\frac{6\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)cd}{b}-\frac{6\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)ad^2}{b^2}+\frac{\left(4\,(bx+a)^3+6\,(bx+a)\cos(2\,bx+2\,a)+3\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)\right)d^2}{b^2}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^2 - 12*(2*b*x + 2*a + sin(2*b*x
+ 2*a))*a*c*d/b + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*d^2/b^2 + 6*(2*(b*
x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c*d/b - 6*(2*(b
*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a*d^2/b^2 + (4
*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b
*x + 2*a))*d^2/b^2)/b
```

Fricas [**A**]
time = 0.36, size = 113, normalized size = 1.19

$$\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,(bd^2x+bcd)\cos{(bx+a)}^2+3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos{(bx+a)}\sin{(bx+a)}+3\,(2\,b^3c^2-bd^2)x}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 +
3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a)
 + 3*(2*b^3*c^2 - b*d^2)*x)/b^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(85) = 170$.
time = 0.24, size = 264, normalized size = 2.78

$$\begin{cases} \frac{c^2x\sin^2{(a+bx)}}{2}+\frac{c^2x\cos^2{(a+bx)}}{2}+\frac{cdx^2\sin^2{(a+bx)}}{2}+\frac{cdx^2\cos^2{(a+bx)}}{2}+\frac{d^2x^3\sin^2{(a+bx)}}{6}+\frac{d^2x^3\cos^2{(a+bx)}}{6}+\frac{c^2\sin{(a+bx)}\cos{(a+bx)}}{2b}+\frac{cdx\sin{(a+bx)}\cos{(a+bx)}}{b}+\frac{d^2x^2\sin{(a+bx)}\cos{(a+bx)}}{2b}-\frac{cd\sin^2{(a+bx)}}{2b^2}-\frac{d^2x\sin^2{(a+bx)}}{4b^2}+\frac{d^2x\cos^2{(a+bx)}}{4b^2}-\frac{d^2\sin{(a+bx)}\cos{(a+bx)}}{4b^3} & \text{for}\ b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\cos^2{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**2/2 + c**2*x*cos(a + b*x)**2/2 + c*d*x**2*s
in(a + b*x)**2/2 + c*d*x**2*cos(a + b*x)**2/2 + d**2*x**3*sin(a + b*x)**2/6
 + d**2*x**3*cos(a + b*x)**2/6 + c**2*sin(a + b*x)*cos(a + b*x)/(2*b) + c*d
*x*sin(a + b*x)*cos(a + b*x)/b + d**2*x**2*sin(a + b*x)*cos(a + b*x)/(2*b)
- c*d*sin(a + b*x)**2/(2*b**2) - d**2*x*sin(a + b*x)**2/(4*b**2) + d**2*x*c
os(a + b*x)**2/(4*b**2) - d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)
), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cos(a)**2, True))
```

Giac [A]
time = 0.41, size = 94, normalized size = 0.99

$$\frac{1}{6}\,d^2x^3+\frac{1}{2}\,cdx^2+\frac{1}{2}\,c^2x+\frac{(bd^2x+bcd)\cos\left(2\,bx+2\,a\right)}{4\,b^3}+\frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\sin\left(2\,bx+2\,a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x + 1/4*(b*d^2*x + b*c*d)*cos(2*b*x + 2
*a)/b^3 + 1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*sin(2*b*x + 2
*a)/b^3
```

Mupad [B]
time = 0.21, size = 179, normalized size = 1.88

$$x\left(\frac{c^2}{4}-\frac{d^2}{8\,b^2}\right)+x\left(\frac{c^2}{4}+\frac{d^2}{8\,b^2}\right)+\frac{d^2\,x^3}{6}-\frac{\sin\left(2\,a+2\,b\,x\right)\,\left(d^2-2\,b^2\,c^2\right)}{8\,b^3}-\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{c^2}{2}-\frac{d^2}{4\,b^2}\right)}{2}+\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{c^2}{2}+\frac{d^2}{4\,b^2}\right)}{2}+\frac{c\,d\,x^2}{2}+\frac{d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{c\,d\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^2}+\frac{c\,d\,x\,\sin\left(2\,a+2\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x*(c^2/4 - d^2/(8*b^2)) + x*(c^2/4 + d^2/(8*b^2)) + (d^2*x^3)/6 - (sin(2*a
+ 2*b*x)*(d^2 - 2*b^2*c^2))/(8*b^3) - (x*cos(2*a + 2*b*x)*(c^2/2 - d^2/(4*b
^2)))/2 + (x*cos(2*a + 2*b*x)*(c^2/2 + d^2/(4*b^2)))/2 + (c*d*x^2)/2 + (d^2
*x^2*sin(2*a + 2*b*x))/(4*b) + (c*d*cos(2*a + 2*b*x))/(4*b^2) + (c*d*x*sin(
2*a + 2*b*x))/(2*b)
```

3.12 $\int (c+dx)\cos^2(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{cx}{2}+\frac{dx^2}{4}+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] 1/2*c*x+1/4*d*x^2+1/4*d*cos(b*x+a)^2/b^2+1/2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/
b
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3391}

$$\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2,x]
```

```
[Out] (c*x)/2 + (d*x^2)/4 + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/(2*b)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^2(a+bx)\,dx &= \frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}+\frac{1}{2}\int (c+dx)\,dx\\ &= \frac{cx}{2}+\frac{dx^2}{4}+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 50, normalized size = 0.91

$$\frac{d\cos(2(a+bx))+2b(2ac+bx(2c+dx)+(c+dx)\sin(2(a+bx)))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2,x]
```

```
[Out] (d*Cos[2*(a + b*x)] + 2*b*(2*a*c + b*x*(2*c + d*x) + (c + d*x)*Sin[2*(a + b
*x)]))/(8*b^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.
time = 0.06, size = 112, normalized size = 2.04

method	result
risch	$\frac{d\,x^2}{4}+\frac{cx}{2}+\frac{d\cos(2bx+2a)}{8b^2}+\frac{(dx+c)\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{-\frac{da\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{d\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\right.}{b}}{b}$
default	$\frac{-\frac{da\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{d\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\right.}{b}}{b}$
norman	$\frac{\frac{c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+cx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{cx}{2}+\frac{d\,x^2}{4}-\frac{c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{cx\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c*(1/2*cos(b*x+a)*s
in(b*x+a)+1/2*b*x+1/2*a)+1/b*d*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+
1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2))
```

Maxima [**A**]
time = 0.29, size = 90, normalized size = 1.64

$$\frac{2\,(2\,bx+2\,a+\sin\left(2\,bx+2\,a\right))c-\frac{2\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ad}{b}+\frac{\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)d}{b}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*b*x + 2*a + sin(2*b*x + 2*a))*c - 2*(2*b*x + 2*a + sin(2*b*x + 2*
a))*a*d/b + (2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
))*d/b)/b
```

Fricas [**A**]
time = 0.39, size = 53, normalized size = 0.96

$$\frac{b^2dx^2+2\,b^2cx+d\cos\left(bx+a\right)^2+2\,(bdx+bc)\cos\left(bx+a\right)\sin\left(bx+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^2 + 2*b^2*c*x + d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(b*x + a
)*sin(b*x + a))/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(49) = 98$.
time = 0.12, size = 126, normalized size = 2.29

$$\begin{cases} \frac{cx\sin^2(a+bx)}{2} + \frac{cx\cos^2(a+bx)}{2} + \frac{dx^2\sin^2(a+bx)}{4} + \frac{dx^2\cos^2(a+bx)}{4} + \frac{c\sin(a+bx)\cos(a+bx)}{2b} + \frac{dx\sin(a+bx)\cos(a+bx)}{2b} - \frac{d\sin^2(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**2/2 + c*x*cos(a + b*x)**2/2 + d*x**2*sin(a + b
*x)**2/4 + d*x**2*cos(a + b*x)**2/4 + c*sin(a + b*x)*cos(a + b*x)/(2*b) + d
*x*sin(a + b*x)*cos(a + b*x)/(2*b) - d*sin(a + b*x)**2/(4*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*cos(a)**2, True))
```

Giac [A]
time = 0.55, size = 48, normalized size = 0.87

$$\frac{1}{4}dx^2 + \frac{1}{2}cx + \frac{d\cos(2bx + 2a)}{8b^2} + \frac{(bdx + bc)\sin(2bx + 2a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*x^2 + 1/2*c*x + 1/8*d*cos(2*b*x + 2*a)/b^2 + 1/4*(b*d*x + b*c)*sin(2*
b*x + 2*a)/b^2
```

Mupad [B]
time = 0.10, size = 57, normalized size = 1.04

$$\frac{c\,x}{2} + \frac{d\,x^2}{4} + \frac{d\,\cos(2\,a + 2\,b\,x)}{8\,b^2} + \frac{c\,\sin(2\,a + 2\,b\,x)}{4\,b} + \frac{d\,x\,\sin(2\,a + 2\,b\,x)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c + d*x),x)
```

```
[Out] (c*x)/2 + (d*x^2)/4 + (d*cos(2*a + 2*b*x))/(8*b^2) + (c*sin(2*a + 2*b*x))/(
4*b) + (d*x*sin(2*a + 2*b*x))/(4*b)
```

3.13 $\int \frac{\cos^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=78

$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] 1/2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d+1/2*ln(d*x+c)/d-1/2*Si(2*b*c/d+2*b
*x)*sin(2*a-2*b*c/d)/d
```

Rubi [A]
time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3380, 3383}

$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]/
(2*d) - (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{2(c+dx)} + \frac{\cos(2a+2bx)}{2(c+dx)}\right) dx \\
&= \frac{\log(c+dx)}{2d} + \frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{2d} + \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx - \frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}\right.}{c+} \\
&= \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{\log(c+dx)}{2d} - \frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 65, normalized size = 0.83

$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)-\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - Sin[2
*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Maple [A]
time = 0.08, size = 114, normalized size = 1.46

method	result
risch	$\frac{\ln(dx+c)}{2d} - \frac{e^{-\frac{2i(da-bc)}{d}}\,\text{expIntegral}\left(1,2ibx+2ia-\frac{2i(da-bc)}{d}\right)}{4d} - \frac{e^{\frac{2i(da-bc)}{d}}\,\text{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{4d}$
derivativedivides	$\frac{\frac{b\left(-\frac{2\,\text{sinIntegral}\left(-2bx-2a-\frac{2(-da+bc)}{d}\right)\sin\left(\frac{-2da+2bc}{d}\right)}{d}+\frac{2\,\text{cosineIntegral}\left(2bx+2a+\frac{-2da+2bc}{d}\right)\cos\left(\frac{-2da+2bc}{d}\right)}{d}\right)}{4}+\frac{b\ln(-da+bc}{2d}}{b}$
default	$\frac{\frac{b\left(-\frac{2\,\text{sinIntegral}\left(-2bx-2a-\frac{2(-da+bc)}{d}\right)\sin\left(\frac{-2da+2bc}{d}\right)}{d}+\frac{2\,\text{cosineIntegral}\left(2bx+2a+\frac{-2da+2bc}{d}\right)\cos\left(\frac{-2da+2bc}{d}\right)}{d}\right)}{4}+\frac{b\ln(-da+bc}{2d}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

[Out] 1/b*(1/4*b*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b
*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)+1/2*b*ln(-d*a+b*c+d*(b*x+a))/
d)

Maxima [C] Result contains complex when optimal does not.
time = 0.38, size = 163, normalized size = 2.09

$$-\frac{b\left(E_1\left(\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,b\log\left(bc+(bx+a)d-ad\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] -1/4*(b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_inte
gral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) - b
*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integr
al_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) - 2*b
*log(b*c + (b*x + a)*d - a*d))/(b*d)

Fricas [A]
time = 0.38, size = 88, normalized size = 1.13

$$\frac{\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] 1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*c
os(-2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)
/d) + 2*log(d*x + c))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2/(d*x+c),x)

[Out] Integral(cos(a + b*x)**2/(c + d*x), x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 610, normalized size = 7.82

$$\frac{2\log\left(|dx+c|\right)\tan\left(a\right)^2\tan\left(\frac{bc}{d}\right)^2+\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^2\tan\left(\frac{bc}{d}\right)^2+\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^2\tan\left(\frac{bc}{d}\right)^2-2\,\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^2\tan\left(\frac{bc}{d}\right)+2\,\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^2\tan\left(\frac{bc}{d}\right)-4\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan\left(a\right)^2\tan\left(\frac{bc}{d}\right)+2\,\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)^2-2\,\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)^2+4\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)^2+2\log\left(|dx+c|\right)\tan\left(a\right)^2-\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^2-\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^2+4\,\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)+4\,\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)+2\log\left(|dx+c|\right)\tan\left(\frac{bc}{d}\right)^2-\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(\frac{bc}{d}\right)^2-\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(\frac{bc}{d}\right)^2-2\,\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)+2\,\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)-4\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan\left(a\right)+2\,\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(\frac{bc}{d}\right)-2\,\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(\frac{bc}{d}\right)+4\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan\left(\frac{bc}{d}\right)+2\log\left(|dx+c|\right)+\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)+\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)}{4\left(d\tan\left(a\right)^2\tan\left(\frac{bc}{d}\right)^2+d\tan\left(a\right)^2+d\tan\left(\frac{bc}{d}\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(d*x + c))*tan(a)^2*tan(b*c/d)^2 + real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)^2*tan(b*c/d) + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*ta
n(b*c/d) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) + 2*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)*tan(b*c/d)^2 + 2*log(abs(d*x + c))*tan(a)^2 - real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2 - real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 2*log(ab
s(d*x + c))*tan(b*c/d)^2 - real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c
/d)^2 - real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*imag_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(a) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a) + 2*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 2*
log(abs(d*x + c)) + real_part(cos_integral(2*b*x + 2*b*c/d)) + real_part(co
s_integral(-2*b*x - 2*b*c/d)))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*ta
n(b*c/d)^2 + d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x), x)
```

3.14 $\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\cos^2(a+bx)}{d(c+dx)}-\frac{b\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2}-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

```
[Out] -cos(b*x+a)^2/d/(d*x+c)-b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-b*Ci(2*b*c
/d+2*b*x)*sin(2*a-2*b*c/d)/d^2
```

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3394, 12, 3384, 3380, 3383}

$$-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] -(Cos[a + b*x]^2/(d*(c + d*x))) - (b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a
 - (2*b*c)/d])/d^2 - (b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x]
)/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx &= -\frac{\cos^2(a+bx)}{d(c+dx)} + \frac{(2b)\int -\frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d} \\
&= -\frac{\cos^2(a+bx)}{d(c+dx)} - \frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d} \\
&= -\frac{\cos^2(a+bx)}{d(c+dx)} - \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} - \frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,d}{d} \\
&= -\frac{\cos^2(a+bx)}{d(c+dx)} - \frac{b\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.41, size = 75, normalized size = 0.90

$$-\frac{\frac{d\cos^2(a+bx)}{c+dx} + b\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) + b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] -(((d*Cos[a + b*x]^2)/(c + d*x) + b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a
- (2*b*c)/d] + b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2)
```

Maple [A]
time = 0.12, size = 156, normalized size = 1.88

method	result
risch	$-\frac{1}{2d(dx+c)} + \frac{ib\,e^{-\frac{2i(da-bc)}{d}}\,\text{expIntegral}\left(1,2ibx+2ia-\frac{2i(da-bc)}{d}\right)}{2d^2} - \frac{ib\,e^{\frac{2i(da-bc)}{d}}\,\text{expIntegral}\left(1,-2ibx-2ia-\frac{2(-ia}{}\right.}{2d^2}$

derivativedivides	$\frac{\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-da+bc+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-da+bc)}{d}\right)\cos\left(\frac{-2da+2bc}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2bx+2a+\frac{-2da+2bc}{d}\right)\mathrm{si}}{d}\right.}{d}\right.}{4}}{b}$
default	$\frac{\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-da+bc+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-da+bc)}{d}\right)\cos\left(\frac{-2da+2bc}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2bx+2a+\frac{-2da+2bc}{d}\right)\mathrm{si}}{d}\right.}{d}\right.}{4}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*b^2*(-2*cos(2*b*x+2*a)/(-d*a+b*c+d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-
2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*
(-a*d+b*c)/d)/d)/d)-1/2*b^2/(-d*a+b*c+d*(b*x+a))/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.38, size = 171, normalized size = 2.06

$$-\frac{b^2\left(E_2\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_2\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)-i\,E_2\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+2\,b^2}{4\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) +
 b^2*(I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_int
egral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) +
2*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A]
time = 0.38, size = 127, normalized size = 1.53

$$-\frac{2\,d\cos\left(bx+a\right)^2+2\,(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)}{2\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integr
al(2*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b
*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^3*x
 + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(83) = 166.
time = 0.50, size = 534, normalized size = 6.43

$$-\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\,\mathrm{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\,b^3c\,\mathrm{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,ab^2d\,\mathrm{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)-2\,b^3c\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)+2\,ab^2d\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)+b^2d\cos\left(-\frac{2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)+b^2d\right)d^2}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(2*((
d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c -
a*d)/d) + 2*b^3*c*cos_integral(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
 c)) + b*c - a*d)/d)*sin(-2*(b*c - a*d)/d) - 2*a*b^2*d*cos_integral(2*((d*x
 + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c - a*d
)/d) - 2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-2*(b*c - a*
d)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d) - 2*b^3*c*cos(-2*(b*c - a*d)/d)*sin_integral(-2*((d*x + c)*(b - b
*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 2*a*b^2*d*cos(-2*(b*c - a*d
)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
 a*d)/d) + b^2*d*cos(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) +
b^2*d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 -
a*d^5)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^2, x)
```

3.15 $\int \frac{\cos^2(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=112

$$-\frac{\cos^2(a+bx)}{2d(c+dx)^2}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)}+\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b}{d}\right.}{d^3}$$

```
[Out] -b^2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^3-1/2*cos(b*x+a)^2/d/(d*x+c)^2+b^
2*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3+b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c
)
```

Rubi [A]

time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3395, 31, 3393, 3384, 3380, 3383}

$$-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\cos^2(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] -1/2*Cos[a + b*x]^2/(d*(c + d*x)^2) - (b^2*Cos[2*a - (2*b*c)/d]*CosIntegral
[(2*b*c)/d + 2*b*x])/d^3 + (b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*x)) +
(b^2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2} - \frac{(2b^2)\int \frac{\cos^2(a+bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\log(c+dx)}{d^3} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{(2b^2)\int \left(\frac{1}{2(c+dx)} + \frac{\cos(2a+\ldots}{2(c+\ldots}\right.}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{b^2\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} + \frac{(b^2\ldots}{}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{b^2\sin(\ldots}{}$$

Mathematica [A]
time = 0.64, size = 102, normalized size = 0.91

$$\frac{-2b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \frac{d\left(-d\cos^2(a+bx)+b(c+dx)\sin(2(a+bx))\right)}{(c+dx)^2} + 2b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (-2*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(-(d*Cos[a
 + b*x]^2) + b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sin[2*a - (
2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [A]
time = 0.20, size = 193, normalized size = 1.72

method	result
derivativedivides	$\frac{\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-da+bc+d(bx+a))^2 d}-\frac{-\frac{2\sin(2bx+2a)}{(-da+bc+d(bx+a))d}+\frac{-\frac{4\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-da+bc)}{d}\right)\sin\left(\frac{-2da+2bc}{d}\right)}{d}+\frac{4\,\mathrm{cosineIntegral}(\cdots}{d}}{d}}{d}\right.}{4}}{b}$
default	$\frac{\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-da+bc+d(bx+a))^2 d}-\frac{-\frac{2\sin(2bx+2a)}{(-da+bc+d(bx+a))d}+\frac{-\frac{4\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-da+bc)}{d}\right)\sin\left(\frac{-2da+2bc}{d}\right)}{d}+\frac{4\,\mathrm{cosineIntegral}(\cdots}{d}}{d}}{d}\right.}{4}}{b}$
risch	$-\frac{1}{4d(dx+c)^2}+\frac{b^2 \mathrm{e}^{-\frac{2i(da-bc)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(da-bc)}{d}\right)}{2d^3}+\frac{b^2 \mathrm{e}^{\frac{2i(da-bc)}{d}}\,\mathrm{expIntegral}\left(1,-2ibx-2ia-\frac{2(\cdots}{}\right.}{2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*b^3*(-cos(2*b*x+2*a)/(-d*a+b*c+d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(
-d*a+b*c+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/
d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)-1/4*b^3/(-
d*a+b*c+d*(b*x+a))^2/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.42, size = 204, normalized size = 1.82

$$-\frac{b^3\left(E_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(iE_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)-iE_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+b^3}{4\left(b^2c^2d-2abcd^2+(bx+a)^2d^3+a^2d^3+2(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) +
 b^3*(I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_int
egral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) +
b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*b)
```

Fricas [A]
time = 0.37, size = 218, normalized size = 1.95

$$-\frac{d^{2}\cos\left(bx+a\right)^{2}-2\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-2\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)}{2\left(d^{5}x^{2}+2\,cd^{4}x+c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) -
2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integral(
2*(b*d*x + b*c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*
(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b
*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(a+bx\right)}{\left(c+dx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 5136, normalized size = 45.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x
^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2
- 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)
*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_in
```

```
tegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b^2*d^2*x^2*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*d^2*x^2*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^
2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*
c/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*t
an(a)^2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^2*c*d*x*sin_integr
al(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*c^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^
2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
) + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^
2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^
2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b
^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c
/d) + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan
(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*
tan(a)*tan(b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
a)^2*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integr
al(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b^2*c^2*imag_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*si
n_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*
x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b^2*d^2*x^2*real
```

```
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^2*c*d*x*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^2*c*d*x*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^2*c*d*x*sin_integral(2*
(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b^2*d^2*x^2*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)^2 - b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2
*tan(a)^2 - b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*ta
n(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2
*tan(b*c/d) + 8*b^2*c*d*x*sin_integral(2*(b*d*x...
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^3, x)
```

3.16 $\int (c+dx)^4 \cos^3(a+bx)\,dx$

Optimal. Leaf size=225

$$-\frac{160d^3(c+dx)\cos(a+bx)}{9b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}$$

```
[Out] -160/9*d^3*(d*x+c)*cos(b*x+a)/b^4+8/3*d*(d*x+c)^3*cos(b*x+a)/b^2-8/27*d^3*(
d*x+c)*cos(b*x+a)^3/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)^3/b^2+488/27*d^4*sin(b*x
+a)/b^5-80/9*d^2*(d*x+c)^2*sin(b*x+a)/b^3+2/3*(d*x+c)^4*sin(b*x+a)/b-4/9*d^
2*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b^3+1/3*(d*x+c)^4*cos(b*x+a)^2*sin(b*x+
a)/b-8/81*d^4*sin(b*x+a)^3/b^5
```

Rubi [A]
time = 0.16, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2717, 2713}

$$-\frac{8d^4\sin^3(a+bx)}{81b^5}+\frac{488d^4\sin(a+bx)}{27b^5}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\cos(a+bx)}{9b^4}-\frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3}-\frac{4d^2(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{9b^3}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}+\frac{2(c+dx)^4\sin(a+bx)}{3b}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3,x]
```

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])/
(3*b^2) - (8*d^3*(c + d*x)*Cos[a + b*x]^3)/(27*b^4) + (4*d*(c + d*x)^3*Cos[
a + b*x]^3)/(9*b^2) + (488*d^4*Sin[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2
*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^4*Sin[a + b*x])/(3*b) - (4*d^2*(c + d
*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^3) + ((c + d*x)^4*Cos[a + b*x]^2*Si
n[a + b*x])/(3*b) - (8*d^4*Sin[a + b*x]^3)/(81*b^5)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4\cos^3(a+bx)\,dx &= \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b}+\frac{2}{3}\int (c+dx)^{4} \\
&= -\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2(c+dx)^4\sin(a+b \\
&= \frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}+\frac{4d(c+dx)^3\cos^3(a+b \\
&= -\frac{16d^3(c+dx)\cos(a+bx)}{9b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a+ \\
&= -\frac{160d^3(c+dx)\cos(a+bx)}{9b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a- \\
&= -\frac{160d^3(c+dx)\cos(a+bx)}{9b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a-
\end{aligned}$$

Mathematica [A]
time = 0.59, size = 385, normalized size = 1.71

$$\frac{972bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)+12bd(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))+243b^4c^4\sin(a+bx)-2916b^2c^2d^2\sin(a+bx)+5832d^4\sin(a+bx)+972b^4c^3dx\sin(a+bx)-5832b^2cd^3x\sin(a+bx)+1458b^4c^2d^2x^2\sin(a+bx)-2916b^2d^4x^2\sin(a+bx)+972b^4cd^3x^3\sin(a+bx)+243b^4d^4x^4\sin(a+bx)+27b^4c^4\sin(3(a+bx))-36b^2c^2d^2\sin(3(a+bx))+8d^4\sin(3(a+bx))+108b^4c^3dx\sin(3(a+bx))-72b^2cd^3x\sin(3(a+bx))+162b^4c^2d^2x^2\sin(3(a+bx))-36b^2d^4x^2\sin(3(a+bx))+108b^4cd^3x^3\sin(3(a+bx))+27b^4d^4x^4\sin(3(a+bx))}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3,x]
```

```
[Out] (972*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 12*b*d*(c + d*
x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 243*b^4*c^4*Sin[a + b*x]
 - 2916*b^2*c^2*d^2*Sin[a + b*x] + 5832*d^4*Sin[a + b*x] + 972*b^4*c^3*d*x*
Sin[a + b*x] - 5832*b^2*c*d^3*x*Sin[a + b*x] + 1458*b^4*c^2*d^2*x^2*Sin[a +
 b*x] - 2916*b^2*d^4*x^2*Sin[a + b*x] + 972*b^4*c*d^3*x^3*Sin[a + b*x] + 24
3*b^4*d^4*x^4*Sin[a + b*x] + 27*b^4*c^4*Sin[3*(a + b*x)] - 36*b^2*c^2*d^2*S
in[3*(a + b*x)] + 8*d^4*Sin[3*(a + b*x)] + 108*b^4*c^3*d*x*Sin[3*(a + b*x)]
 - 72*b^2*c*d^3*x*Sin[3*(a + b*x)] + 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)] -
 36*b^2*d^4*x^2*Sin[3*(a + b*x)] + 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] + 27*
b^4*d^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(205) = 410$.
time = 0.18, size = 1023, normalized size = 4.55

method	result
risch	$\frac{3d(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\cos(bx+a)}{b^4} + \frac{3(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+c^4b^4-}{4b}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/b^4*a^4*d^4*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3/b^3*a^3*c*d^3*(2+cos(b
*x+a)^2)*sin(b*x+a)-4/b^4*a^3*d^4*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+
1/9*cos(b*x+a)^3+2/3*cos(b*x+a))+2/b^2*a^2*c^2*d^2*(2+cos(b*x+a)^2)*sin(b*x
+a)+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a
)^3+2/3*cos(b*x+a))+6/b^4*a^2*d^4*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a
)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+co
s(b*x+a)^2)*sin(b*x+a))-4/3/b*a*c^3*d*(2+cos(b*x+a)^2)*sin(b*x+a)-12/b^2*a*
c^2*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos(b
*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*
x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*
sin(b*x+a))-4/b^4*a*d^4*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2*(b*x+a
)^2*cos(b*x+a)-40/9*cos(b*x+a)-4*(b*x+a)*sin(b*x+a)+1/3*(b*x+a)^2*cos(b*x+a
)^3-2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)-2/27*cos(b*x+a)^3)+1/3*c^4*(2+c
os(b*x+a)^2)*sin(b*x+a)+4/b*c^3*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+
1/9*cos(b*x+a)^3+2/3*cos(b*x+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)
^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)
^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+4/b^3*c*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+
a)^2)*sin(b*x+a)+2*(b*x+a)^2*cos(b*x+a)-40/9*cos(b*x+a)-4*(b*x+a)*sin(b*x+a
)+1/3*(b*x+a)^2*cos(b*x+a)^3-2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)-2/27*c
os(b*x+a)^3)+1/b^4*d^4*(1/3*(b*x+a)^4*(2+cos(b*x+a)^2)*sin(b*x+a)+8/3*(b*x+
a)^3*cos(b*x+a)-8*(b*x+a)^2*sin(b*x+a)+160/9*sin(b*x+a)-160/9*(b*x+a)*cos(b
*x+a)+4/9*(b*x+a)^3*cos(b*x+a)^3-4/9*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-
8/27*(b*x+a)*cos(b*x+a)^3+8/81*(2+cos(b*x+a)^2)*sin(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(205) = 410$.
time = 0.34, size = 925, normalized size = 4.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/324*(108*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^4 - 432*(sin(b*x + a)^3 - 3
*sin(b*x + a))*a*c^3*d/b + 648*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*c^2*d^
2/b^2 - 432*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^3*c*d^3/b^3 + 108*(sin(b*x
+ a)^3 - 3*sin(b*x + a))*a^4*d^4/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) + 2
7*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*c^3*d/b + 10
8*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3
*a) + 27*cos(b*x + a))*a*c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) +
27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a^2*c*d^3/b
^3 + 36*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b
*x + 3*a) + 27*cos(b*x + a))*a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a)
 + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*(
(b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*
a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81
*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x +
3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) +
81*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*co
s(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*
b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*
c*d^3/b^3 + 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 243*((b*x + a)^2 - 2
)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*
x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) + 972*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) +
(27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*sin(3*b*x + 3*a) + 243*((b*x + a)^4 -
 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A]
time $= 0.39$, size $= 350$, normalized size $= 1.56$

$$\frac{12\,(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 3\,b^3c^3d - 2\,bcd^3 + (9\,b^3c^2d^2 - 2\,bd^4)x)\cos(bx+a)^3 + 72\,(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 3\,b^3c^3d - 20\,bcd^3 + (9\,b^3c^2d^2 - 20\,bd^4)x)\cos(bx+a) + (54\,b^4d^4x^4 + 216\,b^4cd^3x^3 + 54\,b^4c^4 - 720\,b^2c^2d^2 + 1456\,d^4 + 36\,(9\,b^4c^2d^2 - 20\,b^2d^4)x^2 + (27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 27\,b^4c^4 - 36\,b^2c^2d^2 + 8\,d^4 + 18\,(9\,b^4c^2d^2 - 2\,b^2d^4)x^2 + 36\,(3\,b^4c^3d - 2\,b^2cd^3)x)\cos(bx+a)^2 + 72\,(3\,b^4c^3d - 20\,b^2cd^3)x)\sin(bx+a)}{81\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b^
3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 + 72*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^
2 + 3*b^3*c^3*d - 20*b*c*d^3 + (9*b^3*c^2*d^2 - 20*b*d^4)*x)*cos(b*x + a) +
 (54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 720*b^2*c^2*d^2 + 1456*
d^4 + 36*(9*b^4*c^2*d^2 - 20*b^2*d^4)*x^2 + (27*b^4*d^4*x^4 + 108*b^4*c*d^3
*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)
*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 72*(3*b^4*c^3*d -
 20*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.
time $= 0.78$, size $= 772$, normalized size $= 3.43$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**4*sin(a + b*x)**3/(3*b) + c**4*sin(a + b*x)*cos(a + b*x)**2
/b + 8*c**3*d*x*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)*cos(a + b*x
)**2/b + 4*c**2*d**2*x**2*sin(a + b*x)**3/b + 6*c**2*d**2*x**2*sin(a + b*x)
*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**3/(3*b) + 4*c*d**3*x**3*si
n(a + b*x)*cos(a + b*x)**2/b + 2*d**4*x**4*sin(a + b*x)**3/(3*b) + d**4*x**
4*sin(a + b*x)*cos(a + b*x)**2/b + 8*c**3*d*sin(a + b*x)**2*cos(a + b*x)/(3
*b**2) + 28*c**3*d*cos(a + b*x)**3/(9*b**2) + 8*c**2*d**2*x*sin(a + b*x)**2
*cos(a + b*x)/b**2 + 28*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 8*c*d**3*x**
2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 28*c*d**3*x**2*cos(a + b*x)**3/(3*b**
2) + 8*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 28*d**4*x**3*cos(a
 + b*x)**3/(9*b**2) - 80*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 28*c**2*d**2*
sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 160*c*d**3*x*sin(a + b*x)**3/(9*b**
3) - 56*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 80*d**4*x**2*sin(a
 + b*x)**3/(9*b**3) - 28*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) -
160*c*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 488*c*d**3*cos(a + b*x)*
*3/(27*b**4) - 160*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 488*d**4*
x*cos(a + b*x)**3/(27*b**4) + 1456*d**4*sin(a + b*x)**3/(81*b**5) + 488*d**
4*sin(a + b*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x*
*2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a)**3, True))
```

Giac [A]
time = 0.50, size = 351, normalized size = 1.56

$$\frac{(3b^3d^4x^3+9b^3cd^3x^2+9b^3c^2d^2x+3b^3c^3d-2bd^4x-2bcd^3)\cos(3bx+3a)}{27b^5}+\frac{3(b^3d^4x^3+3b^3cd^3x^2+3b^3c^2d^2x+b^3c^3d-6bd^4x-6bcd^3)\cos(bx+a)}{b^5}+\frac{(27b^4d^4x^4+108b^4cd^3x^3+162b^4c^2d^2x^2+108b^4c^3dx+27b^4c^4-36b^2d^4x^2-72b^2cd^3x-36b^2c^2d^2+8d^4)\sin(3bx+3a)}{324b^5}+\frac{3(b^4d^4x^4+4b^4cd^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2cd^3x-12b^2c^2d^2+24d^4)\sin(bx+a)}{4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b
*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + 3*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 +
1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c
^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*
d^4)*sin(3*b*x + 3*a)/b^5 + 3/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*
d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^
2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

Mupad [B]
time = 1.14, size = 532, normalized size = 2.36

$$\frac{2\sin(a+bx)^3(27b^4c^4-360b^2c^2d^2+728d^4)}{81b^5}-\frac{4\cos(a+bx)^3(122cd^3-21b^2c^3d)}{27b^4}+\frac{\cos(a+bx)^2\sin(a+bx)(27b^4c^4-252b^2c^2d^2+488d^4)}{27b^5}-\frac{8\cos(a+bx)\sin(a+bx)^2(20cd^3-3b^2c^3d)}{9b^4}+\frac{28d^4x^3\cos(a+bx)^3}{9b^2}-\frac{4x\cos(a+bx)^3(122d^4-63b^2c^2d^2)}{27b^4}+\frac{2d^4x^4\sin(a+bx)^3}{3b}-\frac{8x\sin(a+bx)^3(20cd^3-3b^2c^3d)}{9b^3}-\frac{4x^2\sin(a+bx)^3(20d^4-9b^2c^2d^2)}{9b^3}-\frac{2x^2\cos(a+bx)^2\sin(a+bx)(14d^4-9b^2c^2d^2)}{3b^3}+\frac{28cd^3x^2\cos(a+bx)^3}{3b^2}+\frac{d^4x^4\cos(a+bx)^2\sin(a+bx)}{b}+\frac{8d^4x^3\cos(a+bx)\sin(a+bx)^2}{3b^2}+\frac{8cd^3x^3\sin(a+bx)^3}{3b}-\frac{8x\cos(a+bx)\sin(a+bx)^2(20d^4-9b^2c^2d^2)}{9b^4}-\frac{4x\cos(a+bx)^2\sin(a+bx)(14cd^3-3b^2c^3d)}{3b^3}+\frac{4cd^3x^3\cos(a+bx)^2\sin(a+bx)}{b}+\frac{8cd^3x^2\cos(a+bx)\sin(a+bx)^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] (2*sin(a + b*x)^3*(728*d^4 + 27*b^4*c^4 - 360*b^2*c^2*d^2))/(81*b^5) - (4*c
os(a + b*x)^3*(122*c*d^3 - 21*b^2*c^3*d))/(27*b^4) + (cos(a + b*x)^2*sin(a
+ b*x)*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(27*b^5) - (8*cos(a + b*x)
*sin(a + b*x)^2*(20*c*d^3 - 3*b^2*c^3*d))/(9*b^4) + (28*d^4*x^3*cos(a + b*x
)^3)/(9*b^2) - (4*x*cos(a + b*x)^3*(122*d^4 - 63*b^2*c^2*d^2))/(27*b^4) + (
2*d^4*x^4*sin(a + b*x)^3)/(3*b) - (8*x*sin(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3
*d))/(9*b^3) - (4*x^2*sin(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(9*b^3) - (2
*x^2*cos(a + b*x)^2*sin(a + b*x)*(14*d^4 - 9*b^2*c^2*d^2))/(3*b^3) + (28*c*
d^3*x^2*cos(a + b*x)^3)/(3*b^2) + (d^4*x^4*cos(a + b*x)^2*sin(a + b*x))/b +
 (8*d^4*x^3*cos(a + b*x)*sin(a + b*x)^2)/(3*b^2) + (8*c*d^3*x^3*sin(a + b*x
)^3)/(3*b) - (8*x*cos(a + b*x)*sin(a + b*x)^2*(20*d^4 - 9*b^2*c^2*d^2))/(9*
b^4) - (4*x*cos(a + b*x)^2*sin(a + b*x)*(14*c*d^3 - 3*b^2*c^3*d))/(3*b^3) +
 (4*c*d^3*x^3*cos(a + b*x)^2*sin(a + b*x))/b + (8*c*d^3*x^2*cos(a + b*x)*si
n(a + b*x)^2)/b^2
```

3.17 $\int (c+dx)^3 \cos^3(a+bx)\,dx$

Optimal. Leaf size=175

$$-\frac{40d^3\cos(a+bx)}{9b^4}+\frac{2d(c+dx)^2\cos(a+bx)}{b^2}-\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}-\frac{40d^2(c+dx)\sin}{9b^3}$$

```
[Out] -40/9*d^3*cos(b*x+a)/b^4+2*d*(d*x+c)^2*cos(b*x+a)/b^2-2/27*d^3*cos(b*x+a)^3
/b^4+1/3*d*(d*x+c)^2*cos(b*x+a)^3/b^2-40/9*d^2*(d*x+c)*sin(b*x+a)/b^3+2/3*(
d*x+c)^3*sin(b*x+a)/b-2/9*d^2*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^3+1/3*(d*x+
c)^3*cos(b*x+a)^2*sin(b*x+a)/b
```

Rubi [A]
time = 0.11, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2718, 3391}

$$-\frac{2d^3\cos^3(a+bx)}{27b^4}-\frac{40d^3\cos(a+bx)}{9b^4}-\frac{40d^2(c+dx)\sin(a+bx)}{9b^3}-\frac{2d^2(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^3}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\cos(a+bx)}{b^2}+\frac{2(c+dx)^3\sin(a+bx)}{3b}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3,x]
```

```
[Out] (-40*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/b^2 - (2*d^
3*Cos[a + b*x]^3)/(27*b^4) + (d*(c + d*x)^2*Cos[a + b*x]^3)/(3*b^2) - (40*d
^2*(c + d*x)*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^3*Sin[a + b*x])/(3*b) - (
2*d^2*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^3) + ((c + d*x)^3*Cos[a +
 b*x]^2*Sin[a + b*x])/(3*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cos^3(a+bx)\,dx &= \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b}+\frac{2}{3}\int (c+dx)^3 \\ &= -\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2(c+dx)^3\sin(a+bx)}{3b}-\frac{2d^2(}{} \\ &= \frac{2d(c+dx)^2\cos(a+bx)}{b^2}-\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}-\frac{4d^2(}{} \\ &= -\frac{4d^3\cos(a+bx)}{9b^4}+\frac{2d(c+dx)^2\cos(a+bx)}{b^2}-\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)^2}{3} \\ &= -\frac{40d^3\cos(a+bx)}{9b^4}+\frac{2d(c+dx)^2\cos(a+bx)}{b^2}-\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)}{}\end{aligned}$$

Mathematica [**A**]
time = 0.61, size = 121, normalized size = 0.69

$$\frac{243d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+6b(c+dx)\left(-82d^2+15b^2(c+dx)^2+(-2d^2+3b^2(c+dx)^2)\cos(2(a+bx))\right)\sin(a+bx)}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3,x]
```

```
[Out] (243*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(c + d*x
)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d
^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(161) = 322$.
time = 0.18, size = 560, normalized size = 3.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/b^3*a^3*d^3*(2+cos(b*x+a)^2)*sin(b*x+a)+1/b^2*a^2*c*d^2*(2+cos(b*
x+a)^2)*sin(b*x+a)+3/b^3*a^2*d^3*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1
```

```
/9*cos(b*x+a)^3+2/3*cos(b*x+a))-1/b*a*c^2*d*(2+cos(b*x+a)^2)*sin(b*x+a)-6/b
^2*a*c*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*co
s(b*x+a))-3/b^3*a*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*
x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*
sin(b*x+a))+1/3*c^3*(2+cos(b*x+a)^2)*sin(b*x+a)+3/b*c^2*d*(1/3*(b*x+a)*(2+c
os(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos(b*x+a))+3/b^2*c*d^2*(1/3*(
b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+
2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+1/b^3*d^3*(1/3*(
b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2*(b*x+a)^2*cos(b*x+a)-40/9*cos(b*x+a)
-4*(b*x+a)*sin(b*x+a)+1/3*(b*x+a)^2*cos(b*x+a)^3-2/9*(b*x+a)*(2+cos(b*x+a)^
2)*sin(b*x+a)-2/27*cos(b*x+a)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(161) = 322$.
time = 0.32, size = 535, normalized size = 3.06

$$-\frac{36\left(\sin(bx+a)^3-3\sin(bx+a)\right)c^3-\frac{108\left(\sin(bx+a)^3-3\sin(bx+a)\right)ac^2d}{b}+\frac{108\left(\sin(bx+a)^3-3\sin(bx+a)\right)a^2cd^2}{b^2}-\frac{36\left(\sin(bx+a)^3-3\sin(bx+a)\right)a^3d^3}{b^3}-\frac{9\left(3(bx+a)\sin(3bx+3a)+27(bx+a)\sin(bx+a)+\cos(3bx+3a)+27\cos(bx+a)\right)c^2d}{b}+\frac{18\left(3(bx+a)\sin(3bx+3a)+27(bx+a)\sin(bx+a)+\cos(3bx+3a)+27\cos(bx+a)\right)acd^2}{b^2}-\frac{9\left(3(bx+a)\sin(3bx+3a)+27(bx+a)\sin(bx+a)+\cos(3bx+3a)+27\cos(bx+a)\right)a^2d^3}{b^3}-\frac{3\left(6(bx+a)\cos(3bx+3a)+162(bx+a)\cos(bx+a)+\left(9(bx+a)^2-2\right)\sin(3bx+3a)+81\left((bx+a)^2-2\right)\sin(bx+a)\right)cd^2}{b^2}+\frac{3\left(6(bx+a)\cos(3bx+3a)+162(bx+a)\cos(bx+a)+\left(9(bx+a)^2-2\right)\sin(3bx+3a)+81\left((bx+a)^2-2\right)\sin(bx+a)\right)ad^3}{b^3}-\frac{\left(\left(9(bx+a)^2-2\right)\cos(3bx+3a)+243\left((bx+a)^2-2\right)\cos(bx+a)+3\left(3(bx+a)^3-2bx-2a\right)\sin(3bx+3a)+81\left((bx+a)^3-6bx-6a\right)\sin(bx+a)\right)d^3}{b^3}}{108\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(36*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^3 - 108*(sin(b*x + a)^3 - 3*
sin(b*x + a))*a*c^2*d/b + 108*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*c*d^2/b
^2 - 36*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^3*d^3/b^3 - 9*(3*(b*x + a)*sin(
3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x +
a))*c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) + 27*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*
x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*
a^2*d^3/b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a)
+ (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))
*c*d^2/b^2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) +
 (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*
a*d^3/b^3 - ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*c
os(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A]
time = 0.38, size = 227, normalized size = 1.30

$$\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3+6\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-20\,d^3\right)\cos\left(bx+a\right)+3\left(6\,b^3d^3x^3+18\,b^3cd^2x^2+6\,b^3c^3-40\,bcd^2+\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+\left(9\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^2+2\left(9\,b^3c^2d-20\,bd^3\right)x\right)\sin\left(bx+a\right)}{27\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^3
 + 6*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 20*d^3)*cos(b*x + a) +
```

```
3*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - 40*b*c*d^2 + (3*b^3*d^3*
x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)*x)*
cos(b*x + a)^2 + 2*(9*b^3*c^2*d - 20*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.
time = 0.53, size = 495, normalized size = 2.83

$$\begin{cases} \frac{2c^3\sin^3(a+bx)}{3b} + \frac{c^3\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2c^2dx\sin^3(a+bx)}{b} + \frac{3c^2dx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2cd^2x^2\sin^3(a+bx)}{b} + \frac{3cd^2x^2\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2d^3x^3\sin^3(a+bx)}{3b} + \frac{d^3x^3\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2c^2d\sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{7c^2d\cos^3(a+bx)}{3b^2} + \frac{4cd^2x\sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{14cd^2x\cos^3(a+bx)}{3b^2} + \frac{2d^3x^2\sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{7d^3x^2\cos^3(a+bx)}{3b^2} - \frac{40cd^2\sin^3(a+bx)}{9b^3} - \frac{14cd^2\sin(a+bx)\cos^2(a+bx)}{3b^3} - \frac{40d^3x\sin^3(a+bx)}{9b^3} - \frac{14d^3x\sin(a+bx)\cos^2(a+bx)}{3b^3} - \frac{40d^3\sin^2(a+bx)\cos(a+bx)}{9b^4} - \frac{122d^3\cos^3(a+bx)}{27b^4} & \text{for}\ b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**3*sin(a + b*x)**3/(3*b) + c**3*sin(a + b*x)*cos(a + b*x)**2
/b + 2*c**2*d*x*sin(a + b*x)**3/b + 3*c**2*d*x*sin(a + b*x)*cos(a + b*x)**2
/b + 2*c*d**2*x**2*sin(a + b*x)**3/b + 3*c*d**2*x**2*sin(a + b*x)*cos(a + b
*x)**2/b + 2*d**3*x**3*sin(a + b*x)**3/(3*b) + d**3*x**3*sin(a + b*x)*cos(a
 + b*x)**2/b + 2*c**2*d*sin(a + b*x)**2*cos(a + b*x)/b**2 + 7*c**2*d*cos(a
+ b*x)**3/(3*b**2) + 4*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)/b**2 + 14*c*d*
*2*x*cos(a + b*x)**3/(3*b**2) + 2*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b*
*2 + 7*d**3*x**2*cos(a + b*x)**3/(3*b**2) - 40*c*d**2*sin(a + b*x)**3/(9*b*
*3) - 14*c*d**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 40*d**3*x*sin(a + b
*x)**3/(9*b**3) - 14*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 40*d**3
*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 122*d**3*cos(a + b*x)**3/(27*b**4)
, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a)
**3, True))
```

Giac [A]
time = 0.48, size = 231, normalized size = 1.32

$$\frac{(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\cos(3bx + 3a)}{108b^4} + \frac{9(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cos(bx + a)}{4b^4} + \frac{(3b^3d^3x^3 + 9b^3cd^2x^2 + 9b^3c^2dx + 3b^3c^3 - 2bd^3x - 2bcd^2)\sin(3bx + 3a)}{36b^4} + \frac{3(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3 - 6bd^3x - 6bcd^2)\sin(bx + a)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3*
a)/b^4 + 9/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)
/b^4 + 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 -
2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 3/4*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

Mupad [B]
time = 0.72, size = 364, normalized size = 2.08

$$\frac{7d^3x^2\cos(a+bx)^3}{3b^2} - \frac{2\sin(a+bx)^3(20cd^2 - 3b^2c^3)}{9b^3} - \frac{\cos(a+bx)^2\sin(a+bx)\,(14cd^2 - 3b^2c^3)}{3b^3} - \frac{2\cos(a+bx)\sin(a+bx)^2(20d^3 - 9b^2c^2d)}{9b^4} - \frac{2x\sin(a+bx)^3(20d^3 - 9b^2c^2d)}{9b^3} - \frac{\cos(a+bx)^3(122d^3 - 63b^2c^2d)}{27b^4} + \frac{2d^3x^3\sin(a+bx)^3}{3b} + \frac{14cd^2x\cos(a+bx)^3}{3b^2} - \frac{x\cos(a+bx)^2\sin(a+bx)\,(14d^3 - 9b^2c^2d)}{3b^3} + \frac{d^3x^3\cos(a+bx)^2\sin(a+bx)}{b} + \frac{2d^3x^2\cos(a+bx)\sin(a+bx)^2}{b^2} + \frac{2cd^2x^2\sin(a+bx)^3}{b} + \frac{3cd^2x^2\cos(a+bx)^2\sin(a+bx)}{b} + \frac{4cd^2x\cos(a+bx)\sin(a+bx)^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] (7*d^3*x^2*cos(a + b*x)^3)/(3*b^2) - (2*sin(a + b*x)^3*(20*c*d^2 - 3*b^2*c^
3))/(9*b^3) - (cos(a + b*x)^2*sin(a + b*x)*(14*c*d^2 - 3*b^2*c^3))/(3*b^3)
- (2*cos(a + b*x)*sin(a + b*x)^2*(20*d^3 - 9*b^2*c^2*d))/(9*b^4) - (2*x*sin
(a + b*x)^3*(20*d^3 - 9*b^2*c^2*d))/(9*b^3) - (cos(a + b*x)^3*(122*d^3 - 63
*b^2*c^2*d))/(27*b^4) + (2*d^3*x^3*sin(a + b*x)^3)/(3*b) + (14*c*d^2*x*cos(
a + b*x)^3)/(3*b^2) - (x*cos(a + b*x)^2*sin(a + b*x)*(14*d^3 - 9*b^2*c^2*d)
)/(3*b^3) + (d^3*x^3*cos(a + b*x)^2*sin(a + b*x))/b + (2*d^3*x^2*cos(a + b*
x)*sin(a + b*x)^2)/b^2 + (2*c*d^2*x^2*sin(a + b*x)^3)/b + (3*c*d^2*x^2*cos(
a + b*x)^2*sin(a + b*x))/b + (4*c*d^2*x*cos(a + b*x)*sin(a + b*x)^2)/b^2
```

3.18 $\int (c+dx)^2 \cos^3(a+bx)\,dx$

Optimal. Leaf size=123

$$\frac{4d(c+dx)\cos(a+bx)}{3b^2}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}-\frac{14d^2\sin(a+bx)}{9b^3}+\frac{2(c+dx)^2\sin(a+bx)}{3b}+\frac{(c+dx)^2\cos^2(}{}$$

```
[Out] 4/3*d*(d*x+c)*cos(b*x+a)/b^2+2/9*d*(d*x+c)*cos(b*x+a)^3/b^2-14/9*d^2*sin(b*
x+a)/b^3+2/3*(d*x+c)^2*sin(b*x+a)/b+1/3*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b
+2/27*d^2*sin(b*x+a)^3/b^3
```

Rubi [A]

time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2717, 2713}

$$\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{14d^2\sin(a+bx)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{4d(c+dx)\cos(a+bx)}{3b^2}+\frac{2(c+dx)^2\sin(a+bx)}{3b}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3,x]
```

```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(3*b^2) + (2*d*(c + d*x)*Cos[a + b*x]^3)/(9*b^
2) - (14*d^2*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^2*Sin[a + b*x])/(3*b) + (
(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*d^2*Sin[a + b*x]^3)/(27
*b^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^2\cos^3(a+bx)\,dx = \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} + \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2}{3}\int (c+dx)$$

$$= \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} + \frac{2(c+dx)^2\sin(a+bx)}{3b} + \frac{(c+dx)^2\cos^2(a+bx)\,\mathrm{si}}{3b}$$

$$= \frac{4d(c+dx)\cos(a+bx)}{3b^2} + \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} - \frac{2d^2\sin(a+bx)}{9b^3} + \frac{2(c+}{}$$

$$= \frac{4d(c+dx)\cos(a+bx)}{3b^2} + \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} - \frac{14d^2\sin(a+bx)}{9b^3} + \frac{2(c}{}$$

Mathematica [A]
time = 0.39, size = 93, normalized size = 0.76

$$\frac{162bd(c+dx)\cos(a+bx)+6bd(c+dx)\cos(3(a+bx))+2(-82d^2+45b^2(c+dx)^2+(-2d^2+9b^2(c+dx)^2)\cos(2(a+bx)))\sin(a+bx)}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3,x]
```

```
[Out] (162*b*d*(c + d*x)*Cos[a + b*x] + 6*b*d*(c + d*x)*Cos[3*(a + b*x)] + 2*(-82
*d^2 + 45*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*
Sin[a + b*x])/(108*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(111) = 222$.
time = 0.14, size = 265, normalized size = 2.15

method	result
risch	$\frac{3d(dx+c)\cos(bx+a)}{2b^2} + \frac{3(d^2x^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{4b^3} + \frac{d(dx+c)\cos(3bx+3a)}{18b^2} + \frac{(9d^2x^2b^2+18b^2cdx+9}{10}$
derivativedivides	$\frac{\frac{a^2d^2(2+\cos^2(bx+a))\sin(bx+a)}{3b^2} - \frac{2acd(2+\cos^2(bx+a))\sin(bx+a)}{3b} - \frac{2a\,d^2\left(\frac{(bx+a)(2+\cos^2(bx+a))\sin(bx+a)}{3}+\frac{(\cos^3(bx+a))}{9}+\frac{2c}{}\right.}{b^2}}{}$
default	$\frac{\frac{a^2d^2(2+\cos^2(bx+a))\sin(bx+a)}{3b^2} - \frac{2acd(2+\cos^2(bx+a))\sin(bx+a)}{3b} - \frac{2a\,d^2\left(\frac{(bx+a)(2+\cos^2(bx+a))\sin(bx+a)}{3}+\frac{(\cos^3(bx+a))}{9}+\frac{2c}{}\right.}{b^2}}{}$

norman	$\frac{4cd\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{28cd}{9b^2}+\frac{14d^2x}{9b^2}+\frac{4\left(9b^2c^2-38d^2\right)\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{27b^3}+\frac{2\left(9b^2c^2-14d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{9b^3}+\frac{2\left(9b^2c^2-14d^2\right)\left(\tan^5\left(\frac{bx}{2}+\right.\right.}{9b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/b^2*a^2*d^2*(2+cos(b*x+a)^2)*sin(b*x+a)-2/3/b*a*c*d*(2+cos(b*x+a)^
2)*sin(b*x+a)-2/b^2*a*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(
b*x+a)^3+2/3*cos(b*x+a))+1/3*c^2*(2+cos(b*x+a)^2)*sin(b*x+a)+2/b*c*d*(1/3*(
b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos(b*x+a))+1/b^2*d
^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*co
s(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(111) = 222$.
time = 0.31, size = 267, normalized size = 2.17

$$-\frac{36\left(\sin\left(bx+a\right)^3-3\sin\left(bx+a\right)\right)c^2-\frac{72\left(\sin(bx+a)^3-3\sin(bx+a)\right)acd}{b}+\frac{36\left(\sin(bx+a)^3-3\sin(bx+a)\right)a^2d^2}{b^2}-\frac{6\left(3\left(bx+a\right)\sin(3bx+3a)+27\left(bx+a\right)\sin(bx+a)+\cos(3bx+3a)+27\cos(bx+a)\right)cd}{b}+\frac{6\left(3\left(bx+a\right)\sin(3bx+3a)+27\left(bx+a\right)\sin(bx+a)+\cos(3bx+3a)+27\cos(bx+a)\right)ad^2}{b^2}-\frac{\left(6\left(bx+a\right)\cos(3bx+3a)+162\left(bx+a\right)\cos(bx+a)+\left(9\left(bx+a\right)^2-2\right)\sin(3bx+3a)+81\left(\left(bx+a\right)^2-2\right)\sin(bx+a)\right)d^2}{b^2}}{108b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(36*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^2 - 72*(sin(b*x + a)^3 - 3*s
in(b*x + a))*a*c*d/b + 36*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*d^2/b^2 - 6
*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*
a) + 27*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a
)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a*d^2/b^2 - (6*(b*x +
a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A]
time = 0.38, size = 128, normalized size = 1.04

$$\frac{6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+36\left(bd^2x+bcd\right)\cos\left(bx+a\right)+\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2+\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^2-40\,d^2\right)\sin\left(bx+a\right)}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 + 36*(b*d^2*x + b*c*d)*cos(b*x + a
) + (18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 + (9*b^2*d^2*x^2 + 18*b^2*c
*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 40*d^2)*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.
time = 0.34, size = 284, normalized size = 2.31

$$\begin{cases}\frac{2c^2\sin^3\left(a+bx\right)}{3b}+\frac{c^2\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{b}+\frac{4cdx\sin^3\left(a+bx\right)}{3b}+\frac{2cdx\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{b}+\frac{2d^2x^2\sin^3\left(a+bx\right)}{3b}+\frac{d^2x^2\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{b}+\frac{4cd\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{3b^2}+\frac{14cd\cos^3\left(a+bx\right)}{9b^2}+\frac{4d^2x\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{3b^2}+\frac{14d^2x\cos^3\left(a+bx\right)}{9b^2}-\frac{40d^2\sin^3\left(a+bx\right)}{27b^3}-\frac{14d^2\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{9b^3} & \text{for}\ b\neq 0\\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\cos^3\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**2*sin(a + b*x)**3/(3*b) + c**2*sin(a + b*x)*cos(a + b*x)**2
/b + 4*c*d*x*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)*cos(a + b*x)**2/b
 + 2*d**2*x**2*sin(a + b*x)**3/(3*b) + d**2*x**2*sin(a + b*x)*cos(a + b*x)*
*2/b + 4*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 14*c*d*cos(a + b*x)**3
/(9*b**2) + 4*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 14*d**2*x*cos(
a + b*x)**3/(9*b**2) - 40*d**2*sin(a + b*x)**3/(27*b**3) - 14*d**2*sin(a +
b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3
)*cos(a)**3, True))
```

Giac [A]
time = 0.46, size = 137, normalized size = 1.11

$$\frac{(bd^2x+bcd)\cos(3\,bx+3\,a)}{18\,b^3}+\frac{3\,(bd^2x+bcd)\cos(bx+a)}{2\,b^3}+\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\sin(3\,bx+3\,a)}{108\,b^3}+\frac{3\,(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 3/2*(b*d^2*x + b*c*d)*cos(b*x
 + a)/b^3 + 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*
b*x + 3*a)/b^3 + 3/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x
+ a)/b^3
```

Mupad [B]
time = 0.59, size = 173, normalized size = 1.41

$$\frac{\frac{d^2\,x\,\cos(3\,a+3\,b\,x)}{18}+\frac{3\,c\,d\,\cos(a+b\,x)}{2}+\frac{c\,d\,\cos(3\,a+3\,b\,x)}{18}+\frac{3\,d^2\,x\,\cos(a+b\,x)}{2}}{b^2}+\frac{\frac{3\,c^2\,\sin(a+b\,x)}{4}+\frac{c^2\,\sin(3\,a+3\,b\,x)}{12}+\frac{3\,d^2\,x^2\,\sin(a+b\,x)}{4}+\frac{d^2\,x^2\,\sin(3\,a+3\,b\,x)}{12}+\frac{3\,c\,d\,x\,\sin(a+b\,x)}{2}+\frac{c\,d\,x\,\sin(3\,a+3\,b\,x)}{6}}{b}-\frac{3\,d^2\,\sin\left(a+b\,x\right)}{2\,b^3}-\frac{d^2\,\sin\left(3\,a+3\,b\,x\right)}{54\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] ((d^2*x*cos(3*a + 3*b*x))/18 + (3*c*d*cos(a + b*x))/2 + (c*d*cos(3*a + 3*b*
x))/18 + (3*d^2*x*cos(a + b*x))/2)/b^2 + ((3*c^2*sin(a + b*x))/4 + (c^2*sin
(3*a + 3*b*x))/12 + (3*d^2*x^2*sin(a + b*x))/4 + (d^2*x^2*sin(3*a + 3*b*x))
/12 + (3*c*d*x*sin(a + b*x))/2 + (c*d*x*sin(3*a + 3*b*x))/6)/b - (3*d^2*sin
(a + b*x))/(2*b^3) - (d^2*sin(3*a + 3*b*x))/(54*b^3)
```

3.19 $\int (c+dx)\cos^3(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{2d\cos(a+bx)}{3b^2}+\frac{d\cos^3(a+bx)}{9b^2}+\frac{2(c+dx)\sin(a+bx)}{3b}+\frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b}$$

```
[Out] 2/3*d*cos(b*x+a)/b^2+1/9*d*cos(b*x+a)^3/b^2+2/3*(d*x+c)*sin(b*x+a)/b+1/3*(d
*x+c)*cos(b*x+a)^2*sin(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3391, 3377, 2718}

$$\frac{d\cos^3(a+bx)}{9b^2}+\frac{2d\cos(a+bx)}{3b^2}+\frac{2(c+dx)\sin(a+bx)}{3b}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3,x]
```

```
[Out] (2*d*Cos[a + b*x])/(3*b^2) + (d*Cos[a + b*x]^3)/(9*b^2) + (2*(c + d*x)*Sin[
a + b*x])/(3*b) + ((c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\,dx = \frac{d\cos^3(a+bx)}{9b^2} + \frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2}{3}\int (c+dx)\cos(a+bx$$
$$= \frac{d\cos^3(a+bx)}{9b^2} + \frac{2(c+dx)\sin(a+bx)}{3b} + \frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b} -$$
$$= \frac{2d\cos(a+bx)}{3b^2} + \frac{d\cos^3(a+bx)}{9b^2} + \frac{2(c+dx)\sin(a+bx)}{3b} + \frac{(c+dx)\cos^2(a+}{3b}$$

Mathematica [A]
time = 0.13, size = 52, normalized size = 0.69

$$\frac{27d\cos(a+bx) + d\cos(3(a+bx)) + 3b(c+dx)(9\sin(a+bx) + \sin(3(a+bx)))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3,x]
```

```
[Out] (27*d*Cos[a + b*x] + d*Cos[3*(a + b*x)] + 3*b*(c + d*x)*(9*Sin[a + b*x] + S
in[3*(a + b*x)]))/(36*b^2)
```

Maple [A]
time = 0.09, size = 95, normalized size = 1.27

method	result
risch	$\frac{3d\cos(bx+a)}{4b^2} + \frac{3(dx+c)\sin(bx+a)}{4b} + \frac{d\cos(3bx+3a)}{36b^2} + \frac{(dx+c)\sin(3bx+3a)}{12b}$
derivativedivides	$\frac{-\frac{da\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b} + \frac{c\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3} + \frac{d\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3} + \frac{\left(\cos^3(bx+a)\right)}{9} + \frac{2\cos(bx+a}{3}\right.}{b}}{b}$
default	$\frac{-\frac{da\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b} + \frac{c\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3} + \frac{d\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3} + \frac{\left(\cos^3(bx+a)\right)}{9} + \frac{2\cos(bx+a}{3}\right.}{b}}{b}$
norman	$\frac{\frac{2d\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2} + \frac{14d}{9b^2} + \frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b} + \frac{4c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b} + \frac{2c\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b} + \frac{8d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2} + \frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b} + 4d}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/b*d*a*(2+cos(b*x+a)^2)*sin(b*x+a)+1/3*c*(2+cos(b*x+a)^2)*sin(b*x+
a)+1/b*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos(
b*x+a)))
```

Maxima [A]
time = 0.31, size = 103, normalized size = 1.37

$$-\frac{12\left(\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)\right)c-\frac{12\left(\sin(bx+a)^{3}-3\,\sin(bx+a)\right)ad}{b}-\frac{(3\,(bx+a)\sin(3\,bx+3\,a)+27\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)+27\,\cos(bx+a))d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*(12*(sin(b*x + a)^3 - 3*sin(b*x + a))*c - 12*(sin(b*x + a)^3 - 3*sin(
b*x + a))*a*d/b - (3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a)
 + cos(3*b*x + 3*a) + 27*cos(b*x + a))*d/b)/b
```

Fricas [A]
time = 0.38, size = 60, normalized size = 0.80

$$\frac{d\cos\left(bx+a\right)^{3}+6\,d\cos\left(bx+a\right)+3\,\left(2\,bdx+\left(bdx+bc\right)\cos\left(bx+a\right)^{2}+2\,bc\right)\sin\left(bx+a\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(d*cos(b*x + a)^3 + 6*d*cos(b*x + a) + 3*(2*b*d*x + (b*d*x + b*c)*cos(b
*x + a)^2 + 2*b*c)*sin(b*x + a))/b^2
```

Sympy [A]
time = 0.20, size = 126, normalized size = 1.68

$$\begin{cases}\frac{2c\sin^{3}\left(a+bx\right)}{3b}+\frac{c\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)}{b}+\frac{2dx\sin^{3}\left(a+bx\right)}{3b}+\frac{dx\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)}{b}+\frac{2d\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)}{3b^{2}}+\frac{7d\cos^{3}\left(a+bx\right)}{9b^{2}} & \text{for}\: b\neq 0\\\left(cx+\frac{dx^{2}}{2}\right)\cos^{3}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c*sin(a + b*x)**3/(3*b) + c*sin(a + b*x)*cos(a + b*x)**2/b + 2
*d*x*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)*cos(a + b*x)**2/b + 2*d*sin(a
 + b*x)**2*cos(a + b*x)/(3*b**2) + 7*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*cos(a)**3, True))
```

Giac [A]
time = 0.51, size = 69, normalized size = 0.92

$$\frac{d\cos\left(3\,bx+3\,a\right)}{36\,b^{2}}+\frac{3\,d\cos\left(bx+a\right)}{4\,b^{2}}+\frac{\left(bdx+bc\right)\sin\left(3\,bx+3\,a\right)}{12\,b^{2}}+\frac{3\,\left(bdx+bc\right)\sin\left(bx+a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/36*d*cos(3*b*x + 3*a)/b^2 + 3/4*d*cos(b*x + a)/b^2 + 1/12*(b*d*x + b*c)*s
in(3*b*x + 3*a)/b^2 + 3/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

Mupad [B]
time = 0.26, size = 77, normalized size = 1.03

$$\frac{\frac{3\,c\,\sin(a+b\,x)}{4}+\frac{c\,\sin(3\,a+3\,b\,x)}{12}+\frac{d\,x\,\sin(3\,a+3\,b\,x)}{12}+\frac{3\,d\,x\,\sin(a+b\,x)}{4}}{b}+\frac{d\,\cos\left(3\,a+3\,b\,x\right)}{36\,b^2}+\frac{3\,d\,\cos\left(a+b\,x\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c + d*x),x)
```

```
[Out] ((3*c*sin(a + b*x))/4 + (c*sin(3*a + 3*b*x))/12 + (d*x*sin(3*a + 3*b*x))/12
 + (3*d*x*sin(a + b*x))/4)/b + (d*cos(3*a + 3*b*x))/(36*b^2) + (3*d*cos(a +
 b*x))/(4*b^2)
```

3.20 $\int \frac{\cos^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\sin($$

```
[Out] 1/4*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d+3/4*Ci(b*c/d+b*x)*cos(a-b*c/d)/d-1
/4*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-3/4*Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A]

time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3380, 3383}

$$\frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/
d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (3*Sin[a - (b*c)/d]*SinIntegral[
(b*c)/d + b*x])/(4*d) - (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x
])/(4*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{3\cos(a+bx)}{4(c+dx)}+\frac{\cos(3a+3bx)}{4(c+dx)}\right)dx\\ &= \frac{1}{4}\int \frac{\cos(3a+3bx)}{c+dx}\,dx+\frac{3}{4}\int \frac{\cos(a+bx)}{c+dx}\,dx\\ &= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx+\frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx-\\ &= \frac{3\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{4d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 103, normalized size = 0.85

$$\frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-3\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)-\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*CosInte
gral[(3*b*(c + d*x))/d] - 3*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - Sin
[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A]
time = 0.10, size = 171, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{b\left(-\frac{3\,\text{sinIntegral}\left(-3bx-3a-\frac{3(-da+bc)}{d}\right)\sin\left(\frac{-3da+3bc}{d}\right)}{d}+\frac{3\,\text{cosineIntegral}\left(3bx+3a+\frac{-3da+3bc}{d}\right)\cos\left(\frac{-3da+3bc}{d}\right)}{d}\right)}{12}+\frac{3b\left(-\frac{\text{sinI}}{}\right.}{}}{b}$
default	$\frac{\frac{b\left(-\frac{3\,\text{sinIntegral}\left(-3bx-3a-\frac{3(-da+bc)}{d}\right)\sin\left(\frac{-3da+3bc}{d}\right)}{d}+\frac{3\,\text{cosineIntegral}\left(3bx+3a+\frac{-3da+3bc}{d}\right)\cos\left(\frac{-3da+3bc}{d}\right)}{d}\right)}{12}+\frac{3b\left(-\frac{\text{sinI}}{}\right.}{}}{b}$
risch	$-\frac{e^{-\frac{3i(da-bc)}{d}}\,\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(da-bc)}{d}\right)}{8d}-\frac{3\,e^{-\frac{i(da-bc)}{d}}\,\text{expIntegral}\left(1,ibx+ia-\frac{i(da-bc)}{d}\right)}{8d}-\frac{3\,e^{\frac{i(da-bc}{d}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*
b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)+3/4*b*(-Si(-b*x-a-(-a*d+b*c)
/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.40, size = 278, normalized size = 2.30

$$-\frac{3\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+3\,b\left(-i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b\left(-i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(exp_
integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b*(-I*exp_in
tegral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*b*
c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(-I*exp_integral_e(1
, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I*b*c -
I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 0.36, size = 153, normalized size = 1.26

$$\frac{3\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-6\,\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos
(-(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*
x + b*c)/d))*cos(-3*(b*c - a*d)/d) - 2*sin(-3*(b*c - a*d)/d)*sin_integral(3
*(b*d*x + b*c)/d) - 6*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^3\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 6075, normalized size = 50.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x + b*c/
d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*imag_part
(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1
/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*
a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x
 - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) + 12*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos
_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/
d)^2 + 4*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
```

```
3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3
*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x +
b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 6*imag_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*imag_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*sin_in
tegral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2
 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(
3/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^
2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*imag_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*sin_integral((b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*imag_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(-...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b x)^3}{c+d x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/(c + d*x),x)
[Out] int(cos(a + b*x)^3/(c + d*x), x)
```

3.21 $\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{\cos^3(a+bx)}{d(c+dx)}-\frac{3b\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2}-\frac{3b\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d^2}-\frac{3b\cos\left(a-\right.}{}$$

```
[Out] -cos(b*x+a)^3/d/(d*x+c)-3/4*b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2-3/4*b*cos(3*a-
3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2-3/4*b*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2
-3/4*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

Rubi [A]
time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3394, 3384, 3380, 3383}

$$-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\cos^3(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] -(Cos[a + b*x]^3/(d*(c + d*x))) - (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3
*a - (3*b*c)/d])/(4*d^2) - (3*b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/(4*d^2) - (3*b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) - (3*
b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx &= -\frac{\cos^3(a+bx)}{d(c+dx)}+\frac{(3b)\int \left(-\frac{\sin(a+bx)}{4(c+dx)}-\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx}{d}\\ &= -\frac{\cos^3(a+bx)}{d(c+dx)}-\frac{(3b)\int \frac{\sin(a+bx)}{c+dx}\,dx}{4d}-\frac{(3b)\int \frac{\sin(3a+3bx)}{c+dx}\,dx}{4d}\\ &= -\frac{\cos^3(a+bx)}{d(c+dx)}-\frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}-\frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,d\ldots}{4d}\\ &= -\frac{\cos^3(a+bx)}{d(c+dx)}-\frac{3b\,\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2}-\frac{3b\,\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d^2}-\frac{3b\cos\ldots}{\ldots}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 200, normalized size = 1.38

$$-\frac{3d\cos(a+bx)+d\cos(3(a+bx))+3b(c+dx)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+3b(c+dx)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+3bc\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+3bdx\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+3bc\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)+3bdx\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] -1/4*(3*d*Cos[a + b*x] + d*Cos[3*(a + b*x)] + 3*b*(c + d*x)*CosIntegral[(3*
b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 3*b*(c + d*x)*CosIntegral[b*(c/d + x
)]*Sin[a - (b*c)/d] + 3*b*c*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b
*d*x*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*c*Cos[3*a - (3*b*c)/d]
*SinIntegral[(3*b*(c + d*x))/d] + 3*b*d*x*Cos[3*a - (3*b*c)/d]*SinIntegral[
(3*b*(c + d*x))/d])/(d^2*(c + d*x))
```

Maple [A]
time = 0.16, size = 247, normalized size = 1.70

method	result

derivativedivides	$\frac{b^2\left(-\frac{3\cos(3bx+3a)}{(-da+bc+d(bx+a))d}-\frac{3\left(-\frac{3\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+bc)}{d}\right)\cos\left(\frac{-3da+3bc}{d}\right)}{d}-\frac{3\,\mathrm{cosineIntegral}\left(3bx+3a+\frac{-3da+3bc}{d}\right)\mathrm{si}}{d}\right.}{d}\right.}{12}$
default	$\frac{b^2\left(-\frac{3\cos(3bx+3a)}{(-da+bc+d(bx+a))d}-\frac{3\left(-\frac{3\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+bc)}{d}\right)\cos\left(\frac{-3da+3bc}{d}\right)}{d}-\frac{3\,\mathrm{cosineIntegral}\left(3bx+3a+\frac{-3da+3bc}{d}\right)\mathrm{si}}{d}\right.}{d}\right.}{12}$
risch	$\frac{3ib\,\mathrm{e}^{-\frac{3i(da-bc)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(da-bc)}{d}\right)}{8d^2}+\frac{3ib\,\mathrm{e}^{-\frac{i(da-bc)}{d}}\,\mathrm{expIntegral}\left(1,ibx+ia-\frac{i(da-bc)}{d}\right)}{8d^2}-\frac{3ib\,\mathrm{e}^{\frac{i(d}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b^2*(-3*cos(3*b*x+3*a)/(-d*a+b*c+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a
-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3
*(-a*d+b*c)/d)/d)/d)+3/4*b^2*(-cos(b*x+a)/(-d*a+b*c+d*(b*x+a))/d-(-Si(-b*x-
a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d
)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.44, size = 304, normalized size = 2.10

$$-\frac{3\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+3\,b^2\left(-i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b^2\left(i\,E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_2\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2*(
exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b^2*(-I*
exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(2,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(I*exp_integ
ral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a
)*d^2 - a*d^2)*b)
```

Fricas [A]
time = 0.46, size = 227, normalized size = 1.57

$$-\frac{8\,d\cos\left(bx+a\right)^3+6\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+6\,(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*d*cos(b*x + a)^3 + 6*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integr
al(3*(b*d*x + b*c)/d) + 6*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b
*d*x + b*c)/d) + 3*((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x +
b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 3*((b*d*x + b*c)
*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b
*c)/d))*sin(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(137) = 274.
time = 0.57, size = 1000, normalized size = 6.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(((d*
x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)
/d) + 3*b^3*c*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) +
 b*c - a*d)/d)*sin(-(b*c - a*d)/d) - 3*a*b^2*d*cos_integral(((d*x + c)*(b -
 b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) + 3*(d*
x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(3*((d*x + c)*(b
 - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-3*(b*c - a*d)/d) + 3
*b^3*c*cos_integral(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d)*sin(-3*(b*c - a*d)/d) - 3*a*b^2*d*cos_integral(3*((d*x + c)*(b -
b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-3*(b*c - a*d)/d) - 3*(d
*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)/d)*sin_int
egral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*b
^3*c*cos(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*cos(-(b*c - a*d)/d)*sin_integral(-((
d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*(d*x + c)*
```

```
(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-3*(b*c - a*d)/d)*sin_integral(
-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*b^3*c
*cos(-3*(b*c - a*d)/d)*sin_integral(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*cos(-3*(b*c - a*d)/d)*sin_integral(-
3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*b^2*d*
cos(-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + b^2*d*cos(-3*(d*x +
 c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x +
 c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)^3/(c + d*x)^2, x)
```

3.22 $\int \frac{\cos^3(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=184

$$-\frac{\cos^3(a+bx)}{2d(c+dx)^2}-\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b\cos^2(a}{2d}$$

```
[Out] -9/8*b^2*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^3-3/8*b^2*Ci(b*c/d+b*x)*cos(a
-b*c/d)/d^3-1/2*cos(b*x+a)^3/d/(d*x+c)^2+9/8*b^2*Si(3*b*c/d+3*b*x)*sin(3*a-
3*b*c/d)/d^3+3/8*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+3/2*b*cos(b*x+a)^2*sin(
b*x+a)/d^2/(d*x+c)
```

Rubi [A]
time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3395, 3384, 3380, 3383, 3393}

$$-\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] -1/2*Cos[a + b*x]^3/(d*(c + d*x)^2) - (3*b^2*Cos[a - (b*c)/d]*CosIntegral[(
b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d
+ 3*b*x])/(8*d^3) + (3*b*Cos[a + b*x]^2*Sin[a + b*x])/(2*d^2*(c + d*x)) + (
3*b^2*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos^3(a+bx)}{2d(c+dx)^2} + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} + \frac{(3b^2)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d^2} - \frac{(9b^2)\int \frac{\cos^3(a+bx)}{c+dx}}{2d^2}$$

$$= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} - \frac{(9b^2)\int \left(\frac{3\cos(a+bx)}{4(c+dx)} + \frac{\cos(3a+3bx)}{4(c+dx)}\right)dx}{2d^2} +$$

$$= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^3} + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} - \frac{3b^2\sin}{}$$

$$= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^3} + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} - \frac{3b^2\sin}{}$$

$$= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3} - \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^3} +$$

Mathematica [A]

time = 0.53, size = 221, normalized size = 1.20

$$\frac{6d\cos(bx)(-d\cos(a)+b(c+dx)\sin(a))+2d\cos(3bx)(-d\cos(3a)+3b(c+dx)\sin(3a))+6d(b(c+dx)\cos(a)+d\sin(a))\sin(bx)+2d(3b(c+dx)\cos(3a)+d\sin(3a))\sin(3bx)-6b^2(c+dx)^2\left(\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)-3\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)\right)}{16d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^3,x]

[Out] (6*d*Cos[b*x]*(-(d*Cos[a]) + b*(c + d*x)*Sin[a]) + 2*d*Cos[3*b*x]*(-(d*Cos[
3*a]) + 3*b*(c + d*x)*Sin[3*a]) + 6*d*(b*(c + d*x)*Cos[a] + d*Sin[a])*Sin[b
```

```
*x] + 2*d*(3*b*(c + d*x)*Cos[3*a] + d*Sin[3*a])*Sin[3*b*x] - 6*b^2*(c + d*x
)^2*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*Cos[3*a - (3*b*c)/d]*Cos
Integral[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3
*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]))/(16*d^3*(c + d*x)^2)
```

Maple [A]
time = 0.29, size = 316, normalized size = 1.72

method	result
derivativedivides	$\frac{b^3\left(-\frac{3\cos(3bx+3a)}{2(-da+bc+d(bx+a))^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-da+bc+d(bx+a))d}+\frac{-\frac{9\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+bc)}{d}\right)\sin\left(\frac{-3da+3bc}{d}\right)}{d}+\frac{9\,\mathrm{cosineIntegra}}{}}{d}\right.}{2d}}{12}$
default	$\frac{b^3\left(-\frac{3\cos(3bx+3a)}{2(-da+bc+d(bx+a))^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-da+bc+d(bx+a))d}+\frac{-\frac{9\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+bc)}{d}\right)\sin\left(\frac{-3da+3bc}{d}\right)}{d}+\frac{9\,\mathrm{cosineIntegra}}{}}{d}\right.}{2d}}{12}$
risch	$\frac{9b^2 e^{-\frac{3i(da-bc)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(da-bc)}{d}\right)}{16d^3}+\frac{3b^2 e^{-\frac{i(da-bc)}{d}}\,\mathrm{expIntegral}\left(1,ibx+ia-\frac{i(da-bc)}{d}\right)}{16d^3}+\frac{3b^2 e^{\frac{i(da-}{d}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b^3*(-3/2*cos(3*b*x+3*a)/(-d*a+b*c+d*(b*x+a))^2/d-3/2*(-3*sin(3*b
*x+3*a)/(-d*a+b*c+d*(b*x+a))/d+3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-
a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)+3
/4*b^3*(-1/2*cos(b*x+a)/(-d*a+b*c+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-d*a+b*c
+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+
b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 339, normalized size = 1.84

$$-\frac{3\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+3\,b^3\left(-i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b^3\left(i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_3\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3*(
```

```
exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b^3*(-I*
exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(3,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(I*exp_integ
ral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*
b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(172) = 344$.
time = 0.43, size = 375, normalized size = 2.04

$$-\frac{8\,d^2\cos\left(bx+a\right)^3-24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2\sin\left(bx+a\right)-18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-6\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)+3\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+9\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)}{16\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*d^2*cos(b*x + a)^3 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2*sin(b*x +
 a) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_in
tegral(3*(b*d*x + b*c)/d) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b
*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 3*((b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)
/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.71, size = 115446, normalized size = 627.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

[Out] -1/16*(9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b
^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2
*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x
^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_p
art(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral(
(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x
 + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^
2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_
part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a
)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral
(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2
*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 +

```
12*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2
*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_p
art(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c
)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d) - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(
cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
 + 36*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36
*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b\,x)^3}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)^3/(c + d*x)^3, x)
```

3.23 $\int x^3 \cos^4(a+bx)\,dx$

Optimal. Leaf size=172

$$-\frac{45x^2}{128b^2}+\frac{3x^4}{32}-\frac{45\cos^2(a+bx)}{128b^4}+\frac{9x^2\cos^2(a+bx)}{16b^2}-\frac{3\cos^4(a+bx)}{128b^4}+\frac{3x^2\cos^4(a+bx)}{16b^2}-\frac{45x\cos(a+bx)\sin(a+}{64b^3}$$

```
[Out] -45/128*x^2/b^2+3/32*x^4-45/128*cos(b*x+a)^2/b^4+9/16*x^2*cos(b*x+a)^2/b^2-
3/128*cos(b*x+a)^4/b^4+3/16*x^2*cos(b*x+a)^4/b^2-45/64*x*cos(b*x+a)*sin(b*x
+a)/b^3+3/8*x^3*cos(b*x+a)*sin(b*x+a)/b-3/32*x*cos(b*x+a)^3*sin(b*x+a)/b^3+
1/4*x^3*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A]

time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 30, 3391}

$$-\frac{3\cos^4(a+bx)}{128b^4}-\frac{45\cos^2(a+bx)}{128b^4}-\frac{3x\sin(a+bx)\cos^3(a+bx)}{32b^3}-\frac{45x\sin(a+bx)\cos(a+bx)}{64b^3}+\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{9x^2\cos^2(a+bx)}{16b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3x^3\sin(a+bx)\cos(a+bx)}{8b}-\frac{45x^2}{128b^2}+\frac{3x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[a + b*x]^4,x]
```

```
[Out] (-45*x^2)/(128*b^2) + (3*x^4)/32 - (45*Cos[a + b*x]^2)/(128*b^4) + (9*x^2*C
os[a + b*x]^2)/(16*b^2) - (3*Cos[a + b*x]^4)/(128*b^4) + (3*x^2*Cos[a + b*x
]^4)/(16*b^2) - (45*x*Cos[a + b*x]*Sin[a + b*x])/(64*b^3) + (3*x^3*Cos[a +
b*x]*Sin[a + b*x])/(8*b) - (3*x*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^3) + (x^
3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^3\cos^4(a+bx)\,dx = \frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{x^3\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{3}{4}\int x^3\cos^2(a+bx)\,dx-\frac{3\int x\,\mathrm{co}}{}$$
$$=\frac{9x^2\cos^2(a+bx)}{16b^2}-\frac{3\cos^4(a+bx)}{128b^4}+\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{3x^3\cos(a+bx)\sin(a+bx}{8b}$$
$$=\frac{3x^4}{32}-\frac{45\cos^2(a+bx)}{128b^4}+\frac{9x^2\cos^2(a+bx)}{16b^2}-\frac{3\cos^4(a+bx)}{128b^4}+\frac{3x^2\cos^4(a+bx)}{16b^2}-$$
$$=-\frac{45x^2}{128b^2}+\frac{3x^4}{32}-\frac{45\cos^2(a+bx)}{128b^4}+\frac{9x^2\cos^2(a+bx)}{16b^2}-\frac{3\cos^4(a+bx)}{128b^4}+\frac{3x^2\cos^4(a}{16b^2}$$

Mathematica [A]
time = 0.28, size = 100, normalized size = 0.58

$$\frac{192(-1+2b^2x^2)\cos(2(a+bx))+3(-1+8b^2x^2)\cos(4(a+bx))+4bx(24b^3x^3+32(-3+2b^2x^2)\sin(2(a+bx))+(-3+8b^2x^2)\sin(4(a+bx)))}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[a + b*x]^4,x]
```

```
[Out] (192*(-1 + 2*b^2*x^2)*Cos[2*(a + b*x)] + 3*(-1 + 8*b^2*x^2)*Cos[4*(a + b*x)
] + 4*b*x*(24*b^3*x^3 + 32*(-3 + 2*b^2*x^2)*Sin[2*(a + b*x)] + (-3 + 8*b^2*
x^2)*Sin[4*(a + b*x)]))/(1024*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(152) = 304$.
time = 0.12, size = 432, normalized size = 2.51

method	result
risch	$\frac{3x^4}{32}+\frac{3(8x^2b^2-1)\cos(4bx+4a)}{1024b^4}+\frac{x(8x^2b^2-3)\sin(4bx+4a)}{256b^3}+\frac{3(2x^2b^2-1)\cos(2bx+2a)}{16b^4}+\frac{x(2x^2b^2-3)\sin(2bx-}{8b^3}$
derivativedivides	$\frac{-a^3\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)+3a^2\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{}$
default	$\frac{-a^3\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)+3a^2\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(-a^3*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3*
a^2*((b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3
/16*(b*x+a)^2+1/64*(2*cos(b*x+a)^2+3)^2)-3*a*((b*x+a)^2*(1/4*(cos(b*x+a)^3+
3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+1/8*(b*x+a)*cos(b*x+a)^4-1/32*(co
s(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-15/64*b*x-15/64*a+3/8*(b*x+a)*cos(b*x
+a)^2-3/16*cos(b*x+a)*sin(b*x+a)-1/4*(b*x+a)^3)+(b*x+a)^3*(1/4*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/16*(b*x+a)^2*cos(b*x+a)^4-3/8
*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+45/12
8*(b*x+a)^2-3/512*(2*cos(b*x+a)^2+3)^2+9/16*(b*x+a)^2*cos(b*x+a)^2-9/8*(b*x
+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*sin(b*x+a)^2-9/32*(b*x+a
)^4)
```

Maxima [A]
time = 0.30, size = 303, normalized size = 1.76

$$\frac{96\,(bx+a)^4-32\,(12\,bx+12\,a+\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))a^3+24\,(24\,(bx+a)^2+4\,(bx+a)\sin(4\,bx+4\,a)+32\,(bx+a)\sin(2\,bx+2\,a)+\cos(4\,bx+4\,a)+16\,\cos(2\,bx+2\,a))a^2-12\,(32\,(bx+a)^3+4\,(bx+a)\cos(4\,bx+4\,a)+64\,(bx+a)\cos(2\,bx+2\,a)+(8\,(bx+a)^2-1)\sin(4\,bx+4\,a)+32\,(2\,(bx+a)^2-1)\sin(2\,bx+2\,a))a+3\,(8\,(bx+a)^2-1)\cos(4\,bx+4\,a)+192\,(2\,(bx+a)^2-1)\cos(2\,bx+2\,a)+4\,(8\,(bx+a)^3-3\,bx-3\,a)\sin(4\,bx+4\,a)+128\,(2\,(bx+a)^3-3\,bx-3\,a)\sin(2\,bx+2\,a)}{1024\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/1024*(96*(b*x + a)^4 - 32*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x
 + 2*a))*a^3 + 24*(24*(b*x + a)^2 + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x
+ a)*sin(2*b*x + 2*a) + cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))*a^2 - 12*(3
2*(b*x + a)^3 + 4*(b*x + a)*cos(4*b*x + 4*a) + 64*(b*x + a)*cos(2*b*x + 2*a
) + (8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 32*(2*(b*x + a)^2 - 1)*sin(2*b*x
 + 2*a))*a + 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 192*(2*(b*x + a)^2 -
1)*cos(2*b*x + 2*a) + 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) + 12
8*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))/b^4
```

Fricas [A]
time = 0.39, size = 115, normalized size = 0.67

$$\frac{12\,b^4x^4+3\,(8\,b^2x^2-1)\cos(bx+a)^4-45\,b^2x^2+9\,(8\,b^2x^2-5)\cos(bx+a)^2+2\,(2\,(8\,b^3x^3-3\,bx)\cos(bx+a)^3+3\,(8\,b^3x^3-15\,bx)\cos(bx+a))\sin(bx+a)}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(12*b^4*x^4 + 3*(8*b^2*x^2 - 1)*cos(b*x + a)^4 - 45*b^2*x^2 + 9*(8*b^
2*x^2 - 5)*cos(b*x + a)^2 + 2*(2*(8*b^3*x^3 - 3*b*x)*cos(b*x + a)^3 + 3*(8*
b^3*x^3 - 15*b*x)*cos(b*x + a))*sin(b*x + a))/b^4
```

Sympy [A]
time = 0.64, size = 253, normalized size = 1.47

$$\begin{cases} \frac{3x^4\sin^4(a+bx)}{32} + \frac{3x^4\sin^2(a+bx)\cos^2(a+bx)}{16} + \frac{3x^4\cos^4(a+bx)}{32} + \frac{3x^3\sin^3(a+bx)\cos(a+bx)}{8b} + \frac{5x^3\sin(a+bx)\cos^3(a+bx)}{8b} - \frac{45x^2\sin^4(a+bx)}{128b^2} - \frac{9x^2\sin^2(a+bx)\cos^2(a+bx)}{64b^2} + \frac{51x^2\cos^4(a+bx)}{128b^2} - \frac{45x\sin^3(a+bx)\cos(a+bx)}{64b^3} - \frac{51x\sin(a+bx)\cos^3(a+bx)}{64b^3} + \frac{45\sin^4(a+bx)}{256b^4} - \frac{51\cos^4(a+bx)}{256b^4} & \text{for } b \neq 0 \\ \frac{x^4\cos^4(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**4*sin(a + b*x)**4/32 + 3*x**4*sin(a + b*x)**2*cos(a + b*x)*
*2/16 + 3*x**4*cos(a + b*x)**4/32 + 3*x**3*sin(a + b*x)**3*cos(a + b*x)/(8*
b) + 5*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 45*x**2*sin(a + b*x)**4/(1
28*b**2) - 9*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2) + 51*x**2*cos(a
 + b*x)**4/(128*b**2) - 45*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) - 51*x*
sin(a + b*x)*cos(a + b*x)**3/(64*b**3) + 45*sin(a + b*x)**4/(256*b**4) - 51
*cos(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*cos(a)**4/4, True))
```

Giac [A]
time = 0.46, size = 108, normalized size = 0.63

$$\frac{3}{32}x^4 + \frac{3\,(8\,b^2x^2-1)\cos(4\,bx+4\,a)}{1024\,b^4} + \frac{3\,(2\,b^2x^2-1)\cos(2\,bx+2\,a)}{16\,b^4} + \frac{(8\,b^3x^3-3\,bx)\sin(4\,bx+4\,a)}{256\,b^4} + \frac{(2\,b^3x^3-3\,bx)\sin(2\,bx+2\,a)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/32*x^4 + 3/1024*(8*b^2*x^2 - 1)*cos(4*b*x + 4*a)/b^4 + 3/16*(2*b^2*x^2 -
1)*cos(2*b*x + 2*a)/b^4 + 1/256*(8*b^3*x^3 - 3*b*x)*sin(4*b*x + 4*a)/b^4 +
1/8*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a)/b^4
```

Mupad [B]
time = 0.80, size = 138, normalized size = 0.80

$$\frac{\frac{3\sin(2\,a+2\,b\,x)^2}{512} - b^2\left(\frac{3\,x^2\left(2\sin(2\,a+2\,b\,x)^2-1\right)}{128} + \frac{3\,x^2\left(2\sin(a+b\,x)^2-1\right)}{8}\right) - b\left(\frac{3\,x\,\sin(2\,a+2\,b\,x)}{8} + \frac{3\,x\,\sin(4\,a+4\,b\,x)}{256}\right) + b^3\left(\frac{x^3\,\sin(2\,a+2\,b\,x)}{4} + \frac{x^3\,\sin(4\,a+4\,b\,x)}{32}\right) + \frac{3\sin(a+b\,x)^2}{8}}{b^4} + \frac{3\,x^4}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(a + b*x)^4,x)
```

```
[Out] ((3*sin(2*a + 2*b*x)^2)/512 - b^2*((3*x^2*(2*sin(2*a + 2*b*x)^2 - 1))/128 +
 (3*x^2*(2*sin(a + b*x)^2 - 1))/8) - b*((3*x*sin(2*a + 2*b*x))/8 + (3*x*sin
(4*a + 4*b*x))/256) + b^3*((x^3*sin(2*a + 2*b*x))/4 + (x^3*sin(4*a + 4*b*x)
)/32) + (3*sin(a + b*x)^2)/8)/b^4 + (3*x^4)/32
```

3.24 $\int x^2 \cos^4(a + bx)\, dx$

Optimal. Leaf size=134

$$-\frac{15x}{64b^2}+\frac{x^3}{8}+\frac{3x\cos^2(a+bx)}{8b^2}+\frac{x\cos^4(a+bx)}{8b^2}-\frac{15\cos(a+bx)\sin(a+bx)}{64b^3}+\frac{3x^2\cos(a+bx)\sin(a+bx)}{8b}-\cos^3$$

```
[Out] -15/64*x/b^2+1/8*x^3+3/8*x*cos(b*x+a)^2/b^2+1/8*x*cos(b*x+a)^4/b^2-15/64*co
s(b*x+a)*sin(b*x+a)/b^3+3/8*x^2*cos(b*x+a)*sin(b*x+a)/b-1/32*cos(b*x+a)^3*s
in(b*x+a)/b^3+1/4*x^2*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3392, 30, 2715, 8}

$$-\frac{\sin(a+bx)\cos^3(a+bx)}{32b^3}-\frac{15\sin(a+bx)\cos(a+bx)}{64b^3}+\frac{x\cos^4(a+bx)}{8b^2}+\frac{3x\cos^2(a+bx)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3x^2\sin(a+bx)\cos(a+bx)}{8b}-\frac{15x}{64b^2}+\frac{x^3}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x]^4,x]
```

```
[Out] (-15*x)/(64*b^2) + x^3/8 + (3*x*Cos[a + b*x]^2)/(8*b^2) + (x*Cos[a + b*x]^4
)/(8*b^2) - (15*Cos[a + b*x]*Sin[a + b*x])/(64*b^3) + (3*x^2*Cos[a + b*x]*S
in[a + b*x])/(8*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(32*b^3) + (x^2*Cos[a +
b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
```

```
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^2\cos^4(a+bx)\,dx = \frac{x\cos^4(a+bx)}{8b^2} + \frac{x^2\cos^3(a+bx)\sin(a+bx)}{4b} + \frac{3}{4}\int x^2\cos^2(a+bx)\,dx - \frac{\int\cos^4(a-}{8b}$$
$$= \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} + \frac{3x^2\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin(}{32b^3}$$
$$= \frac{x^3}{8} + \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} - \frac{15\cos(a+bx)\sin(a+bx)}{64b^3} + \frac{3x^2\cos(a+}{}$$
$$= -\frac{15x}{64b^2} + \frac{x^3}{8} + \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} - \frac{15\cos(a+bx)\sin(a+bx)}{64b^3} + \frac{3x}{}$$

Mathematica [A]
time = 0.12, size = 92, normalized size = 0.69

$$\frac{32b^3x^3 + 64bx\cos(2(a+bx)) + 4bx\cos(4(a+bx)) - 32\sin(2(a+bx)) + 64b^2x^2\sin(2(a+bx)) - \sin(4(a+bx)) + 8b^2x^2\sin(4(a+bx))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x]^4,x]
```

```
[Out] (32*b^3*x^3 + 64*b*x*Cos[2*(a + b*x)] + 4*b*x*Cos[4*(a + b*x)] - 32*Sin[2*(
a + b*x)] + 64*b^2*x^2*Sin[2*(a + b*x)] - Sin[4*(a + b*x)] + 8*b^2*x^2*Sin[
4*(a + b*x)])/(256*b^3)
```

Maple [A]
time = 0.12, size = 237, normalized size = 1.77

method	result
risch	$\frac{x^3}{8} + \frac{x\cos(4bx+4a)}{64b^2} + \frac{(8x^2b^2-1)\sin(4bx+4a)}{256b^3} + \frac{x\cos(2bx+2a)}{4b^2} + \frac{(2x^2b^2-1)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-2a\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-3\right.}{}$
default	$\frac{a^2\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-2a\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-3\right.}{}$
norman	$\frac{\frac{x^3}{8}-\frac{17\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{32b^3}-\frac{9\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{9\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{17\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{17x}{64b^2}+\frac{x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{3x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(a^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-2*a
*((b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/16
*(b*x+a)^2+1/64*(2*cos(b*x+a)^2+3)^2)+(b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(
b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+1/8*(b*x+a)*cos(b*x+a)^4-1/32*(cos(b*x+a)
^3+3/2*cos(b*x+a))*sin(b*x+a)-15/64*b*x-15/64*a+3/8*(b*x+a)*cos(b*x+a)^2-3/
16*cos(b*x+a)*sin(b*x+a)-1/4*(b*x+a)^3)
```

Maxima [A]
time = 0.31, size = 188, normalized size = 1.40

$$\frac{32\,(bx+a)^3+8\,(12\,bx+12\,a+\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))a^2-4\,\left(24\,(bx+a)^2+4\,(bx+a)\sin(4\,bx+4\,a)+32\,(bx+a)\sin(2\,bx+2\,a)+\cos(4\,bx+4\,a)+16\,\cos(2\,bx+2\,a)\right)a+4\,(bx+a)\cos(4\,bx+4\,a)+64\,(bx+a)\cos(2\,bx+2\,a)+\left(8\,(bx+a)^2-1\right)\sin(4\,bx+4\,a)+32\,\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)}{256\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/256*(32*(b*x + a)^3 + 8*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x +
 2*a))*a^2 - 4*(24*(b*x + a)^2 + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a
)*sin(2*b*x + 2*a) + cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))*a + 4*(b*x + a
)*cos(4*b*x + 4*a) + 64*(b*x + a)*cos(2*b*x + 2*a) + (8*(b*x + a)^2 - 1)*si
n(4*b*x + 4*a) + 32*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

Fricas [A]
time = 0.43, size = 88, normalized size = 0.66

$$\frac{8\,b^3x^3+8\,bx\cos(bx+a)^4+24\,bx\cos(bx+a)^2-15\,bx+\left(2\,(8\,b^2x^2-1)\cos(bx+a)^3+3\,(8\,b^2x^2-5)\cos(bx+a)\right)\sin(bx+a)}{64\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(8*b^3*x^3 + 8*b*x*cos(b*x + a)^4 + 24*b*x*cos(b*x + a)^2 - 15*b*x + (
2*(8*b^2*x^2 - 1)*cos(b*x + a)^3 + 3*(8*b^2*x^2 - 5)*cos(b*x + a))*sin(b*x
+ a))/b^3
```

Sympy [A]
time = 0.45, size = 209, normalized size = 1.56

$$\begin{cases} \frac{x^3\sin^4(a+bx)}{8}+\frac{x^3\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{x^3\cos^4(a+bx)}{8}+\frac{3x^2\sin^3(a+bx)\cos(a+bx)}{8b}+\frac{5x^2\sin(a+bx)\cos^3(a+bx)}{8b}-\frac{15x\sin^4(a+bx)}{64b^2}-\frac{3x\sin^2(a+bx)\cos^2(a+bx)}{32b^2}+\frac{17x\cos^4(a+bx)}{64b^2}-\frac{15\sin^3(a+bx)\cos(a+bx)}{64b^3}-\frac{17\sin(a+bx)\cos^3(a+bx)}{64b^3} & \text{for}\ b\neq 0 \\ \frac{x^3\cos^4(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**4/8 + x**3*sin(a + b*x)**2*cos(a + b*x)**2/4
+ x**3*cos(a + b*x)**4/8 + 3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 5*x*
*2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 15*x*sin(a + b*x)**4/(64*b**2) - 3*
x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) + 17*x*cos(a + b*x)**4/(64*b**2
) - 15*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) - 17*sin(a + b*x)*cos(a + b*x
)**3/(64*b**3), Ne(b, 0)), (x**3*cos(a)**4/3, True))
```

Giac [A]
time = 0.44, size = 84, normalized size = 0.63

$$\frac{1}{8}\,x^3+\frac{x\cos\left(4\,bx+4\,a\right)}{64\,b^2}+\frac{x\cos\left(2\,bx+2\,a\right)}{4\,b^2}+\frac{\left(8\,b^2x^2-1\right)\sin\left(4\,bx+4\,a\right)}{256\,b^3}+\frac{\left(2\,b^2x^2-1\right)\sin\left(2\,bx+2\,a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*x^3 + 1/64*x*cos(4*b*x + 4*a)/b^2 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/256*
(8*b^2*x^2 - 1)*sin(4*b*x + 4*a)/b^3 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

Mupad [B]
time = 0.54, size = 104, normalized size = 0.78

$$\frac{x^3}{8}-\frac{\frac{\sin(2\,a+2\,b\,x)}{8}+\frac{\sin(4\,a+4\,b\,x)}{256}+b\left(\frac{x\,\left(2\,\sin(a+b\,x)^2-1\right)}{4}+\frac{x\,\left(2\,\sin(2\,a+2\,b\,x)^2-1\right)}{64}\right)-b^2\left(\frac{x^2\,\sin(2\,a+2\,b\,x)}{4}+\frac{x^2\,\sin(4\,a+4\,b\,x)}{32}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*x)^4,x)
```

```
[Out] x^3/8 - (sin(2*a + 2*b*x)/8 + sin(4*a + 4*b*x)/256 + b*((x*(2*sin(a + b*x)^
2 - 1))/4 + (x*(2*sin(2*a + 2*b*x)^2 - 1))/64) - b^2*((x^2*sin(2*a + 2*b*x)
)/4 + (x^2*sin(4*a + 4*b*x))/32))/b^3
```

3.25 $\int x\cos^4(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{3x^2}{16}+\frac{3\cos^2(a+bx)}{16b^2}+\frac{\cos^4(a+bx)}{16b^2}+\frac{3x\cos(a+bx)\sin(a+bx)}{8b}+\frac{x\cos^3(a+bx)\sin(a+bx)}{4b}$$

```
[Out] 3/16*x^2+3/16*cos(b*x+a)^2/b^2+1/16*cos(b*x+a)^4/b^2+3/8*x*cos(b*x+a)*sin(b
*x+a)/b+1/4*x*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3391, 30}

$$\frac{\cos^4(a+bx)}{16b^2}+\frac{3\cos^2(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3x\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^4,x]
```

```
[Out] (3*x^2)/16 + (3*Cos[a + b*x]^2)/(16*b^2) + Cos[a + b*x]^4/(16*b^2) + (3*x*C
os[a + b*x]*Sin[a + b*x])/(8*b) + (x*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\cos^4(a+bx)\,dx&=\frac{\cos^4(a+bx)}{16b^2}+\frac{x\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{3}{4}\int x\cos^2(a+bx)\,dx\\&=\frac{3\cos^2(a+bx)}{16b^2}+\frac{\cos^4(a+bx)}{16b^2}+\frac{3x\cos(a+bx)\sin(a+bx)}{8b}+\frac{x\cos^3(a+bx)\sin(a+}{4b}\\&=\frac{3x^2}{16}+\frac{3\cos^2(a+bx)}{16b^2}+\frac{\cos^4(a+bx)}{16b^2}+\frac{3x\cos(a+bx)\sin(a+bx)}{8b}+\frac{x\cos^3(a+bx)\,\mathrm{si}}{4b}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 53, normalized size = 0.66

$$\frac{16\cos(2(a+bx))+\cos(4(a+bx))+4bx(6bx+8\sin(2(a+bx))+\sin(4(a+bx)))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^4,x]
```

```
[Out] (16*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 4*b*x*(6*b*x + 8*Sin[2*(a + b*x)]
 + Sin[4*(a + b*x)]))/(128*b^2)
```

Maple [A]
time = 0.09, size = 106, normalized size = 1.32

method	result
risch	$\frac{3x^2}{16}+\frac{\cos(4bx+4a)}{128b^2}+\frac{x\sin(4bx+4a)}{32b}+\frac{\cos(2bx+2a)}{8b^2}+\frac{x\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{(bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{3(bx+a)^2}{16}+\frac{\left(2\left(\cos^2(bx+a)\right)+3\right)^2}{64}-a\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a}{2}}{4}\right.}{b^2}$
default	$\frac{(bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{3(bx+a)^2}{16}+\frac{\left(2\left(\cos^2(bx+a)\right)+3\right)^2}{64}-a\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a}{2}}{4}\right.}{b^2}$
norman	$\frac{\frac{3x^2}{16}+\frac{3x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{9x^2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8}+\frac{3x^2\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{3x^2\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16}+\frac{5x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{3x\left(\tan^3\left(\frac{bx}{2}+\right.\right.}{4b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*((b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)
-3/16*(b*x+a)^2+1/64*(2*cos(b*x+a)^2+3)^2-a*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+
a))*sin(b*x+a)+3/8*b*x+3/8*a))
```

Maxima [A]
time = 0.30, size = 98, normalized size = 1.22

$$\frac{24\,(bx+a)^2-4\,(12\,bx+12\,a+\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))a+4\,(bx+a)\sin(4\,bx+4\,a)+32\,(bx+a)\sin(2\,bx+2\,a)+\cos(4\,bx+4\,a)+16\,\cos(2\,bx+2\,a)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/128*(24*(b*x + a)^2 - 4*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x +
 2*a))*a + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a) + c
os(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))/b^2
```

Fricas [A]
time = 0.40, size = 63, normalized size = 0.79

$$\frac{3\,b^2x^2+\cos\left(bx+a\right)^4+3\,\cos\left(bx+a\right)^2+2\,\left(2\,bx\cos\left(bx+a\right)^3+3\,bx\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*(3*b^2*x^2 + cos(b*x + a)^4 + 3*cos(b*x + a)^2 + 2*(2*b*x*cos(b*x + a)
^3 + 3*b*x*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A]
time = 0.27, size = 138, normalized size = 1.72

$$\begin{cases} \frac{3x^2\sin^4\left(a+bx\right)}{16}+\frac{3x^2\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{8}+\frac{3x^2\cos^4\left(a+bx\right)}{16}+\frac{3x\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{8b}+\frac{5x\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{8b}-\frac{3\sin^4\left(a+bx\right)}{32b^2}+\frac{5\cos^4\left(a+bx\right)}{32b^2} & \text{for}\ b\neq 0 \\ \frac{x^2\cos^4\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**2*sin(a + b*x)**4/16 + 3*x**2*sin(a + b*x)**2*cos(a + b*x)*
*2/8 + 3*x**2*cos(a + b*x)**4/16 + 3*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) +
 5*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 3*sin(a + b*x)**4/(32*b**2) + 5*c
os(a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*cos(a)**4/2, True))
```

Giac [A]
time = 0.41, size = 64, normalized size = 0.80

$$\frac{3}{16}\,x^2+\frac{x\sin\left(4\,bx+4\,a\right)}{32\,b}+\frac{x\sin\left(2\,bx+2\,a\right)}{4\,b}+\frac{\cos\left(4\,bx+4\,a\right)}{128\,b^2}+\frac{\cos\left(2\,bx+2\,a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x^2 + 1/32*x*sin(4*b*x + 4*a)/b + 1/4*x*sin(2*b*x + 2*a)/b + 1/128*cos
(4*b*x + 4*a)/b^2 + 1/8*cos(2*b*x + 2*a)/b^2
```

Mupad [B]
time = 0.34, size = 63, normalized size = 0.79

$$\frac{3\,x^2}{16}-\frac{\frac{\sin(2\,a+2\,b\,x)^2}{64}-b\,\left(\frac{x\,\sin(2\,a+2\,b\,x)}{4}+\frac{x\,\sin(4\,a+4\,b\,x)}{32}\right)+\frac{\sin(a+b\,x)^2}{4}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^4,x)
```

```
[Out] (3*x^2)/16 - (sin(2*a + 2*b*x)^2/64 - b*((x*sin(2*a + 2*b*x))/4 + (x*sin(4*
a + 4*b*x))/32) + sin(a + b*x)^2/4)/b^2
```

3.26 $\int \frac{\cos^4(a+bx)}{x} \, dx$

Optimal. Leaf size=59

$$\frac{1}{2}\cos(2a)\text{CosIntegral}(2bx)+\frac{1}{8}\cos(4a)\text{CosIntegral}(4bx)+\frac{3\log(x)}{8}-\frac{1}{2}\sin(2a)\text{Si}(2bx)-\frac{1}{8}\sin(4a)\text{Si}(4bx)$$

```
[Out] 1/2*Ci(2*b*x)*cos(2*a)+1/8*Ci(4*b*x)*cos(4*a)+3/8*ln(x)-1/2*Si(2*b*x)*sin(2
*a)-1/8*Si(4*b*x)*sin(4*a)
```

Rubi [A]
time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3384, 3380, 3383}

$$\frac{1}{2}\cos(2a)\text{CosIntegral}(2bx)+\frac{1}{8}\cos(4a)\text{CosIntegral}(4bx)-\frac{1}{2}\sin(2a)\text{Si}(2bx)-\frac{1}{8}\sin(4a)\text{Si}(4bx)+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x])/2 + (Cos[4*a]*CosIntegral[4*b*x])/8 + (3*Log[
x])/8 - (Sin[2*a]*SinIntegral[2*b*x])/2 - (Sin[4*a]*SinIntegral[4*b*x])/8
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(a+bx)}{x}\,dx &= \int \left(\frac{3}{8x}+\frac{\cos(2a+2bx)}{2x}+\frac{\cos(4a+4bx)}{8x}\right)dx\\ &= \frac{3\log(x)}{8}+\frac{1}{8}\int \frac{\cos(4a+4bx)}{x}\,dx+\frac{1}{2}\int \frac{\cos(2a+2bx)}{x}\,dx\\ &= \frac{3\log(x)}{8}+\frac{1}{2}\cos(2a)\int \frac{\cos(2bx)}{x}\,dx+\frac{1}{8}\cos(4a)\int \frac{\cos(4bx)}{x}\,dx-\frac{1}{2}\sin(2a)\int \frac{\sin(2bx)}{x}\\ &= \frac{1}{2}\cos(2a)\text{Ci}(2bx)+\frac{1}{8}\cos(4a)\text{Ci}(4bx)+\frac{3\log(x)}{8}-\frac{1}{2}\sin(2a)\text{Si}(2bx)-\frac{1}{8}\sin(4a)\text{Si}(4bx)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.88

$$\frac{1}{8}(4\cos(2a)\text{CosIntegral}(2bx)+\cos(4a)\text{CosIntegral}(4bx)+3\log(x)-4\sin(2a)\text{Si}(2bx)-\sin(4a)\text{Si}(4bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4/x,x]
```

```
[Out] (4*Cos[2*a]*CosIntegral[2*b*x] + Cos[4*a]*CosIntegral[4*b*x] + 3*Log[x] - 4
*Sin[2*a]*SinIntegral[2*b*x] - Sin[4*a]*SinIntegral[4*b*x])/8
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.88

method	result
derivativedivides	$-\frac{\text{sinIntegral}(4bx)\sin(4a)}{8}+\frac{\text{cosineIntegral}(4bx)\cos(4a)}{8}-\frac{\text{sinIntegral}(2bx)\sin(2a)}{2}+\frac{\text{cosineIntegral}(2bx)\cos(2a)}{2}$
default	$-\frac{\text{sinIntegral}(4bx)\sin(4a)}{8}+\frac{\text{cosineIntegral}(4bx)\cos(4a)}{8}-\frac{\text{sinIntegral}(2bx)\sin(2a)}{2}+\frac{\text{cosineIntegral}(2bx)\cos(2a)}{2}$
risch	$\frac{3\ln(x)}{8}+\frac{i\mathrm{e}^{-4ia}\pi\,\text{csgn}(bx)}{16}-\frac{i\mathrm{e}^{-4ia}\,\text{sinIntegral}(4bx)}{8}-\frac{\mathrm{e}^{-4ia}\,\text{expIntegral}(1,-4ibx)}{16}+\frac{i\pi\,\text{csgn}(bx)\mathrm{e}^{-2ia}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Si(4*b*x)*sin(4*a)+1/8*Ci(4*b*x)*cos(4*a)-1/2*Si(2*b*x)*sin(2*a)+1/2*C
i(2*b*x)*cos(2*a)+3/8*ln(b*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 91, normalized size = 1.54

$$-\frac{1}{16}\left(E_1(4i\,bx)+E_1(-4i\,bx)\right)\cos(4\,a)-\frac{1}{4}\left(E_1(2i\,bx)+E_1(-2i\,bx)\right)\cos(2\,a)+\frac{1}{16}\left(i\,E_1(4i\,bx)-i\,E_1(-4i\,bx)\right)\sin(4\,a)-\frac{1}{4}\left(-i\,E_1(2i\,bx)+i\,E_1(-2i\,bx)\right)\sin(2\,a)+\frac{3}{8}\log(bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="maxima")
```

```
[Out] -1/16*(exp_integral_e(1, 4*I*b*x) + exp_integral_e(1, -4*I*b*x))*cos(4*a) -
 1/4*(exp_integral_e(1, 2*I*b*x) + exp_integral_e(1, -2*I*b*x))*cos(2*a) +
1/16*(I*exp_integral_e(1, 4*I*b*x) - I*exp_integral_e(1, -4*I*b*x))*sin(4*a
) - 1/4*(-I*exp_integral_e(1, 2*I*b*x) + I*exp_integral_e(1, -2*I*b*x))*sin
(2*a) + 3/8*log(b*x)
```

Fricas [A]
time = 0.42, size = 61, normalized size = 1.03

$$\frac{1}{16}\left(\mathrm{Ci}\left(4\,bx\right)+\mathrm{Ci}\left(-4\,bx\right)\right)\cos\left(4\,a\right)+\frac{1}{4}\left(\mathrm{Ci}\left(2\,bx\right)+\mathrm{Ci}\left(-2\,bx\right)\right)\cos\left(2\,a\right)-\frac{1}{8}\sin\left(4\,a\right)\mathrm{Si}\left(4\,bx\right)-\frac{1}{2}\sin\left(2\,a\right)\mathrm{Si}\left(2\,bx\right)+\frac{3}{8}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="fricas")
```

```
[Out] 1/16*(cos_integral(4*b*x) + cos_integral(-4*b*x))*cos(4*a) + 1/4*(cos_integ
ral(2*b*x) + cos_integral(-2*b*x))*cos(2*a) - 1/8*sin(4*a)*sin_integral(4*b
*x) - 1/2*sin(2*a)*sin_integral(2*b*x) + 3/8*log(x)
```

Sympy [A]
time = 1.30, size = 60, normalized size = 1.02

$$\frac{3\log\left(x\right)}{8}-\frac{\sin\left(2a\right)\mathrm{Si}\left(2bx\right)}{2}-\frac{\sin\left(4a\right)\mathrm{Si}\left(4bx\right)}{8}+\frac{\cos\left(2a\right)\mathrm{Ci}\left(2bx\right)}{2}+\frac{\cos\left(4a\right)\mathrm{Ci}\left(4bx\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/x,x)
```

```
[Out] 3*log(x)/8 - sin(2*a)*Si(2*b*x)/2 - sin(4*a)*Si(4*b*x)/8 + cos(2*a)*Ci(2*b*
x)/2 + cos(4*a)*Ci(4*b*x)/8
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 428, normalized size = 7.25

$$\frac{6\log\left(|x|\right)\tan\left(2a\right)^2\tan\left(a\right)^2-\Re(\mathrm{Ci}\left(4bx\right))\tan\left(2a\right)^2\tan\left(a\right)^2-4\Re(\mathrm{Ci}\left(2bx\right))\tan\left(2a\right)^2\tan\left(a\right)^2-4\Re(\mathrm{Ci}\left(-2bx\right))\tan\left(2a\right)^2\tan\left(a\right)^2-\Re(\mathrm{Ci}\left(-4bx\right))\tan\left(2a\right)^2\tan\left(a\right)^2-8\Im(\mathrm{Ci}\left(2bx\right))\tan\left(2a\right)^2\tan\left(a\right)+8\Im(\mathrm{Ci}\left(-2bx\right))\tan\left(2a\right)^2\tan\left(a\right)-16\,\mathrm{Si}\left(2bx\right)\tan\left(2a\right)^2\tan\left(a\right)-2\Im(\mathrm{Ci}\left(4bx\right))\tan\left(2a\right)\tan\left(a\right)^2+2\Im(\mathrm{Ci}\left(-4bx\right))\tan\left(2a\right)\tan\left(a\right)^2-4\,\mathrm{Si}\left(4bx\right)\tan\left(2a\right)\tan\left(a\right)^2+6\log\left(|x|\right)\tan\left(2a\right)^2-\Re(\mathrm{Ci}\left(4bx\right))\tan\left(2a\right)^2+4\Re(\mathrm{Ci}\left(2bx\right))\tan\left(2a\right)^2+4\Re(\mathrm{Ci}\left(-2bx\right))\tan\left(2a\right)^2-\Re(\mathrm{Ci}\left(-4bx\right))\tan\left(2a\right)^2+6\log\left(|x|\right)\tan\left(a\right)^2+\Re(\mathrm{Ci}\left(4bx\right))\tan\left(a\right)^2-4\Re(\mathrm{Ci}\left(2bx\right))\tan\left(a\right)^2-4\Re(\mathrm{Ci}\left(-2bx\right))\tan\left(a\right)^2+\Re(\mathrm{Ci}\left(-4bx\right))\tan\left(a\right)^2-2\Im(\mathrm{Ci}\left(4bx\right))\tan\left(2a\right)+2\Im(\mathrm{Ci}\left(-4bx\right))\tan\left(2a\right)-4\,\mathrm{Si}\left(4bx\right)\tan\left(2a\right)-8\Im(\mathrm{Ci}\left(2bx\right))\tan\left(a\right)+8\Im(\mathrm{Ci}\left(-2bx\right))\tan\left(a\right)-16\,\mathrm{Si}\left(2bx\right)\tan\left(a\right)+6\log\left(|x|\right)+\Re(\mathrm{Ci}\left(4bx\right))+4\Re(\mathrm{Ci}\left(2bx\right))+4\Re(\mathrm{Ci}\left(-2bx\right))+\Re(\mathrm{Ci}\left(-4bx\right))}{16\left(\tan\left(2a\right)^2\tan\left(a\right)^2+\tan\left(2a\right)^2+\tan\left(a\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="giac")
```

```
[Out] 1/16*(6*log(abs(x))*tan(2*a)^2*tan(a)^2 - real_part(cos_integral(4*b*x))*ta
n(2*a)^2*tan(a)^2 - 4*real_part(cos_integral(2*b*x))*tan(2*a)^2*tan(a)^2 -
4*real_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 - real_part(cos_integ
ral(-4*b*x))*tan(2*a)^2*tan(a)^2 - 8*imag_part(cos_integral(2*b*x))*tan(2*a
)^2*tan(a) + 8*imag_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a) - 16*sin_i
```

```
ntegral(2*b*x)*tan(2*a)^2*tan(a) - 2*imag_part(cos_integral(4*b*x))*tan(2*a
)*tan(a)^2 + 2*imag_part(cos_integral(-4*b*x))*tan(2*a)*tan(a)^2 - 4*sin_in
tegral(4*b*x)*tan(2*a)*tan(a)^2 + 6*log(abs(x))*tan(2*a)^2 - real_part(cos_
integral(4*b*x))*tan(2*a)^2 + 4*real_part(cos_integral(2*b*x))*tan(2*a)^2 +
 4*real_part(cos_integral(-2*b*x))*tan(2*a)^2 - real_part(cos_integral(-4*b
*x))*tan(2*a)^2 + 6*log(abs(x))*tan(a)^2 + real_part(cos_integral(4*b*x))*t
an(a)^2 - 4*real_part(cos_integral(2*b*x))*tan(a)^2 - 4*real_part(cos_integ
ral(-2*b*x))*tan(a)^2 + real_part(cos_integral(-4*b*x))*tan(a)^2 - 2*imag_p
art(cos_integral(4*b*x))*tan(2*a) + 2*imag_part(cos_integral(-4*b*x))*tan(2
*a) - 4*sin_integral(4*b*x)*tan(2*a) - 8*imag_part(cos_integral(2*b*x))*tan
(a) + 8*imag_part(cos_integral(-2*b*x))*tan(a) - 16*sin_integral(2*b*x)*tan
(a) + 6*log(abs(x)) + real_part(cos_integral(4*b*x)) + 4*real_part(cos_inte
gral(2*b*x)) + 4*real_part(cos_integral(-2*b*x)) + real_part(cos_integral(-
4*b*x)))/(tan(2*a)^2*tan(a)^2 + tan(2*a)^2 + tan(a)^2 + 1)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/x,x)
```

```
[Out] int(cos(a + b*x)^4/x, x)
```

3.27 $\int \frac{\cos^4(a+bx)}{x^2} \, dx$

Optimal. Leaf size=66

$$-\frac{\cos^4(a+bx)}{x}-b\text{CosIntegral}(2bx)\sin(2a)-\frac{1}{2}b\text{CosIntegral}(4bx)\sin(4a)-b\cos(2a)\text{Si}(2bx)-\frac{1}{2}b\cos(4a)\text{Si}(4b$$

```
[Out] -cos(b*x+a)^4/x-b*cos(2*a)*Si(2*b*x)-1/2*b*cos(4*a)*Si(4*b*x)-b*Ci(2*b*x)*s
in(2*a)-1/2*b*Ci(4*b*x)*sin(4*a)
```

Rubi [A]
time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3394, 3384, 3380, 3383}

$$-b\sin(2a)\text{CosIntegral}(2bx)-\frac{1}{2}b\sin(4a)\text{CosIntegral}(4bx)-b\cos(2a)\text{Si}(2bx)-\frac{1}{2}b\cos(4a)\text{Si}(4bx)-\frac{\cos^4(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4/x^2,x]
```

```
[Out] -(Cos[a + b*x]^4/x) - b*CosIntegral[2*b*x]*Sin[2*a] - (b*CosIntegral[4*b*x]
*Sin[4*a])/2 - b*Cos[2*a]*SinIntegral[2*b*x] - (b*Cos[4*a]*SinIntegral[4*b*
x])/2
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
```

```
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(a+bx)}{x^2}\,dx &= -\frac{\cos^4(a+bx)}{x} + (4b)\int \left(-\frac{\sin(2a+2bx)}{4x} - \frac{\sin(4a+4bx)}{8x}\right)dx \\ &= -\frac{\cos^4(a+bx)}{x} - \frac{1}{2}b\int \frac{\sin(4a+4bx)}{x}\,dx - b\int \frac{\sin(2a+2bx)}{x}\,dx \\ &= -\frac{\cos^4(a+bx)}{x} - (b\cos(2a))\int \frac{\sin(2bx)}{x}\,dx - \frac{1}{2}(b\cos(4a))\int \frac{\sin(4bx)}{x}\,dx - (b\sin(2a)) \\ &= -\frac{\cos^4(a+bx)}{x} - b\text{Ci}(2bx)\sin(2a) - \frac{1}{2}b\text{Ci}(4bx)\sin(4a) - b\cos(2a)\text{Si}(2bx) - \frac{1}{2}b\cos(4a\end{aligned}$$

Mathematica [A]
time = 0.15, size = 79, normalized size = 1.20

$$-\frac{3+4\cos(2(a+bx))+\cos(4(a+bx))+8bx\text{CosIntegral}(2bx)\sin(2a)+4bx\text{CosIntegral}(4bx)\sin(4a)+8bx\cos(2a)\text{Si}(2bx)+4bx\cos(4a)\text{Si}(4bx)}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4/x^2,x]
[Out] -1/8*(3 + 4*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 8*b*x*CosIntegral[2*b*x]*
Sin[2*a] + 4*b*x*CosIntegral[4*b*x]*Sin[4*a] + 8*b*x*Cos[2*a]*SinIntegral[2
*b*x] + 4*b*x*Cos[4*a]*SinIntegral[4*b*x])/x
```

Maple [A]
time = 0.12, size = 90, normalized size = 1.36

method	result
derivativedivides	$b\Big(-\frac{\cos(4bx+4a)}{8bx} - \frac{\text{sinIntegral}(4bx)\cos(4a)}{2} - \frac{\text{cosineIntegral}(4bx)\sin(4a)}{2} - \frac{\cos(2bx+2a)}{2bx} - \text{sinIntegral}\,(2b$
default	$b\Big(-\frac{\cos(4bx+4a)}{8bx} - \frac{\text{sinIntegral}(4bx)\cos(4a)}{2} - \frac{\text{cosineIntegral}(4bx)\sin(4a)}{2} - \frac{\cos(2bx+2a)}{2bx} - \text{sinIntegral}\,(2b$
risch	$\frac{\pi\,\text{csgn}(bx)\mathrm{e}^{-2ia}b}{2} - \text{sinIntegral}\,(2bx)\,\mathrm{e}^{-2ia}b + \frac{i\,\text{expIntegral}(1,-2ibx)\mathrm{e}^{-2ia}b}{2} - \frac{ib\,\text{expIntegral}(1,-2ibx)\mathrm{e}^{2ia}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/x^2,x,method=_RETURNVERBOSE)
[Out] b*(-1/8*cos(4*b*x+4*a)/b/x-1/2*Si(4*b*x)*cos(4*a)-1/2*Ci(4*b*x)*sin(4*a)-1/
2*cos(2*b*x+2*a)/b/x-Si(2*b*x)*cos(2*a)-Ci(2*b*x)*sin(2*a)-3/8/b/x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.38, size = 717, normalized size = 10.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="maxima")
```

```
[Out] 1/32*(((exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*cos(2*a)^
2 + (exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*
cos(4*a)^3 + ((-I*exp_integral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*b*x
))*cos(2*a)^2 + (-I*exp_integral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*b
*x))*sin(2*a)^2)*sin(4*a)^3 + 4*((exp_integral_e(2, 2*I*b*x) + exp_integral
_e(2, -2*I*b*x))*cos(2*a)^3 + (-I*exp_integral_e(2, 2*I*b*x) + I*exp_integr
al_e(2, -2*I*b*x))*sin(2*a)^3 + ((exp_integral_e(2, 2*I*b*x) + exp_integral
_e(2, -2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + (exp_integral_e(2, 2*I*b*x) + e
xp_integral_e(2, -2*I*b*x))*cos(2*a) + 3*cos(2*a)^2 + ((-I*exp_integral_e(2
, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*cos(2*a)^2 - I*exp_integral_e(2
, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*sin(2*a))*cos(4*a)^2 + (4*(exp_
integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*cos(2*a)^3 + 4*(-I*ex
p_integral_e(2, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*sin(2*a)^3 + 4*((
exp_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*cos(2*a) + 3)*sin
(2*a)^2 + ((exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*cos(2
*a)^2 + (exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*sin(2*a)
^2)*cos(4*a) + 4*(exp_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))
*cos(2*a) + 12*cos(2*a)^2 + 4*((-I*exp_integral_e(2, 2*I*b*x) + I*exp_integ
ral_e(2, -2*I*b*x))*cos(2*a)^2 - I*exp_integral_e(2, 2*I*b*x) + I*exp_integ
ral_e(2, -2*I*b*x))*sin(2*a))*sin(4*a)^2 + ((exp_integral_e(2, 4*I*b*x) + e
xp_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (exp_integral_e(2, 4*I*b*x) + exp_
integral_e(2, -4*I*b*x))*sin(2*a)^2)*cos(4*a) + (((-I*exp_integral_e(2, 4*I
*b*x) + I*exp_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (-I*exp_integral_e(2, 4
*I*b*x) + I*exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*cos(4*a)^2 + (-I*exp_i
ntegral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (-I*exp
_integral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*sin(4*
a))*b/((a*cos(2*a)^2 + a*sin(2*a)^2)*cos(4*a)^2 + (a*cos(2*a)^2 + a*sin(2*a
)^2)*sin(4*a)^2 - ((cos(2*a)^2 + sin(2*a)^2)*cos(4*a)^2 + (cos(2*a)^2 + sin
(2*a)^2)*sin(4*a)^2)*(b*x + a))
```

Fricas [A]
time = 0.36, size = 87, normalized size = 1.32

$$-\frac{4\cos\left(bx+a\right)^{4}+2\,bx\cos\left(4\,a\right)\operatorname{Si}\left(4\,bx\right)+4\,bx\cos\left(2\,a\right)\operatorname{Si}\left(2\,bx\right)+\left(bx\operatorname{Ci}\left(4\,bx\right)+bx\operatorname{Ci}\left(-4\,bx\right)\right)\sin\left(4\,a\right)+2\left(bx\operatorname{Ci}\left(2\,bx\right)+bx\operatorname{Ci}\left(-2\,bx\right)\right)\sin\left(2\,a\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^4 + 2*b*x*cos(4*a)*sin_integral(4*b*x) + 4*b*x*cos(2*a
)*sin_integral(2*b*x) + (b*x*cos_integral(4*b*x) + b*x*cos_integral(-4*b*x)
)*sin(4*a) + 2*(b*x*cos_integral(2*b*x) + b*x*cos_integral(-2*b*x))*sin(2*a
))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/x**2,x)
```

```
[Out] Integral(cos(a + b*x)**4/x**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 3220, normalized size = 48.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b
*x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^
2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*t
an(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(
b*x)^2*tan(2*a)^2*tan(a)^2 - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*ta
n(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 2*b*x*real_part(cos_integral(4*b*
x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*real_part(cos_integra
l(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 + b*x*imag_part(cos_in
tegral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 2*b*x*imag_part(cos_int
egral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_inte
gral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integr
al(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*ta
n(b*x)^2*tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b
*x)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x
)^2*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*
tan(a)^2 - 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b
*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + b*x*imag_part(cos
```

_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*imag_part(cos_in tegral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 - 2*b*x*imag_part(cos_integ ral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2 *b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(2* a)^2*tan(a)^2 + b*x*imag_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2*ta n(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^ 2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x* sin_integral(4*b*x)*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b *x)*tan(b*x)^2*tan(2*a)^2*tan(a)^2 - 2*b*x*real_part(cos_integral(4*b*x))*t an(2*b*x)^2*tan(b*x)^2*tan(2*a) - 2*b*x*real_part(cos_integral(-4*b*x))*tan (2*b*x)^2*tan(b*x)^2*tan(2*a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(2* b*x)^2*tan(b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(2*b*x) ^2*tan(b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*ta n(2*a)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(2* a)^2*tan(a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a)^2*ta n(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a) - 2*b*x*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 2*b* x*real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*re al_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*real_part (cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 4*tan(2*b*x)^2*tan(b* x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2* tan(b*x)^2 - 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2 + 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2 + b*x*imag_p art(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2 - 2*b*x*sin_integral(4*b* x)*tan(2*b*x)^2*tan(b*x)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x)^2 + b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)^2 - 2*b*x*im ag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_ integral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(-4*b *x))*tan(2*b*x)^2*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*tan(2 *a)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(2*a)^2 + b*x*imag_part(c os_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2 - 2*b*x*imag_part(cos_integral(2* b*x))*tan(b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*tan(b*x)^2*tan(2*a)^2 - 4*b*x*sin_integral(2*b* x)*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2* tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a)^2 - 2*b *x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(a)^2 + b*x*imag_part(co s_integral(-4*b*x))*tan(2*b*x)^2*tan(a)^2 - 2*b*x*sin_integral(4*b*x)*tan(2 *b*x)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(a)^2 - b*x*im ag_part(cos_integral(4*b*x))*tan(b*x)^2*tan(a)^2 + 2*b*x*imag_part(cos_inte gral(2*b*x))*tan(b*x)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*ta n(b*x)^2*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(a)^2 - 2*b*x*sin_integral(4*b*x)*tan(b*x)^2*tan(a)^...

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(a+b\,x)^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/x^2,x)
```

```
[Out] int(cos(a + b*x)^4/x^2, x)
```

3.28 $\int \frac{\cos^4(a+bx)}{x^3} \, dx$

Optimal. Leaf size=90

$$-\frac{\cos^4(a+bx)}{2x^2}-b^2\cos(2a)\text{CosIntegral}(2bx)-b^2\cos(4a)\text{CosIntegral}(4bx)+\frac{2b\cos^3(a+bx)\sin(a+bx)}{x}+b^2\,\text{si}$$

```
[Out] -b^2*Ci(2*b*x)*cos(2*a)-b^2*Ci(4*b*x)*cos(4*a)-1/2*cos(b*x+a)^4/x^2+b^2*Si(
2*b*x)*sin(2*a)+b^2*Si(4*b*x)*sin(4*a)+2*b*cos(b*x+a)^3*sin(b*x+a)/x
```

Rubi [A]
time = 0.17, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3395, 3393, 3384, 3380, 3383}

$$-b^2\cos(2a)\text{CosIntegral}(2bx)-b^2\cos(4a)\text{CosIntegral}(4bx)+b^2\sin(2a)\text{Si}(2bx)+b^2\sin(4a)\text{Si}(4bx)-\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\sin(a+bx)\cos^3(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4/x^3,x]
```

```
[Out] -1/2*Cos[a + b*x]^4/x^2 - b^2*Cos[2*a]*CosIntegral[2*b*x] - b^2*Cos[4*a]*Co
sIntegral[4*b*x] + (2*b*Cos[a + b*x]^3*Sin[a + b*x])/x + b^2*Sin[2*a]*SinIn
tegral[2*b*x] + b^2*Sin[4*a]*SinIntegral[4*b*x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(a+bx)}{x^3}\,dx &= -\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\cos^3(a+bx)\sin(a+bx)}{x}+\left(6b^2\right)\int\frac{\cos^2(a+bx)}{x}\,dx-\left(8b^2\right)\int \frac{\cos \\
&= -\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\cos^3(a+bx)\sin(a+bx)}{x}+\left(6b^2\right)\int\left(\frac{1}{2x}+\frac{\cos(2a+2bx)}{2x}\right)dx-(\\
&= -\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\cos^3(a+bx)\sin(a+bx)}{x}-b^2\int\frac{\cos(4a+4bx)}{x}\,dx+\left(3b^2\right)\int\frac{\cos(2 \\
&= -\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\cos^3(a+bx)\sin(a+bx)}{x}+\left(3b^2\cos(2a)\right)\int\frac{\cos(2bx)}{x}\,dx-\left(4b^2\cos\right. \\
&= -\frac{\cos^4(a+bx)}{2x^2}-b^2\cos(2a)\text{Ci}(2bx)-b^2\cos(4a)\text{Ci}(4bx)+\frac{2b\cos^3(a+bx)\sin(a+bx)}{x}-
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 119, normalized size = 1.32

$$-\frac{3+4\cos(2(a+bx))+\cos(4(a+bx))+16b^2x^2\cos(2a)\text{CosIntegral}(2bx)+16b^2x^2\cos(4a)\text{CosIntegral}(4bx)-8bx\sin(2(a+bx))-4bx\sin(4(a+bx))-16b^2x^2\sin(2a)\text{Si}(2bx)-16b^2x^2\sin(4a)\text{Si}(4bx)}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4/x^3,x]
```

```
[Out] -1/16*(3 + 4*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16*b^2*x^2*Cos[2*a]*CosI
ntegral[2*b*x] + 16*b^2*x^2*Cos[4*a]*CosIntegral[4*b*x] - 8*b*x*Sin[2*(a +
b*x)] - 4*b*x*Sin[4*(a + b*x)] - 16*b^2*x^2*Sin[2*a]*SinIntegral[2*b*x] - 1
6*b^2*x^2*Sin[4*a]*SinIntegral[4*b*x])/x^2
```

Maple [A]
time = 0.11, size = 124, normalized size = 1.38

method	result
derivativedivides	$b^2\left(-\frac{\cos(4bx+4a)}{16b^2x^2}+\frac{\sin(4bx+4a)}{4bx}+\text{sinIntegral}\left(4bx\right)\sin\left(4a\right)-\text{cosineIntegral}\left(4bx\right)\cos\left(4a\right)\right.$
default	$b^2\left(-\frac{\cos(4bx+4a)}{16b^2x^2}+\frac{\sin(4bx+4a)}{4bx}+\text{sinIntegral}\left(4bx\right)\sin\left(4a\right)-\text{cosineIntegral}\left(4bx\right)\cos\left(4a\right)\right.$
risch	$-\frac{3}{16x^2}-\frac{i\pi\,\text{csgn}(bx)\,\mathrm{e}^{-4ia}b^2}{2}+i\,\text{sinIntegral}\left(4bx\right)\mathrm{e}^{-4ia}b^2+\frac{\text{expIntegral}(1,-4ibx)\,\mathrm{e}^{-4ia}b^2}{2}-\frac{i\pi\,\text{csgn}(bx}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/16*cos(4*b*x+4*a)/b^2/x^2+1/4*sin(4*b*x+4*a)/b/x+Si(4*b*x)*sin(4*a)
-Ci(4*b*x)*cos(4*a)-1/4*cos(2*b*x+2*a)/b^2/x^2+1/2*sin(2*b*x+2*a)/b/x+Si(2*
b*x)*sin(2*a)-Ci(2*b*x)*cos(2*a)-3/16/x^2/b^2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.38, size = 790, normalized size = 8.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/32*(((exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*cos(2*a)
^2 + (exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)
*cos(4*a)^3 + ((-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*
x))*cos(2*a)^2 + (-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*
b*x))*sin(2*a)^2)*sin(4*a)^3 + 2*(2*(exp_integral_e(3, 2*I*b*x) + exp_integ
ral_e(3, -2*I*b*x))*cos(2*a)^3 + 2*(-I*exp_integral_e(3, 2*I*b*x) + I*exp_i
ntegral_e(3, -2*I*b*x))*sin(2*a)^3 + (2*(exp_integral_e(3, 2*I*b*x) + exp_i
ntegral_e(3, -2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + 2*(exp_integral_e(3, 2*I
*b*x) + exp_integral_e(3, -2*I*b*x))*cos(2*a) + 3*cos(2*a)^2 + 2*((-I*exp_i
ntegral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*cos(2*a)^2 - I*exp_i
ntegral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*sin(2*a))*cos(4*a)^2
 + (4*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x))*cos(2*a)^3
 + 4*(-I*exp_integral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*sin(2*
a)^3 + 2*(2*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x))*cos(
2*a) + 3)*sin(2*a)^2 + ((exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*
I*b*x))*cos(2*a)^2 + (exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b
*x))*sin(2*a)^2)*cos(4*a) + 4*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(
3, -2*I*b*x))*cos(2*a) + 6*cos(2*a)^2 + 4*((-I*exp_integral_e(3, 2*I*b*x) +
 I*exp_integral_e(3, -2*I*b*x))*cos(2*a)^2 - I*exp_integral_e(3, 2*I*b*x) +
 I*exp_integral_e(3, -2*I*b*x))*sin(2*a))*sin(4*a)^2 + ((exp_integral_e(3,
4*I*b*x) + exp_integral_e(3, -4*I*b*x))*cos(2*a)^2 + (exp_integral_e(3, 4*I
```

```
*b*x) + exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)*cos(4*a) + (((-I*exp_integ
ral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*x))*cos(2*a)^2 + (-I*exp_int
egral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)*cos(4*a)^2
 + (-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*x))*cos(2*a)
^2 + (-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*x))*sin(2*
a)^2)*sin(4*a))*b^2/(((cos(2*a)^2 + sin(2*a)^2)*cos(4*a)^2 + (cos(2*a)^2 +
sin(2*a)^2)*sin(4*a)^2)*(b*x + a)^2 + (a^2*cos(2*a)^2 + a^2*sin(2*a)^2)*cos
(4*a)^2 + (a^2*cos(2*a)^2 + a^2*sin(2*a)^2)*sin(4*a)^2 - 2*((a*cos(2*a)^2 +
 a*sin(2*a)^2)*cos(4*a)^2 + (a*cos(2*a)^2 + a*sin(2*a)^2)*sin(4*a)^2)*(b*x
+ a))
```

Fricas [A]
time = 0.37, size = 130, normalized size = 1.44

$$\frac{4\,bx\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)+2\,b^{2}x^{2}\sin\left(4\,a\right)\mathrm{Si}\left(4\,bx\right)+2\,b^{2}x^{2}\sin\left(2\,a\right)\mathrm{Si}\left(2\,bx\right)-\cos\left(bx+a\right)^{4}-\left(b^{2}x^{2}\,\mathrm{Ci}\left(4\,bx\right)+b^{2}x^{2}\,\mathrm{Ci}\left(-4\,bx\right)\right)\cos\left(4\,a\right)-\left(b^{2}x^{2}\,\mathrm{Ci}\left(2\,bx\right)+b^{2}x^{2}\,\mathrm{Ci}\left(-2\,bx\right)\right)\cos\left(2\,a\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*x*cos(b*x + a)^3*sin(b*x + a) + 2*b^2*x^2*sin(4*a)*sin_integral(4*
b*x) + 2*b^2*x^2*sin(2*a)*sin_integral(2*b*x) - cos(b*x + a)^4 - (b^2*x^2*c
os_integral(4*b*x) + b^2*x^2*cos_integral(-4*b*x))*cos(4*a) - (b^2*x^2*cos_
integral(2*b*x) + b^2*x^2*cos_integral(-2*b*x))*cos(2*a))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(a+bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/x**3,x)
```

```
[Out] Integral(cos(a + b*x)**4/x**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 3920, normalized size = 43.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2
*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(
b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(
```

```
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(
-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 8*b^2*x^2*imag_part(
cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 8*b^2*x^2*
imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) +
 16*b^2*x^2*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) +
 8*b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*
tan(a)^2 - 8*b^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2
*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_part(cos_integral(-4*b*x))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x)
)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x
))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b
*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(-4
*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(
4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral
(2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integra
l(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_int
egral(4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_inte
gral(2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 8*b^2*x^2*imag_part(cos_integ
ral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) - 8*b^2*x^2*imag_part(cos_inte
gral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) + 16*b^2*x^2*sin_integral(4*
b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) + 8*b^2*x^2*imag_part(cos_integral(2*
b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*
b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2
*b*x)^2*tan(b*x)^2*tan(a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x))*tan(2*
b*x)^2*tan(2*a)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*b*x))*tan(2*
b*x)^2*tan(2*a)^2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2*b*x)^2*tan(
2*a)^2*tan(a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a
)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^
2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(b*x)^2*tan(2*a)^2*tan(a) + 8*
b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 8*b
^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 + 16*
b^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^2*tan(2*a)*tan(a)^2 + 8*b^2*x^2*imag
_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 8*b^2*x^2*imag_pa
rt(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_inte
gral(4*b*x)*tan(b*x)^2*tan(2*a)*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral
(4*b*x))*tan(2*b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))
*tan(2*b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*
b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*
```

```
tan(b*x)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)
^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2 - 4*b
^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 4*b^2*x^2*
real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_pa
rt(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_int
egral(2*b*x))*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b
*x))*tan(b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(
b*x)^2*tan(2*a)^2 - 8*b*x*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 4*b^2
*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(a)^2 + 4*b^2*x^2*real_
part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_i
ntegral(-2*b*x))*tan(2*b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(-
4*b*x))*tan(2*b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x))*ta
n(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(b*x)^2*tan
(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(a)^2 - 4*b
^2*x^2*real_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(a)^2 - 4*b*x*tan(2*b*
x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x)
)*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*a)^2
*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 +
 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^4}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/x^3,x)
```

```
[Out] int(cos(a + b*x)^4/x^3, x)
```

3.29 $\int (c+dx)^3 \sec(a+bx)\,dx$

Optimal. Leaf size=205

$$-\frac{2i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-6d$$

```
[Out] -2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*
(b*x+a)))/b^2-3*I*d*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)
*polylog(3,-I*exp(I*(b*x+a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))
/b^3-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*
x+a)))/b^4
```

Rubi [A]

time = 0.10, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4266, 2611, 6744, 2320, 6724}

$$-\frac{2i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{6id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 +
(6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4,
 (-I)*E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3 \sec(a+bx)\,dx &= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(3d)\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)\,dx}{b}+\frac{(3d)\int \\
&= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\mathrm{Li}_2}{b^2} \\
&= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\mathrm{Li}_2}{b^2} \\
&= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\mathrm{Li}_2}{b^2} \\
&= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\mathrm{Li}_2}{b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 196, normalized size = 0.96

$$
-\frac{i\left(2b^3(c+dx)^3\mathrm{ArcTan}\left(e^{i(a+bx)}\right)-3d\left(b^2(c+dx)^2\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)+2ibd(c+dx)\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)-2d^2\mathrm{PolyLog}\left(4,-ie^{i(a+bx)}\right)\right)+3d\left(b^2(c+dx)^2\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)+2ibd(c+dx)\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)-2d^2\mathrm{PolyLog}\left(4,ie^{i(a+bx)}\right)\right)\right)}{b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x],x]
```

```
[Out] ((-I)*(2*b^3*(c + d*x)^3*ArcTan[E^(I*(a + b*x))] - 3*d*(b^2*(c + d*x)^2*Pol
yLog[2, (-I)*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a
 + b*x))] - 2*d^2*PolyLog[4, (-I)*E^(I*(a + b*x))]) + 3*d*(b^2*(c + d*x)^2*
PolyLog[2, I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, I*E^(I*(a +
b*x))] - 2*d^2*PolyLog[4, I*E^(I*(a + b*x))])))/b^4
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(180) = 360$.
time = 0.19, size = 685, normalized size = 3.34

method	result
risch	$\frac{d^3 \ln(1-i e^{i(bx+a)}) x^3}{b} - \frac{d^3 \ln(1+i e^{i(bx+a)}) x^3}{b} - \frac{a^3 d^3 \ln(1+i e^{i(bx+a)})}{b^4} + \frac{a^3 d^3 \ln(1-i e^{i(bx+a)})}{b^4} - \frac{2 i c^3 \arctan(e^{i(bx+a)})}{b} + 6$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x
+a)))-6*I/b^2*d^2*c*polylog(2,I*exp(I*(b*x+a)))*x+6*I/b^2*d^2*c*polylog(2,-
I*exp(I*(b*x+a)))*x+6/b^3*d^2*c*polylog(3,I*exp(I*(b*x+a)))+1/b*d^3*ln(1-I*
exp(I*(b*x+a)))*x^3-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3-1/b^4*a^3*d^3*ln(1+I
*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,I
*exp(I*(b*x+a)))*x-6/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))-6/b^3*d^3*polyl
og(3,-I*exp(I*(b*x+a)))*x-2*I/b*c^3*arctan(exp(I*(b*x+a)))-3/b*c^2*d*ln(1+I
*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-3/b^3*a^2*c*d^2*ln(
1-I*exp(I*(b*x+a)))+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*e
xp(I*(b*x+a)))*a+3/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*d^2*c*ln(1+I*exp(
I*(b*x+a)))*x^2+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))+6*I*d^3*polylog(4,I*
exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+3*I/b^2*c^2*d*
polylog(2,-I*exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3
*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*
x+a)))-3*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(167) = 334$.
time = 0.63, size = 722, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*log(sec(b*x + a) + tan(b*x + a)) - 6*a*c^2*d*log(sec(b*x + a) +
tan(b*x + a))/b + 6*a^2*c*d^2*log(sec(b*x + a) + tan(b*x + a))/b^2 - 2*a^3*
d^3*log(sec(b*x + a) + tan(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x
+ I*a)) - 12*I*d^3*polylog(4, -I*e^(I*b*x + I*a)) - 2*(I*(b*x + a)^3*d^3 +
```

```
3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^
2*d^3)*(b*x + a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x + a)^
3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^
2 + I*a^2*d^3)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*(I*b
^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I
*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d + 2*I*a*b*c*d
^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*di
log(-I*e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^
2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*polylog(3, I*e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*polylog(3, -I*e^(I*b*x + I*a)))/b^3)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(167) = 334$.
time = 0.45, size = 970, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d
^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)
) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a)
- sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(
-I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x +
 a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*pol
ylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I
```

```
*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)))/b^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \sec (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a),x)
[Out] Integral((c + d*x)**3*sec(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*sec(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{\cos (a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/cos(a + b*x),x)
[Out] int((c + d*x)^3/cos(a + b*x), x)
```

3.30 $\int (c+dx)^2 \sec(a+bx)\,dx$

Optimal. Leaf size=137

$$-\frac{2i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{Po}}{}$$

```
[Out] -2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+2*I*d*(d*x+c)*polylog(2,-I*exp(I*(b
*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-
I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4266, 2611, 2320, 6724}

$$-\frac{2i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{2d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog
[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog
[3, I*E^(I*(a + b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
```

```
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sec(a+bx)\,dx = -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)\,dx}{b} + \frac{(2d)\int$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2(}{b^2}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2(}{b^2}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2(}{b^2}$$

Mathematica [A]
time = 0.07, size = 130, normalized size = 0.95

$$-\frac{2i\left(b^2(c+dx)^2\mathrm{ArcTan}\left(e^{i(a+bx)}\right)-d\left(b(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)+id\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)\right)+d\left(b(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)+id\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)\right)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(b^2*(c + d*x)^2*ArcTan[E^(I*(a + b*x))] - d*(b*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))] + I*d*PolyLog[3, (-I)*E^(I*(a + b*x))]) + d*(b*(c +
 d*x)*PolyLog[2, I*E^(I*(a + b*x))] + I*d*PolyLog[3, I*E^(I*(a + b*x))])))/
b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(120) = 240.
time = 0.10, size = 392, normalized size = 2.86

method	result
risch	$-\frac{2ic^2\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{2cd\ln\left(1+ie^{i(bx+a)}\right)a}{b^2}+\frac{a^2d^2\ln\left(1+ie^{i(bx+a)}\right)}{b^3}-\frac{2id^2a^2\arctan\left(e^{i(bx+a)}\right)}{b^3}+\frac{2id^2\,\mathrm{polylog}\left(2,-ie^{i(bx}\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x-2/b^2*c*d*ln(1+I*exp(I*(b*x+a)))
*a+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-2*I/b*c^2*arctan(exp(I*(b*x+a)))-2*
I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a))
)+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-I*exp(I
*(b*x+a)))+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+2*d^2*polylog(3,I*exp(I*(b*x+
a)))/b^3+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))+2/b*c*d*ln(1-I*exp(I*(b*x+a))
)*x-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x-2
*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+2/
b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(111) = 222$.
time = 0.59, size = 402, normalized size = 2.93

$$\frac{2c^2\log(\sec(bx+a)+\tan(bx+a)) - \frac{4acd\log(\sec(bx+a)+\tan(bx+a))}{b} + \frac{2a^2d^2\log(\sec(bx+a)+\tan(bx+a))}{b^2} + \frac{4d^2\mathrm{Li}_3(ie^{(ibx+ia)}) - 4d^2\mathrm{Li}_3(-ie^{(ibx+ia)}) - 2\left(i(bx+a)^2d^2+2(ibcd-iad^2)(bx+a)\right)\arctan(\cos(bx+a),\sin(bx+a)+1) - 2\left(i(bx+a)^2d^2+2(ibcd-iad^2)(bx+a)\right)\arctan(\cos(bx+a),-\sin(bx+a)+1) - 4(ibcd+i(bx+a)d^2-iad^2)\mathrm{Li}_2(ie^{(ibx+ia)}) - 4(-ibcd-i(bx+a)d^2+iad^2)\mathrm{Li}_2(-ie^{(ibx+ia)}) + \left((bx+a)^2d^2+2(bcd-ad^2)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\sin(bx+a)+1\right) - \left((bx+a)^2d^2+2(bcd-ad^2)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\sin(bx+a)+1\right)}{b^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*log(sec(b*x + a) + tan(b*x + a)) - 4*a*c*d*log(sec(b*x + a) + ta
n(b*x + a))/b + 2*a^2*d^2*log(sec(b*x + a) + tan(b*x + a))/b^2 + (4*d^2*pol
ylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*b*x + I*a)) - 2*(I*(b
*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a), sin(
b*x + a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*arc
tan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(I*b*c*d + I*(b*x + a)*d^2 - I*a*
d^2)*dilog(I*e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*di
log(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin
(b*x + a) + 1))/b^2)/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(111) = 222$.
time = 0.44, size = 598, normalized size = 4.36

$$-\frac{2d^2\,\mathrm{polylog}(3, i\cos(bx+a)+\sin(bx+a)) - 2d^2\,\mathrm{polylog}(3, i\cos(bx+a)-\sin(bx+a)) + 2d^2\,\mathrm{polylog}(3, -i\cos(bx+a)+\sin(bx+a)) - 2d^2\,\mathrm{polylog}(3, -i\cos(bx+a)-\sin(bx+a)) + 2(ibd^2x+ibcd)\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a)) + 2(ibd^2x+ibcd)\mathrm{Li}_2(i\cos(bx+a)-\sin(bx+a)) + 2(-ibd^2x-ibcd)\mathrm{Li}_2(-i\cos(bx+a)+\sin(bx+a)) + 2(-ibd^2x-ibcd)\mathrm{Li}_2(-i\cos(bx+a)-\sin(bx+a)) - (b^2c^2-2abcd+a^2d^2)\log(\cos(bx+a)+i\sin(bx+a)+i) + (b^2c^2-2abcd+a^2d^2)\log(\cos(bx+a)-i\sin(bx+a)+i) - (b^2d^2x^2+2b^2cdx+2abcd-a^2d^2)\log(i\cos(bx+a)+\sin(bx+a)+1) + (b^2d^2x^2+2b^2cdx+2abcd-a^2d^2)\log(i\cos(bx+a)-\sin(bx+a)+1) - (b^2d^2x^2+2b^2cdx+2abcd-a^2d^2)\log(-i\cos(bx+a)+\sin(bx+a)+1) + (b^2d^2x^2+2b^2cdx+2abcd-a^2d^2)\log(-i\cos(bx+a)-\sin(bx+a)+1) - (b^2c^2-2abcd+a^2d^2)\log(-\cos(bx+a)+i\sin(bx+a)+i) + (b^2c^2-2abcd+a^2d^2)\log(-\cos(bx+a)-i\sin(bx+a)+i)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(I*b*d^2*x + I*
b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*dilog
(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x
 + a) + sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x +
a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)
*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*
b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2
*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) -
 (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) +
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^
3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^2}{\cos (a + b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/cos(a + b*x),x)
```

```
[Out] int((c + d*x)^2/cos(a + b*x), x)
```

3.31 $\int (c + dx) \sec(a + bx) \, dx$

Optimal. Leaf size=75

$$-\frac{2i(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

```
[Out] -2*I*(d*x+c)*arctan(exp(I*(b*x+a)))/b+I*d*polylog(2,-I*exp(I*(b*x+a)))/b^2-
I*d*polylog(2,I*exp(I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4266, 2317, 2438}

$$-\frac{2i(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\sec(a+bx)\,dx = -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-ie^{i(a+bx)}\right)\,dx}{b} + \frac{d\int \log\left(1+ie^{i(a+bx}\right.}{b}$$

$$= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(id)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^2} - \frac{(id)\text{Subs}}{}$$

$$= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}$$

Mathematica [A]
time = 0.01, size = 87, normalized size = 1.16

$$-\frac{2idx\text{ArcTan}\left(e^{ia+ibx}\right)}{b} + \frac{c\tanh^{-1}(\sin(a+bx))}{b} + \frac{id\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{id\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*a + I*b*x)])/b + (c*ArcTanh[Sin[a + b*x]])/b + (I*d
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))]
)/b^2
```

Maple [A]
time = 0.04, size = 128, normalized size = 1.71

method	result
derivativedivides	$\frac{-\frac{da\ln(\sec(bx+a)+\tan(bx+a))}{b}+c\ln(\sec(bx+a)+\tan(bx+a))+\frac{d\left(-(bx+a)\ln\left(1+ie^{i(bx+a)}\right)+(bx+a)\ln\left(1-ie^{i(bx+a)}\right)+i\,\text{dilog}\left(1\right.\right.}{b}}{b}$
default	$\frac{-\frac{da\ln(\sec(bx+a)+\tan(bx+a))}{b}+c\ln(\sec(bx+a)+\tan(bx+a))+\frac{d\left(-(bx+a)\ln\left(1+ie^{i(bx+a)}\right)+(bx+a)\ln\left(1-ie^{i(bx+a)}\right)+i\,\text{dilog}\left(1\right.\right.}{b}}{b}$
risch	$-\frac{2ic\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)x}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)a}{b^2}+\frac{d\ln\left(1-ie^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-ie^{i(bx+a)}\right)a}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*ln(sec(b*x+a)+tan(b*x+a))+c*ln(sec(b*x+a)+tan(b*x+a))+1/b*d*(
-(b*x+a)*ln(1+I*exp(I*(b*x+a)))+(b*x+a)*ln(1-I*exp(I*(b*x+a)))+I*dilog(1+I*
exp(I*(b*x+a)))-I*dilog(1-I*exp(I*(b*x+a)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(4*b*d*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*a)*
sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1), x) + c*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(
b*x + a) + 1) - c*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(60) = 120$.
time = 0.41, size = 306, normalized size = 4.08

$$\frac{-i\,d\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))-i\,d\mathrm{Li}_2(i\,\cos(bx+a)-\sin(bx+a))+i\,d\mathrm{Li}_2(-i\,\cos(bx+a)+\sin(bx+a))+i\,d\mathrm{Li}_2(-i\,\cos(bx+a)-\sin(bx+a))+(bc-ad)\log(\cos(bx+a)+i\,\sin(bx+a)+i)-(bc-ad)\log(\cos(bx+a)-i\,\sin(bx+a)+i)+(bdx+ad)\log(i\,\cos(bx+a)+\sin(bx+a)+1)-(bdx+ad)\log(i\,\cos(bx+a)-\sin(bx+a)+1)+(bdx+ad)\log(-i\,\cos(bx+a)+\sin(bx+a)+1)-(bdx+ad)\log(-i\,\cos(bx+a)-\sin(bx+a)+1)+(bc-ad)\log(-\cos(bx+a)+i\,\sin(bx+a)+i)-(bc-ad)\log(-\cos(bx+a)-i\,\sin(bx+a)+i)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I*c
os(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a*d)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)
- (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-
cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(
b*x + a) + I))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/cos(a + b*x),x)
```

```
[Out] int((c + d*x)/cos(a + b*x), x)
```

3.32 $\int \frac{\sec(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(sec(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(a+bx)}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Sec[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)}{c+dx} \, dx = \int \frac{\sec(a+bx)}{c+dx} \, dx$$

Mathematica [A]
time = 3.04, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Sec[a + b*x]/(c + d*x), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/(d*x+c),x)
[Out] int(sec(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*x+c),x)
[Out] Integral(sec(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)*(c + d*x)), x)
```

3.33 $\int (c+dx)^3 \sec^2(a+bx)\,dx$

Optimal. Leaf size=114

$$-\frac{i(c+dx)^3}{b}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{3id^2(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,-e^{2i(a+b}\right.}{2b^4}$$

```
[Out] -I*(d*x+c)^3/b+3*d*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*I*d^2*(d*x+c)*pol
ylog(2,-exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+(d*x
+c)^3*tan(b*x+a)/b
```

Rubi [A]
time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4269, 3800, 2221, 2611, 2320, 6724}

$$\frac{3d^3\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{i(c+dx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b + (3*d*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^2 -
 ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLo
g[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[a + b*x])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\sec^2(a+bx)\,dx &= \frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\tan(a+bx)\,dx}{b}\\
&= -\frac{i(c+dx)^3}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}+\frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx}{b}\\
&= -\frac{i(c+dx)^3}{b}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(6d^2)}{}\\
&= -\frac{i(c+dx)^3}{b}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{3id^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\\
&= -\frac{i(c+dx)^3}{b}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{3id^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\\
&= -\frac{i(c+dx)^3}{b}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{3id^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+
\end{aligned}$$

Mathematica [A]
time = 0.35, size = 109, normalized size = 0.96

$$\frac{-6ibd^2(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)+3d^3\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)+2b^2(c+dx)^2\left(-ib(c+dx)+3d\log\left(1+e^{2i(a+bx)}\right)+b(c+dx)\tan(a+bx)\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] + 3*d^3*PolyLog[3,
 -E^((2*I)*(a + b*x))] + 2*b^2*(c + d*x)^2*((-I)*b*(c + d*x) + 3*d*Log[1 +
E^((2*I)*(a + b*x))] + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(104) = 208$.
time = 0.13, size = 316, normalized size = 2.77

method	result
risch	$\frac{6id^3a^2x}{b^3} - \frac{6dc^2\ln(\mathrm{e}^{i(bx+a)})}{b^2} + \frac{3dc^2\ln(\mathrm{e}^{2i(bx+a)}+1)}{b^2} - \frac{6d^3a^2\ln(\mathrm{e}^{i(bx+a)})}{b^4} - \frac{3id^3\,\mathrm{polylog}(2,-\mathrm{e}^{2i(bx+a)})x}{b^3} + \frac{6d^2c\ln(\mathrm{e}^{2i(bx}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3*I*d^2/b^3*c*polylog(2,-exp(2*I*(b*x+a)))-6*d/b^2*c^2*ln(exp(I*(b*x+a)))+
3*d/b^2*c^2*ln(exp(2*I*(b*x+a))+1)-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+6*I*d^3
/b^3*a^2*x+6*d^2/b^2*c*ln(exp(2*I*(b*x+a))+1)*x+2*I*(d^3*x^3+3*c*d^2*x^2+3*
c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))+1)+4*I*d^3/b^4*a^3+3*d^3/b^2*ln(exp(2*I*(b
*x+a))+1)*x^2-6*I*d^2/b*c*x^2+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+12*d
^2/b^3*c*a*ln(exp(I*(b*x+a)))-2*I*d^3/b*x^3-3*I*d^3/b^3*polylog(2,-exp(2*I*
(b*x+a)))*x-6*I*d^2/b^3*c*a^2-12*I*d^2/b^2*c*a*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(101) = 202$.
time = 0.61, size = 1059, normalized size = 9.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*tan(b*x + a) - 6*a*c^2*d*tan(b*x + a)/b + 6*a^2*c*d^2*tan(b*x +
a)/b^2 - 2*a^3*d^3*tan(b*x + a)/b^3 + 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) - 6*((cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x +
2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b
*x + 2*a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1)*b^2) + 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*
x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
```

```
*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3) + 2*(6*((b*x + a)^2*d^3 + 2*
(b*c*d^2 - a*d^3)*(b*x + a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x
 + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) -
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - 6*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3 + (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*
b*x + 2*a) + (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(2*b*x + 2*a))*dilo
g(-e^(2*I*b*x + 2*I*a)) - 3*(I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b
*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a
))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) -
3*(I*d^3*cos(2*b*x + 2*a) - d^3*sin(2*b*x + 2*a) + I*d^3)*polylog(3, -e^(2*
I*b*x + 2*I*a)) - 4*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^
2)*sin(2*b*x + 2*a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) -
2*I*b^3))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(101) = 202$.
time = 0.43, size = 790, normalized size = 6.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*
polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3,
-I*cos(b*x + a) - sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*d
ilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a
)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x
+ a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos
(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a
) + I) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x +
a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1
) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*lo
g(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*
a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*s
in(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-
cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\cos(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/cos(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^3/cos(a + b*x)^2, x)
```

3.34 $\int (c+dx)^2 \sec^2(a+bx)\,dx$

Optimal. Leaf size=82

$$-\frac{i(c+dx)^2}{b}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^2\tan(a+bx)}{b}$$

```
[Out] -I*(d*x+c)^2/b+2*d*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^2-I*d^2*polylog(2,-exp(
2*I*(b*x+a)))/b^3+(d*x+c)^2*tan(b*x+a)/b
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4269, 3800, 2221, 2317, 2438}

$$-\frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b + (2*d*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^2 - (
I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^2*Tan[a + b*x])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\sec^2(a+bx)\,dx &= \frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(2d)\int (c+dx)\tan(a+bx)\,dx}{b}\\ &= -\frac{i(c+dx)^2}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}+\frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx}{b}\\ &= -\frac{i(c+dx)^2}{b}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(2d^2}{}\\ &= -\frac{i(c+dx)^2}{b}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}+\frac{(id^2)}{}\\ &= -\frac{i(c+dx)^2}{b}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx}{}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 75, normalized size = 0.91

$$\frac{-id^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)+b(c+dx)\left(-ib(c+dx)+2d\log\left(1+e^{2i(a+bx)}\right)+b(c+dx)\tan(a+bx)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*d^2*PolyLog[2, -E^((2*I)*(a + b*x))] + b*(c + d*x)*((-I)*b*(c + d*x)
+ 2*d*Log[1 + E^((2*I)*(a + b*x))] + b*(c + d*x)*Tan[a + b*x]))/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(76) = 152$.
time = 0.07, size = 170, normalized size = 2.07

method	result
risch	$\frac{2i(d^2x^2+2cdx+c^2)}{b(\mathrm{e}^{2i(bx+a)}+1)}-\frac{4dc\ln(\mathrm{e}^{i(bx+a)})}{b^2}+\frac{2dc\ln(\mathrm{e}^{2i(bx+a)}+1)}{b^2}-\frac{2id^2x^2}{b}-\frac{4id^2ax}{b^2}-\frac{2id^2a^2}{b^3}+\frac{2d^2\ln(\mathrm{e}^{2i(bx+a)}+1)x}{b^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)-4*d/b^2*c*ln(exp(I*(b*x+a)
))+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)-2*I*d^2/b*x^2-4*I*d^2/b^2*a*x-2*I*d^2/b
^3*a^2+2*d^2/b^2*ln(exp(2*I*(b*x+a))+1)*x-I*d^2*polylog(2,-exp(2*I*(b*x+a))
)/b^3+4*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(73) = 146$.
time = 0.58, size = 324, normalized size = 3.95

$$\frac{2\,b^2c^2 + 2\,(bd^2x + bcd + (bd^2x + bcd)\cos(2\,bx + 2\,a) - (-i\,bd^2x - i\,bcd)\sin(2\,bx + 2\,a))\arctan(\sin(2\,bx + 2\,a), \cos(2\,bx + 2\,a) + 1) - 2\,(b^2d^2x^2 + 2\,b^2cdx)\cos(2\,bx + 2\,a) - (d^2\cos(2\,bx + 2\,a) + i\,d^2\sin(2\,bx + 2\,a) + d^2)\mathrm{Li}_2(-e^{(2i\,bx + 2i\,a)}) + (-i\,bd^2x - i\,bcd + (-i\,bd^2x - i\,bcd)\cos(2\,bx + 2\,a) + (bd^2x + bcd)\sin(2\,bx + 2\,a))\log(\cos(2\,bx + 2\,a)^2 + \sin(2\,bx + 2\,a)^2 + 2\,\cos(2\,bx + 2\,a) + 1) - 2\,(i\,b^2d^2x^2 + 2i\,b^2cdx)\sin(2\,bx + 2\,a)}{-i\,b^3\cos(2\,bx + 2\,a) + b^3\sin(2\,bx + 2\,a) - i\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b^2*c^2 + 2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (-I*
b*d^2*x - I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x +
2*a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(2*b*x + 2*a) - (d^2*cos(2*b*x
 + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + (-I*b
*d^2*x - I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c
*d)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1) - 2*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x)*sin(2*b*x + 2*a))/(-I*
b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) - I*b^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(73) = 146$.
time = 0.43, size = 450, normalized size = 5.49

$$\frac{i\,d^2\cos(bx + a)\,\mathrm{Li}_2(i\,\cos(bx + a) + \sin(bx + a)) - i\,d^2\cos(bx + a)\,\mathrm{Li}_2(i\,\cos(bx + a) - \sin(bx + a)) - i\,d^2\cos(bx + a)\,\mathrm{Li}_2(-i\,\cos(bx + a) + \sin(bx + a)) + i\,d^2\cos(bx + a)\,\mathrm{Li}_2(-i\,\cos(bx + a) - \sin(bx + a)) + (bcd - ad^2)\cos(bx + a)\log(\cos(bx + a) + i\,\sin(bx + a) + i) + (bcd - ad^2)\cos(bx + a)\log(\cos(bx + a) - i\,\sin(bx + a) + i) + (bd^2x + ad^2)\cos(bx + a)\log(i\,\cos(bx + a) + \sin(bx + a) + 1) + (bd^2x + ad^2)\cos(bx + a)\log(i\,\cos(bx + a) - \sin(bx + a) + 1) + (bd^2x + ad^2)\cos(bx + a)\log(-i\,\cos(bx + a) + \sin(bx + a) + 1) + (bd^2x + ad^2)\cos(bx + a)\log(-i\,\cos(bx + a) - \sin(bx + a) + 1) + (bcd - ad^2)\cos(bx + a)\log(-\cos(bx + a) + i\,\sin(bx + a) + i) + (bcd - ad^2)\cos(bx + a)\log(-\cos(bx + a) - i\,\sin(bx + a) + i) + (b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\sin(bx + a)}{b^3\cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b*x +
a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b
*x + a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*
x + a)) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) +
I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) +
(b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*
d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d^2
*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x
```

```
 + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c*d - a
*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2
)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\cos(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/cos(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^2/cos(a + b*x)^2, x)
```

3.35 $\int (c+dx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}$$

```
[Out] d*ln(cos(b*x+a))/b^2+(d*x+c)*tan(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4269, 3556}

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2,x]
```

```
[Out] (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec^2(a+bx)\,dx&=\frac{(c+dx)\tan(a+bx)}{b}-\frac{d\int\tan(a+bx)\,dx}{b}\\&=\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.29

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{c\tan(a+bx)}{b}+\frac{dx\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2,x]
```

```
[Out] (d*Log[Cos[a + b*x]])/b^2 + (c*Tan[a + b*x])/b + (d*x*Tan[a + b*x])/b
```

Maple [A]
time = 0.05, size = 52, normalized size = 1.86

method	result	s
derivativedivides	$\frac{-\frac{da\tan(bx+a)}{b}+c\tan(bx+a)+\frac{d((bx+a)\tan(bx+a)+\ln(\cos(bx+a)))}{b}}{b}$	5
default	$\frac{-\frac{da\tan(bx+a)}{b}+c\tan(bx+a)+\frac{d((bx+a)\tan(bx+a)+\ln(\cos(bx+a)))}{b}}{b}$	5
risch	$-\frac{2idx}{b}-\frac{2ida}{b^2}+\frac{2i(dx+c)}{b(\mathrm{e}^{2i(bx+a)}+1)}+\frac{d\ln(\mathrm{e}^{2i(bx+a)}+1)}{b^2}$	5
norman	$\frac{-\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b^2}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b^2}-\frac{d\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}$	[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*tan(b*x+a)+c*tan(b*x+a)+1/b*d*((b*x+a)*tan(b*x+a)+ln(cos(b*x+
a))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(28) = 56$.
time = 0.52, size = 159, normalized size = 5.68

$$\frac{2\,c\tan\left(bx+a\right)-\frac{2\,ad\tan(bx+a)}{b}+\frac{\left(\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)+4\,(bx+a)\sin(2\,bx+2\,a)\right)d}{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*tan(b*x + a) - 2*a*d*tan(b*x + a)/b + ((cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

Fricas [A]
time = 0.38, size = 45, normalized size = 1.61

$$\frac{d\cos\left(bx+a\right)\log\left(-\cos\left(bx+a\right)\right)+\left(bdx+bc\right)\sin\left(bx+a\right)}{b^2\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a)*log(-cos(b*x + a)) + (b*d*x + b*c)*sin(b*x + a))/(b^2*cos(b
*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \sec^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sec(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. 2(28) = 56.
time = 0.66, size = 1459, normalized size = 52.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2*a)^2
 - d*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8
*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a)
 + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1
/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*ta
n(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b
*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*
a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(
1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*c*tan(1/2
*b*x)^2*tan(1/2*a) + 4*b*c*tan(1/2*b*x)*tan(1/2*a)^2 - 4*b*d*x*tan(1/2*b*x)
 + d*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8
*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a)
 + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1
/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*ta
n(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b
*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*
a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(
1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 - 4*b*d*x*tan(1/2*a) + 4*d*l
og(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1
/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*
tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x
```

```
)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*
a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*
tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 +
 tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)
^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + d*log(4*(tan(1/2*b*x)^8
*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)
^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1
/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8
*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x
)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(
1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*ta
n(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2
+ 1))*tan(1/2*a)^2 - 4*b*c*tan(1/2*b*x) - 4*b*c*tan(1/2*a) - d*log(4*(tan(1
/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*t
an(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x
)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2
*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*ta
n(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a)
+ 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)
^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(
1/2*a)^2 + 1)))/(b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*tan(1/2*b*x)^2 - 4*b
^2*tan(1/2*b*x)*tan(1/2*a) - b^2*tan(1/2*a)^2 + b^2)
```

Mupad [B]
time = 0.83, size = 55, normalized size = 1.96

$$\frac{d\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+1\right)}{b^2}+\frac{\left(c+d\,x\right)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}-\frac{d\,x\,2\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/cos(a + b*x)^2,x)
```

```
[Out] (d*log(exp(a*2i)*exp(b*x*2i) + 1))/b^2 + ((c + d*x)*2i)/(b*(exp(a*2i + b*x*
2i) + 1)) - (d*x*2i)/b
```

3.36 $\int \frac{\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(sec(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[a + b*x]^2/(c + d*x),x]
[Out] Defer[Int][Sec[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 3.75, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[a + b*x]^2/(c + d*x),x]
[Out] Integrate[Sec[a + b*x]^2/(c + d*x), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 2*((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c
*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(si
n(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*
c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^
2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + sin(2*b*x + 2
*a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x +
2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sec(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2/(d*x + c), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^2*(c + d*x)), x)
```

3.37 $\int (c+dx)^3 \sec^3(a+bx)\,dx$

Optimal. Leaf size=337

$$-\frac{6id^2(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4}+\frac{3id(c+dx)^2\text{Po}}{}$$

```
[Out] -6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3-I*(d*x+c)^3*arctan(exp(I*(b*x+a
)))/b+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,
-I*exp(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x
+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x
+a)))/b^3+3*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3-3*I*d^3*polylog(4,-
I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/2*d*(d*x+c)
^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A]
time = 0.16, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4271, 4266, 2317, 2438, 2611, 6744, 2320, 6724}

$$-\frac{6id^2(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{3id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^3,x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - (I*(c + d*x)^3*ArcTan[
E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + ((
(3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*P
olyLog[2, I*E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E
^(I*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyL
og[4, (-I)*E^(I*(a + b*x))])/b^4 + ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
)/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*
Tan[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3\sec^3(a+bx)\,dx &= -\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b}+\frac{1}{2}\int (c+dx \\
&= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\operatorname{se}}{2b^2} \\
&= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{L}}{2b} \\
&= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\mathrm{Li}_2\left(-ie^{i(a}\right.}{b^4} \\
&= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\mathrm{Li}_2\left(-ie^{i(a}\right.}{b^4} \\
&= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\mathrm{Li}_2\left(-ie^{i(a}\right.}{b^4}
\end{aligned}
$$

Mathematica [A]
time = 1.82, size = 311, normalized size = 0.92

$$\frac{-2ib^3(c+dx)^3\mathrm{ArcTan}\left(e^{i(a+bx)}\right)-6id^2\left(2b(c+dx)\mathrm{ArcTan}\left(e^{i(a+bx)}\right)-d\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)+d\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)\right)+3id\left(b^2(c+dx)^2\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)+2ibd(c+dx)\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)-2d^2\mathrm{PolyLog}\left(4,-ie^{i(a+bx)}\right)\right)-3id\left(b^2(c+dx)^2\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)+2ibd(c+dx)\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)-2d^2\mathrm{PolyLog}\left(4,ie^{i(a+bx)}\right)\right)-3b^2d(c+dx)^2\sec(a+bx)+b^3(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^3,x]
```

```
[Out] ((-2*I)*b^3*(c + d*x)^3*ArcTan[E^(I*(a + b*x))] - (6*I)*d^2*(2*b*(c + d*x)*
ArcTan[E^(I*(a + b*x))] - d*PolyLog[2, (-I)*E^(I*(a + b*x))] + d*PolyLog[2,
 I*E^(I*(a + b*x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b
*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLo
g[4, (-I)*E^(I*(a + b*x))]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a
 + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))] - 2*d^2*PolyL
og[4, I*E^(I*(a + b*x))]) - 3*b^2*d*(c + d*x)^2*Sec[a + b*x] + b^3*(c + d*x
)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. 2(293) = 586.
time = 0.27, size = 1127, normalized size = 3.34

method	result	size
risch	Expression too large to display	1127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-1/2/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)
))+3/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x+3/b^3*c*d^2*polylog(3,I*exp(I*(b
*x+a)))-3/b^3*c*d^2*polylog(3,-I*exp(I*(b*x+a)))-I/b*c^3*arctan(exp(I*(b*x+
a)))-1/2/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+1/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*
x^3-3/b^3*d^3*ln(1+I*exp(I*(b*x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a+3
/b^3*d^3*ln(1-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-3/b^3*
d^3*polylog(3,-I*exp(I*(b*x+a)))*x+6*I/b^4*d^3*a*arctan(exp(I*(b*x+a)))-6*I
/b^3*c*d^2*arctan(exp(I*(b*x+a)))+3/2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a))
)*x^2-3/2*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog
(2,I*exp(I*(b*x+a)))+I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))-3/2/b^3*a^2*c*d^2
*ln(1-I*exp(I*(b*x+a)))-3/2/b*c*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+3/2/b*c*d^2*
ln(1-I*exp(I*(b*x+a)))*x^2+3/2/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))+3/2/b*c
^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/2/b*
c^2*d*ln(1+I*exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+3/2*I
/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-I/b^2/(exp(2*I*(b*x+a))+1)^2*(d^3*x
^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b
*x+a))-d^3*x^3*b*exp(I*(b*x+a))+c^3*b*exp(3*I*(b*x+a))-3*c*d^2*x^2*b*exp(I*
(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*x+a))-3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*
x*exp(3*I*(b*x+a))-c^3*b*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))-3*I*d^3*
x^2*exp(I*(b*x+a))-6*I*c*d^2*x*exp(I*(b*x+a))-3*I*c^2*d*exp(I*(b*x+a)))-3*I
*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b
^4+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,I*exp(I*(b*x+
a)))/b^4-3*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x+3*I/b^2*c*d^2*polylog(
2,-I*exp(I*(b*x+a)))*x-3*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+3*I/b^2*c^2
*d*a*arctan(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3831 vs. 2(273) = 546.
time = 1.98, size = 3831, normalized size = 11.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
g(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log
(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x + a)
/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
```

```
g(sin(b*x + a) - 1))/b^3 + 4*(2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)
*(b*x + a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(
b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(4*b
*x + 4*a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*
(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(2*
b*x + 2*a) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(I*b*c*d^2 -
I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*
(b*x + a))*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^
3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 +
(I*a^2 + 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b
*x + a) + 1) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a) + ((
b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*
(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(
(b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I
*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x +
 a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x + a))*sin(
4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(I*b*c*d^
2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) +
 4*((b*x + a)^3*d^3 - 3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3 + 3*(b*c*
d^2 - (a + I)*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*(a + I)*b*c*d^2 + (a^2 +
2*I*a)*d^3)*(b*x + a))*cos(3*b*x + 3*a) - 4*((b*x + a)^3*d^3 + 3*I*b^2*c^2*
d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3 + 3*(b*c*d^2 - (a - I)*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*(a - I)*b*c*d^2 + (a^2 - 2*I*a)*d^3)*(b*x + a))*cos(b*x + a
) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 +
 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (
I*a^2 + 2*I)*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*
(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 + 2*I)*d^3 + 2*(I
*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) -
 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2
- a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*cos(2*b*x + 2*a) - (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I
*a^2 - 2*I)*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*
(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 - 2*I)*d^3 + 2*
(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a
)) - (-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3
)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x +
```

```
a) + (-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3
)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(
I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2
+ 2*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*
a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6
*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^
2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) - (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 - 3*
(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*
a^2 - 2*I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 -
3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-
I*a^2 - 2*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)^3*d^3 - 6*I
*b*c*d^2 + 6*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2
*d + 2*I*a*b*c*d^2 + (-I*a^2 - 2*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*
x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(273) = 546$.
time = 0.49, size = 1315, normalized size = 3.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I
*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos
(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^
2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + si
n(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3)*c
os(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*
I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a)
 + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*
d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*
b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(cos(b
*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*
d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I
) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 +
 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) +
sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I
*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^
```

```
2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(
b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d +
 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*
c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2
*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^
2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(
b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos(b*x +
 a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos
(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3,
 -I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*pol
ylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
b^2*c^2*d)*cos(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
+ b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \sec^3 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*sec(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^3, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/cos(a + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

3.38 $\int (c+dx)^2 \sec^3(a+bx)\,dx$

Optimal. Leaf size=193

$$-\frac{i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLo}}{b^2}$$

```
[Out] -I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+d^2*arctanh(sin(b*x+a))/b^3+I*d*(d*x+
c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))
/b^2-d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+d^2*polylog(3,I*exp(I*(b*x+a)))/b
^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A]
time = 0.10, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4271, 3855, 4266, 2611, 2320, 6724}

$$-\frac{i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/
b^3 + (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*(c + d*x)
*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))]
)/b^3 + (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x]
)/b^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\sec^3(a+bx)\,dx = -\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\sec(a+bx)\tan(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^2$$

$$= -\frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+b}{b^2}$$

$$= -\frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(}\right.}{b^2}$$

$$= -\frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(}\right.}{b^2}$$

$$= -\frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(}\right.}{b^2}$$

Mathematica [A]

time = 0.62, size = 184, normalized size = 0.95

$$\frac{-2ib^2(c+dx)^2\mathrm{ArcTan}\left(e^{i(a+bx)}\right)+2d^2\tanh^{-1}(\sin(a+bx))+2ibd(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)-2ibd(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)-2d^2\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)+2d^2\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)-2bd(c+dx)\sec(a+bx)+b^2(c+dx)^2\sec(a+bx)\tan(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^2*ArcTan[E^(I*(a + b*x))] + 2*d^2*ArcTanh[Sin[a + b*x
]] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c +
d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))]
 + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 2*b*d*(c + d*x)*Sec[a + b*x] + b^2
*(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(174) = 348$.
time = 0.16, size = 584, normalized size = 3.03

method	result
risch	$\frac{2icda\arctan\left(e^{i(bx+a)}\right)}{b^2}+\frac{icd\,\mathrm{polylog}\left(2,-ie^{i(bx+a)}\right)}{b^2}-\frac{id^2a^2\arctan\left(e^{i(bx+a)}\right)}{b^3}-\frac{id^2\,\mathrm{polylog}\left(2,ie^{i(bx+a)}\right)x}{b^2}-\frac{ic^2\arctan\left(e^{i(bx+a}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))+
I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-I/b^2*d^2*polylog(2,I*exp(I*(b*x+a
)))*x-I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-d^2*polylog(3,-I*exp(I*(b*x+a)))
/b^3-1/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-1/b^2*c*d*ln(1+I*exp(I*(b*x+a))
)*a+d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-I/b*c^2*arctan(exp(I*(b*x+a)))+I/b^
2*c*d*polylog(2,-I*exp(I*(b*x+a)))-1/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+1/b^2*c
*d*ln(1-I*exp(I*(b*x+a)))*a+1/b*c*d*ln(1-I*exp(I*(b*x+a)))*x+1/2/b^3*a^2*d^
2*ln(1+I*exp(I*(b*x+a)))-2*I/b^3*d^2*arctan(exp(I*(b*x+a)))-I/b^2/(exp(2*I*
(b*x+a))+1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+c^2*b*
exp(3*I*(b*x+a))-d^2*x^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a))-2*I*d^2*
x*exp(3*I*(b*x+a))-c^2*b*exp(I*(b*x+a))-2*I*c*d*exp(3*I*(b*x+a))-2*I*d^2*x*
exp(I*(b*x+a))-2*I*c*d*exp(I*(b*x+a)))-1/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2
+1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1891 vs. $2(165) = 330$.
time = 0.87, size = 1891, normalized size = 9.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
g(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(s
```

```
in(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(sin(
b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2 + 4*(2
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + ((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^
2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*
d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b
*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*b*x + 2*a)
)*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d -
 a*d^2)*(b*x + a) + 2*d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
 + 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a
) + 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b
*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I
*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b
*x + a) + 1) + 4*((b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*a*d^2 + 2*(b*c*d - (a +
 I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) - 4*((b*x + a)^2*d^2 + 2*I*b*c*d - 2*I
*a*d^2 + 2*(b*c*d - (a - I)*d^2)*(b*x + a))*cos(b*x + a) + 4*(b*c*d + (b*x
 + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*
c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2
 - I*a*d^2)*sin(4*b*x + 4*a) + 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2
*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 +
(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2
 - a*d^2)*cos(2*b*x + 2*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b
*x + 4*a) - 2*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilo
g(-I*e^(I*b*x + I*a)) - (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x +
a) - 2*I*d^2 + (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*
d^2)*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
 a) + 2*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a) + 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b
*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
sin(b*x + a) + 1) - (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) +
 2*I*d^2 + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)
*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a
) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
 a) + 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*si
n(b*x + a) + 1) + 4*(I*d^2*cos(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2*a) - d^
2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x + 2*a) + I*d^2)*polylog(3, I*e^(I*b*x
+ I*a)) + 4*(-I*d^2*cos(4*b*x + 4*a) - 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4
*b*x + 4*a) + 2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -I*e^(I*b*x + I*a)
) + 4*(I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(I*b*c*d + (-I*a + 1)*d^2)
*(b*x + a))*sin(3*b*x + 3*a) + 4*(-I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 +
2*(-I*b*c*d + (I*a + 1)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^2*cos(4*b*x +
 4*a) - 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) + 8*b^2*sin(2*b*x
 + 2*a) - 4*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(165) = 330$.
time = 0.44, size = 795, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^
2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x
+ a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*po
lylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*cos(b*x
+ a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*cos(b
*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*c
os(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c
*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b
*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)
 + (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) -
I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos
(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a
) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*l
og(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b
*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b
^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*si
n(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(
-cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b*d^2*x + b*c*d)*cos(b*x + a) - 2*
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/cos(a + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

3.39 $\int (c + dx) \sec^3(a + bx) \, dx$

Optimal. Leaf size=117

$$-\frac{i(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\,\text{se}}{}$$

```
[Out] -I*(d*x+c)*arctan(exp(I*(b*x+a)))/b+1/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/b^
2-1/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c)*
sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A]

time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4270, 4266, 2317, 2438}

$$-\frac{i(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^3,x]
```

```
[Out] ((-I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + ((I/2)*d*PolyLog[2, (-I)*E^(I*
(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a +
 b*x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int (c+dx)\sec^3(a+bx)\,dx = -\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b}+\frac{1}{2}\int (c+dx)\sec(a+bx$$

$$= -\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\sec(a+bx)\tan(a+bx}{2b}$$

$$= -\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\sec(a+bx)\tan(a+bx}{2b}$$

$$= -\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{id\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+}{2b^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 389 vs. 2(117) = 234.
time = 2.42, size = 389, normalized size = 3.32

$$\frac{c\tanh^{-1}(\sin(a+bx))}{2b}+\frac{d\left(\left(-a+\frac{\pi}{2}-bx\right)\left(\log\left(1-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\log\left(1+e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)-\left(-a+\frac{\pi}{2}\right)\log\left(\tan\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-bx\right)\right)\right)+i\left(\mathrm{PolyLog}\left(2,-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\mathrm{PolyLog}\left(2,e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)\right)}{2b^2}+\frac{dx}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}-\frac{d\sin\left(\frac{bx}{2}\right)}{2b^2\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}-\frac{dx}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}+\frac{d\sin\left(\frac{bx}{2}\right)}{2b^2\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}+\frac{c\sec(a+bx)\tan(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^3,x]

[Out] (c*ArcTanh[Sin[a + b*x]])/(2*b) + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a +
 Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-
a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2,
E^(I*(-a + Pi/2 - b*x))])))/(2*b^2) + (d*x)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[
a/2 + (b*x)/2])^2) - (d*Sin[(b*x)/2])/(2*b^2*(Cos[a/2] - Sin[a/2])*(Cos[a/2
 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (d*x)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a
/2 + (b*x)/2])^2) + (d*Sin[(b*x)/2])/(2*b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2
+ (b*x)/2] + Sin[a/2 + (b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(98) = 196.
time = 0.09, size = 267, normalized size = 2.28

method	result

risch	$-\frac{i\left(dxb\,e^{3i(bx+a)}+bc\,e^{3i(bx+a)}-dxb\,e^{i(bx+a)}-bc\,e^{i(bx+a)}-id\,e^{3i(bx+a)}-id\,e^{i(bx+a)}\right)}{b^2\left(e^{2i(bx+a)}+1\right)^2}-\frac{ic\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)x}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I/b^2/(exp(2*I*(b*x+a))+1)^2*(d*x*b*exp(3*I*(b*x+a))+b*c*exp(3*I*(b*x+a))-
d*x*b*exp(I*(b*x+a))-b*c*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))-I*d*exp(I*(b*x
+a)))-I/b*c*arctan(exp(I*(b*x+a)))-1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/2/b^2
*d*ln(1+I*exp(I*(b*x+a)))*a+1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/2/b^2*d*ln(1
-I*exp(I*(b*x+a)))*a+1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-1/2*I/b^2*d*dilo
g(1-I*exp(I*(b*x+a)))+I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*cos(3*b*x + 3*a) + d*cos(b*x + a) - (b*d*x + b*c)*sin(3*b*x + 3*
a) + (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 4*(2*d*cos(2*b*x + 2*a)
 + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 8*(d*cos(b*x +
a) + (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 4*d*cos(b*x + a) - 4*(b
^2*d*cos(4*b*x + 4*a)^2 + 4*b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(4*b*x + 4*
a)^2 + 4*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*d*sin(2*b*x + 2*a)
^2 + 4*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(2*b^2*d*cos(2*b*x + 2*a) + b^2*d
)*cos(4*b*x + 4*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*
x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*cos(2
*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2*b*c*
cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*sin(b*x + a) + 1) + (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*cos(2*b*x + 2*a)
^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b
*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2*b*c*cos(2*b*x +
 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(
b*x + a) + 1) + 4*((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d*x + b*c)*cos(b*x +
 a) + d*sin(3*b*x + 3*a) + d*sin(b*x + a))*sin(4*b*x + 4*a) - 4*(b*d*x + b*
c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*sin(3*b*x + 3*
a) - 8*((b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))*sin(2*b*x + 2*a) + 4*(
b*d*x + b*c)*sin(b*x + a))/(b^2*cos(4*b*x + 4*a)^2 + 4*b^2*cos(2*b*x + 2*a)
^2 + b^2*sin(4*b*x + 4*a)^2 + 4*b^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b
```

```
^2*sin(2*b*x + 2*a)^2 + 4*b^2*cos(2*b*x + 2*a) + b^2 + 2*(2*b^2*cos(2*b*x +
 2*a) + b^2)*cos(4*b*x + 4*a))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(93) = 186$.
time = 0.41, size = 435, normalized size = 3.72

$$\frac{-i\,d\cos\left(bx+a\right)^{2}\mathrm{Li}_2(i\cos\left(bx+a\right)+\sin\left(bx+a\right))-i\,d\cos\left(bx+a\right)^{2}\mathrm{Li}_2(i\cos\left(bx+a\right)-\sin\left(bx+a\right))+i\,d\cos\left(bx+a\right)^{2}\mathrm{Li}_2(-i\cos\left(bx+a\right)+\sin\left(bx+a\right))+i\,d\cos\left(bx+a\right)^{2}\mathrm{Li}_2(-i\cos\left(bx+a\right)-\sin\left(bx+a\right))+(bc-ad)\cos\left(bx+a\right)^{2}\log\left(\cos\left(bx+a\right)+i\sin\left(bx+a\right)+i\right)-(bc-ad)\cos\left(bx+a\right)^{2}\log\left(\cos\left(bx+a\right)-i\sin\left(bx+a\right)+i\right)+(bdx+ad)\cos\left(bx+a\right)^{2}\log\left(i\cos\left(bx+a\right)+\sin\left(bx+a\right)+1\right)-(bdx+ad)\cos\left(bx+a\right)^{2}\log\left(i\cos\left(bx+a\right)-\sin\left(bx+a\right)+1\right)+(bdx+ad)\cos\left(bx+a\right)^{2}\log\left(-i\cos\left(bx+a\right)+\sin\left(bx+a\right)+1\right)-(bdx+ad)\cos\left(bx+a\right)^{2}\log\left(-i\cos\left(bx+a\right)-\sin\left(bx+a\right)+1\right)+(bc-ad)\cos\left(bx+a\right)^{2}\log\left(-\cos\left(bx+a\right)+i\sin\left(bx+a\right)+i\right)-(bc-ad)\cos\left(bx+a\right)^{2}\log\left(-\cos\left(bx+a\right)-i\sin\left(bx+a\right)+i\right)-2\,d\cos\left(bx+a\right)+2\,(bdx+bc)\sin\left(bx+a\right)}{4\,b^{2}\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*cos(b*x
 + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*
cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - s
in(b*x + a)) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) +
 (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d
*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x +
a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)
*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*cos(b*x + a
)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) + 2*(b*d*x +
 b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \sec^{3}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*sec(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/cos(a + b*x)^3,x)
[Out] \text{Hanged}
```

3.40 $\int \frac{\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(sec(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Sec[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Sec[a + b*x]^2/(c + d*x), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 2*((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c
*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(si
n(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*
c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^
2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + sin(2*b*x + 2
*a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x +
2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sec(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^2*(c + d*x)), x)
```

3.41 $\int (c+dx)^{5/2} \cos(a+bx)\, dx$

Optimal. Leaf size=194

$$\frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\cdots}}{\cdots}\right.}{4b^{7/\cdots}}$$

```
[Out] 5/2*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2+(d*x+c)^(5/2)*sin(b*x+a)/b+15/8*d^(5/2)*
cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/
2)*Pi^(1/2)/b^(7/2)+15/8*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/4*d^2*sin(b*x+a)*(d
*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.29, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x],x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(2*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a -
 (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(7/2))
 + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[a - (b*c)/d])/(4*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(4*
b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \frac{(c+dx)^{5/2}\sin(a+bx)}{b} - \frac{(5d)\int (c+dx)^{3/2}\sin(a+bx)\,dx}{2b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} - \frac{(15d^2)\int \sqrt{c+dx}\ \mathrm{co}}{4b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\ \sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+}{b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\ \sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+}{b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\ \sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+}{b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\ \cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right.}{4b^{7/2}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.04, size = 124, normalized size = 0.64

$$-\frac{d^3 e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x],x]
```

```
[Out] -1/2*(d^3*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((-I)*b*(c + d
*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, (I*b*(c + d*
x))/d]))/(b^4*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A]
time = 0.06, size = 232, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}-\frac{d\sqrt{2}}{}\right.}{}\right.}{}}{d}$
default	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}-\frac{d\sqrt{2}}{}\right.}{}\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(5/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-5/2/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 263, normalized size = 1.36

$$\frac{\sqrt{2}\left(40\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-15\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-15\left((i-1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)+4\left(4\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}\,bd^2\right)\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)\right)}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(2)*(40*sqrt(2)*(d*x + c)^(3/2)*b^2*d*cos(((d*x + c)*b - b*c + a*d
)/d) - 15*(-(I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I -
 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) - 15*((I - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) -
(I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(-I*b/d)) + 4*(4*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c
)*b*d^2)*sin(((d*x + c)*b - b*c + a*d)/d))/b^4
```

Fricas [A]
time = 0.39, size = 190, normalized size = 0.98

$$\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+2\sqrt{dx+c}\left(10\left(b^2d^2x+b^2cd\right)\cos\left(bx+a\right)+\left(4b^3d^2x^2+8b^3cdx+4b^3c^2-15bd^2\right)\sin\left(bx+a\right)\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 2*sqrt(d*x
 + c)*(10*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) + (4*b^3*d^2*x^2 + 8*b^3*c*d*x
 + 4*b^3*c^2 - 15*b*d^2)*sin(b*x + a)))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

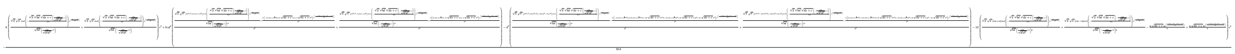
$$\int (c+dx)^{\frac{5}{2}}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(5/2)*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.61, size = 1239, normalized size = 6.39



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(8*(sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)))*c^3 + 6*c*d^2*((sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d
^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*
(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(4*b^2*c^2 -
4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x
 + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*((sqrt(2)*
sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b
^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*
x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(
(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (sqrt(2)*sqrt(pi)*(8*b^3*c^3
 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x
+ c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^
2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b -
I*b*c + I*a*d)/d)/b^3)/d^3) - 12*(sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(2*b*c
- I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 2*I*
sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*I*sqrt(d*x + c)
*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)*(c + d*x)^(5/2), x)
```

3.42 $\int (c+dx)^{3/2}\cos(a+bx)\,dx$

Optimal. Leaf size=169

$$\frac{3d\sqrt{c+dx}\,\cos(a+bx)}{2b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\,\cdots}{\sqrt{d}\cdots}\right.}{2b^{5/\cdots}}$$

```
[Out] (d*x+c)^(3/2)*sin(b*x+a)/b-3/4*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/4*d^(3/2)*Fres
nelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*P
i^(1/2)/b^(5/2)+3/2*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3d\sqrt{c+dx}\,\cos(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x],x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(2*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b
*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*b^(5/2)) +
(3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(2*b^(5/2)) + ((c + d*x)^(3/2)*Sin[a + b*x])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \frac{(c+dx)^{3/2}\sin(a+bx)}{b} - \frac{(3d)\int \sqrt{c+dx}\,\sin(a+bx)\,dx}{2b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b} - \frac{(3d^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{4b^2}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b} - \frac{\left(3d^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos}{\sqrt{}}}{4b^2}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b} - \frac{\left(3d\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{2b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.06, size = 122, normalized size = 0.72

$$\frac{de^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\operatorname{Gamma}\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}+\frac{e^{\frac{2ibc}{d}}\operatorname{Gamma}\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x],x]
```

```
[Out] (d*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[5/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d] + (E^(((2*I)*b*c)/d)*Gamma[5/2, (I*b*(c + d*x))/d])/Sqrt[(
I*b*(c + d*x))/d]))/(2*b^2*E^((I*(b*c + a*d))/d))
```

Maple [A]
time = 0.04, size = 189, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}\cdots}\right.\right.}{\cdots}\right)}{b}}{d}$
default	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}\cdots}\right.\right.}{\cdots}\right)}{b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 242, normalized size = 1.43

$$\frac{\sqrt{2}\left(8\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+12\sqrt{2}\sqrt{dx+c}\,bd\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-3\left(-(i-1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-3\left((i+1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)\right)}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(8*sqrt(2)*(d*x + c)^(3/2)*b^2*sin(((d*x + c)*b - b*c + a*d)/d
) + 12*sqrt(2)*sqrt(d*x + c)*b*d*cos(((d*x + c)*b - b*c + a*d)/d) - 3*(-(I
- 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^
2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 3*(
(I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)
*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/
b^3
```

Fricas [A]
time = 0.37, size = 156, normalized size = 0.92

$$-\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\left(3\,bd\cos\left(bx+a\right)+2\left(b^2dx+b^2c\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_
sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*(3*b*d*co
s(b*x + a) + 2*(b^2*d*x + b^2*c)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.49, size = 773, normalized size = 4.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(4*(sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
```

```
) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)))*c^2 + d^2*((sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x
+ c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b
*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)
*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - 4*(sqrt(2)*sqrt(pi)
*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b) - 2*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)
/d)/b + 2*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*(c + d*x)^(3/2), x)
```

3.43 $\int \sqrt{c+dx} \cos(a+bx) \, dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{b^{3/2}}+$$

```
[Out] -1/2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+sin(b*x+a)
*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x],x]
```

```
[Out] -((Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/P
i]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/b^(3/2) + (Sqrt[c + d*x]*Sin[a
 + b*x])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{c+dx}\,\cos(a+bx)\,dx &= \frac{\sqrt{c+dx}\,\sin(a+bx)}{b} - \frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{2b} \\
&= \frac{\sqrt{c+dx}\,\sin(a+bx)}{b} - \frac{\left(d\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b} - \frac{\left(d\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos}{\sqrt{}}}{2b} \\
&= \frac{\sqrt{c+dx}\,\sin(a+bx)}{b} - \frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{b} - \frac{\sin}{} \\
&= -\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}} - \frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\, C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c}}{\sqrt{d}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.06, size = 124, normalized size = 0.87

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\mathrm{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqr
t[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])
/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d))
```

Maple [A]
time = 0.04, size = 144, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b \cdots}{\sqrt{\pi} \cdots}\right.\right.}{2b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b \cdots}{\sqrt{\pi} \cdots}\right.\right.}{2b\sqrt{\frac{b}{d}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*P
i^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 196, normalized size = 1.38

$$\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{dx+c}\,b\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left(-(i+1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+\left((i-1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(4*sqrt(2)*sqrt(d*x + c)*b*sin(((d*x + c)*b - b*c + a*d)/d) + (
-(I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*
d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((I
 - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d*(
b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

Fricas [A]
time = 0.38, size = 126, normalized size = 0.89

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\sqrt{dx+c}\,b\sin\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*s
qrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*sqrt(d*x + c)*b*s
in(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+dx}\,\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 422, normalized size = 2.97

$$\frac{\frac{\sqrt{2}\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{2}\sqrt{\pi}\,(2\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-2\left(\frac{\sqrt{2}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{2}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{2i\sqrt{dx+c}\,de^{\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{d}\right)}}{b}+\frac{2i\sqrt{dx+c}\,de^{\left(\frac{-i\,(dx+c)b+i\,bc-i\,ad}{d}\right)}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(
```

```
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 2*(sqrt(2)*sqrt(pi)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 2*I*sqrt(d*x + c)*d*e^
((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*I*sqrt(d*x + c)*d*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*(c + d*x)^(1/2), x)
```

3.44 $\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/
2)*Pi^(1/2)/b^(1/2)/d^(1/2)-FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx &= \cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx\\ &= \frac{\left(2\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx,x,\sqrt{c+dx}\right)}{d}-\frac{\left(2\sin\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\right.\right.}{d}\\ &= \frac{\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\,\sqrt{d}}-\frac{\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\right.}{\sqrt{b}\,\sqrt{d}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 124, normalized size = 1.05

$$\frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*
x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*
x))/d]))/(b*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A]
time = 0.04, size = 100, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$	100
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)
/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 159, normalized size = 1.35

$$-\frac{\sqrt{2}\left(\left((i-1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+\left(-(i+1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i-1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(((I - 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I +
1)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b
/d)) + (-(I + 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqr
t(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))
/b
```

Fricas [A]
time = 0.38, size = 108, normalized size = 0.92

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(a + bx \right)}}{\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)/sqrt(c + d*x), x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.45, size = 166, normalized size = 1.41

$$-\frac{\frac{\sqrt{2}\sqrt{\pi}\, d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{2}\sqrt{\pi}\, d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)) + sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)}{\sqrt{c+d\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(1/2), x)
```

3.45 $\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}-\frac{2\sqrt{b}\,\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{b}\,\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] -2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^
(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(a-b*c/d)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b*x+a)/d
/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3387, 3386, 3432, 3385, 3433}

$$-\frac{2\sqrt{2\pi}\,\sqrt{b}\,\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{2\pi}\,\sqrt{b}\,\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(d*Sqrt[c + d*x]) - (2*Sqrt[b]*Sqrt[2*Pi]*Cos[a - (b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (2*Sqrt[b
]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (
b*c)/d])/d^(3/2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{(2b)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{\left(2b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d} - \frac{\left(2b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{\left(4b\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{(4b\sin\left(a-\right.}{}$$

$$= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\,\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\,\sqrt{2\pi}\,C\left(\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 147, normalized size = 1.06

$$\frac{e^{-ia}\left(e^{2ia-\frac{ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{-ibx}\left(-1-e^{2i(a+bx)}+e^{\frac{ib(c+dx)}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (E^((2*I)*a - (I*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c +
 d*x))/d] + (-1 - E^((2*I)*(a + b*x)) + E^((I*b*(c + d*x))/d)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(d*E^(I*a)*Sqrt[c + d*
x])
```

Maple [A]
time = 0.04, size = 140, normalized size = 1.01

method	result
derivativedivides	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right.\right.}{d\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right.\right.}{d\sqrt{\frac{b}{d}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(
b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.62, size = 129, normalized size = 0.93

$$\frac{\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\sqrt{\frac{(dx+c)b}{d}}}{4\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*((-(I + 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma
(-1/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-1/2
, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-(b
*c - a*d)/d))*sqrt((d*x + c)*b/d)/(sqrt(d*x + c)*d)
```

Fricas [A]
time = 0.39, size = 144, normalized size = 1.04

$$-\frac{2\left(\sqrt{2}\left(\pi dx+\pi c\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\left(\pi dx+\pi c\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{dx+c}\cos\left(bx+a\right)\right)}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*
d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) +
 sqrt(d*x + c)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+bx\right)}{\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(3/2), x)
```

3.46 $\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}-\frac{4b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{4b^{3/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right.}{3d^{5/2}}$$

```
[Out] -2/3*cos(b*x+a)/d/(d*x+c)^(3/2)-4/3*b^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+4/3*b^(3/2)
*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1
/2)*Pi^(1/2)/d^(5/2)+4/3*b*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3387, 3386, 3432, 3385, 3433}

$$-\frac{4\sqrt{2\pi}\,b^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{4\sqrt{2\pi}\,b^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*d*(c + d*x)^(3/2)) - (4*b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (
4*b^(3/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[a - (b*c)/d])/(3*d^(5/2)) + (4*b*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} - \frac{(2b)\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{(4b^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(4b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(4b^2\sin\left(a-\frac{b}{c}\right.\right.}{}$$

$$= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(8b^2\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int\cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+d}\right.}{3d^3}$$

$$= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{4b^{3/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{}}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 190, normalized size = 1.13

$$\frac{e^{-ia}\left(-2ie^{2ia-\frac{ibc}{d}}\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)+e^{-ibx}\left(-2d+4ib(c+dx)-4de^{\frac{ib(c+dx)}{d}}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)\right)}{6d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] ((-2*I)*E^((2*I)*a - (I*b*c)/d)*(E^((I*b*(c + d*x))/d)*((-I)*d + 2*b*(c + d
*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-I)*b*(c + d*x))/d
]) + (-2*d + (4*I)*b*(c + d*x) - 4*d*E^((I*b*(c + d*x))/d)*((I*b*(c + d*x))
/d)^(3/2)*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(6*d^2*E^(I*a)*(c + d*x
)^(3/2))
```

Maple [A]
time = 0.04, size = 180, normalized size = 1.07

method	result
derivativedivides	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\cdots\right)\cdots\right)}{d\sqrt{\frac{b}{d}}}\right)}{3d}}{d}$
default	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\cdots\right)\cdots\right)}{d\sqrt{\frac{b}{d}}}\right)}{3d}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3/(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^
(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.63, size = 129, normalized size = 0.77

$$-\frac{\left(\left(\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{i\,(dx+c)b}{d}\right)-\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{i\,(dx+c)b}{d}\right)-\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{i\,(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}}{4\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((I - 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma
(-3/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-3/2
, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-(b
*c - a*d)/d))*((d*x + c)*b/d)^(3/2)/((d*x + c)^(3/2)*d)
```

Fricas [A]
time = 0.40, size = 208, normalized size = 1.24

$$-\frac{2\left(2\sqrt{2}\left(\pi b d^2 x^2+2\pi b c d x+\pi b c^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-2\sqrt{2}\left(\pi b d^2 x^2+2\pi b c d x+\pi b c^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{dx+c}\left(d\cos\left(bx+a\right)-2\left(bdx+bc\right)\sin\left(bx+a\right)\right)\right)}{3\left(d^4x^2+2cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 2*sqrt
(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*(d*c
os(b*x + a) - 2*(b*d*x + b*c)*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)/(c + d*x)^(5/2), x)
```

3.47 $\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}+\frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}}+\frac{8b^{5/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^{5/2}\sqrt{2\pi}\,\text{FresnelC}\Bigg(\cdots}{}$$

```
[Out] -2/5*cos(b*x+a)/d/(d*x+c)^(5/2)+4/15*b*sin(b*x+a)/d^2/(d*x+c)^(3/2)+8/15*b^
(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^2*cos(b*x
+a)/d^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3387, 3386, 3432, 3385, 3433}

$$\frac{8\sqrt{2\pi}\,b^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}\,b^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}}+\frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(5*d*(c + d*x)^(5/2)) + (8*b^2*Cos[a + b*x])/(15*d^3*Sqrt
[c + d*x]) + (8*b^(5/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (8*b^(5/2)*Sqrt[2*Pi]*Fresnel
C[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(15*d^(7/2)
) + (4*b*Sin[a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx &= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} - \frac{(2b)\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx}{5d} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{(4b^2)\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(8b^3)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(8b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}}{15d^3} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(16b^3\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \text{si}\right.}{15d^4} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 228, normalized size = 1.18

$$
\frac{e^{-ia}\left(2e^{2ia}\left(-3d^2e^{ibx}+2be^{-\frac{ibc}{d}}(c+dx)\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)\right)+e^{-ibx}\left(-6d^2+4ibd(c+dx)+8b^2(c+dx)^2+8d^2e^{\frac{ib(c+dx)}{d}}\left(\frac{ib(c+dx)}{d}\right)^{5/2}\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)\right)}{30d^3(c+dx)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (2*E^((2*I)*a)*(-3*d^2*E^(I*b*x) + (2*b*(c + d*x)*(E^((I*b*(c + d*x))/d)*((
-I)*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((
-I)*b*(c + d*x))/d]))/E^((I*b*c)/d)) + (-6*d^2 + (4*I)*b*d*(c + d*x) + 8*b^
2*(c + d*x)^2 + 8*d^2*E^((I*b*(c + d*x))/d)*((I*b*(c + d*x))/d)^(5/2)*Gamma
[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(30*d^3*E^(I*a)*(c + d*x)^(5/2))
```

Maple [A]
time = 0.04, size = 220, normalized size = 1.14

method	result

derivativedivides	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)S\left(\frac{\sqrt{2}\,b \ldots}{\sqrt{\pi}\ldots}\right)\ldots\right)}{\ldots}\right)}{3d\ldots}\right)}{5d}}{d}$
default	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)S\left(\frac{\sqrt{2}\,b \ldots}{\sqrt{\pi}\ldots}\right)\ldots\right)}{\ldots}\right)}{3d\ldots}\right)}{5d}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/5/(d*x+c)^(5/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-2/5*b/d*(-1/3/(d*x+c
)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(1/2)*cos(1/d*b*
(d*x+c)+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fre
snelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.61, size = 129, normalized size = 0.67

$$-\frac{\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}}{4(dx+c)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/4*((-(I + 1)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamm
a(-5/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-5/
2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-(
b*c - a*d)/d))*((d*x + c)*b/d)^(5/2)/((d*x + c)^(5/2)*d)
```

Fricas [A]
time = 0.42, size = 296, normalized size = 1.53

$$\frac{2\left(4\sqrt{2}\left(\pi b^2d^3x^3+3\pi b^2cd^2x^2+3\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+4\sqrt{2}\left(\pi b^2d^3x^3+3\pi b^2cd^2x^2+3\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{dx+c}\left(\left(4b^2d^2x^2+8b^2cdx+4b^2c^2-3d^2\right)\cos\left(bx+a\right)+2\left(bd^2x+bcd\right)\sin\left(bx+a\right)\right)\right)}{15\left(d^6x^3+3cd^5x^2+3c^2d^4x+c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + p
i*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) + 4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*p
i*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*((4*b^2*d^2*x^2 + 8*b
^2*c*d*x + 4*b^2*c^2 - 3*d^2)*cos(b*x + a) + 2*(b*d^2*x + b*c*d)*sin(b*x +
a)))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+bx\right)}{\left(c+dx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(7/2),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(7/2),x)
[Out] int(cos(a + b*x)/(c + d*x)^(7/2), x)
```

3.48 $\int (c+dx)^{5/2}\cos^2(a+bx)\,dx$

Optimal. Leaf size=231

$$-\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}+\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}+\frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+$$

```
[Out] -5/16*d*(d*x+c)^(3/2)/b^2+1/7*(d*x+c)^(7/2)/d+5/8*d*(d*x+c)^(3/2)*cos(b*x+a
)^2/b^2+1/2*(d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)/b+15/128*d^(5/2)*cos(2*a-2*
b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+
15/128*d^(5/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2
*b*c/d)*Pi^(1/2)/b^(7/2)-15/64*d^2*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.30, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 32, 3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}-\frac{15d^2\sqrt{c+dx}\,\sin(2a+2bx)}{64b^3}+\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}-\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2,x]
```

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) + (5*d*(c + d*x)^(3
/2)*Cos[a + b*x]^2)/(8*b^2) + (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^
(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2
*a - (2*b*c)/d])/(128*b^(7/2)) + ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]
)/(2*b) - (15*d^2*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(64*b^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{1}{2}\int (c$$

$$= \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+}{2b}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\,c}{}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\,c}{}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\,c}{}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\,c}{}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{15d^{5/2}\sqrt{\pi}\,c}{}$$

Mathematica [A]
time = 1.37, size = 194, normalized size = 0.84

$$\frac{\sqrt{\frac{b}{d}}\left(105d^3\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+105d^3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(64b^3(c+dx)^3+140bd^2(c+dx)\cos(2(a+bx))+7d(-15d^2+16b^2(c+dx)^2)\sin(2(a+bx))\right)\right)}{896b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(105*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqr
t[c + d*x])/Sqrt[Pi]] + 105*d^3*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x
])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(64*b^3*(c +
d*x)^3 + 140*b*d^2*(c + d*x)*Cos[2*(a + b*x)] + 7*d*(-15*d^2 + 16*b^2*(c +
d*x)^2)*Sin[2*(a + b*x)])))/(896*b^4)
```

Maple [A]
time = 0.07, size = 242, normalized size = 1.05

method	result

derivativedivides	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{7}+\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}\right.}{4b}\right.}{}\right.}{}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{7}+\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}\right.}{4b}\right.}{}\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/14*(d*x+c)^(7/2)+1/8/b*d*(d*x+c)^(5/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)
/d)-5/8/b*d*(-1/4/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/4/b*
d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/
(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 295, normalized size = 1.28

$$\frac{\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d}+1120\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-105\left(-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)-105\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)+56\left(16\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}\,bd^2\right)\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)\right)}{7168\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7168*sqrt(2)*(512*sqrt(2)*(d*x + c)^(7/2)*b^4/d + 1120*sqrt(2)*(d*x + c)^
(3/2)*b^2*d*cos(2*((d*x + c)*b - b*c + a*d)/d) - 105*(-(I + 1)*4^(1/4)*sqrt
(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(pi)*d
^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d))
- 105*((I - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
 (I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)) + 56*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(
2)*sqrt(d*x + c)*b*d^2)*sin(2*((d*x + c)*b - b*c + a*d)/d))/b^4
```

Fricas [A]
time = 0.39, size = 258, normalized size = 1.12

$$\frac{105\,\pi d^4 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2\,(bc-ad)}{d}\right) \mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) + 105\,\pi d^4 \sqrt{\frac{b}{\pi d}}\, \mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2\,(bc-ad)}{d}\right) + 4\,\left(32\,b^4 d^3 x^3 + 96\,b^4 c d^2 x^2 + 32\,b^4 c^3 - 70\,b^2 c d^2 + 140\,\left(b^2 d^3 x + b^2 c d^2\right) \cos\left(bx+a\right)^2 + 7\,\left(16\,b^3 d^3 x^2 + 32\,b^3 c d^2 x + 16\,b^3 c^2 d - 15\,b d^3\right) \cos\left(bx+a\right) \sin\left(bx+a\right) + 2\,\left(48\,b^4 c^2 d - 35\,b^2 d^3\right) x\right) \sqrt{dx+c}}{896\,b^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/896*(105*pi*d^4*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) + 105*pi*d^4*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(32*b^4*d^3*x^3 + 96*b^4*c*d
^2*x^2 + 32*b^4*c^3 - 70*b^2*c*d^2 + 140*(b^2*d^3*x + b^2*c*d^2)*cos(b*x +
a)^2 + 7*(16*b^3*d^3*x^2 + 32*b^3*c*d^2*x + 16*b^3*c^2*d - 15*b*d^3)*cos(b*
x + a)*sin(b*x + a) + 2*(48*b^4*c^2*d - 35*b^2*d^3)*x)*sqrt(d*x + c))/(b^4*
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{5}{2}} \cos^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(5/2)*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.64, size = 1325, normalized size = 5.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8960*(2240*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
 sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x + c
))*c^3 - 28*c*d^2*(64*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)/d^2 - 15*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(4*I*(d*x + c)^(3/2)*b*d -
8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b^2)/d^2 - 15*(sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-4*I*(d*x + c)^(3/2)*
b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*(256*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(
5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 + 35*(sqrt(pi)*
(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*
x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*
d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 35*(sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*
d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b^3) - 2*(-16*I*(d*x + c)^(5/2)*b^2*d + 48*I*(d*x + c)^(3/2)*b^2*c*d - 48*I
*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*
d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)
/d^3) - 560*(3*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)*b) + 3*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b) + 16*(d*x + c)^(3/2) - 48*sqrt(d*x + c)*c + 6*I*sqrt(d*x
+ c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(
-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.49 $\int (c+dx)^{3/2} \cos^2(a+bx)\,dx$

Optimal. Leaf size=203

$$-\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}+\frac{3d\sqrt{c+dx}\,\cos^2(a+bx)}{8b^2}-\frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}$$

```
[Out] 1/5*(d*x+c)^(5/2)/d+1/2*(d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)/b-3/32*d^(3/2)*
cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2
)/b^(5/2)+3/32*d^(3/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*s
in(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)-3/16*d*(d*x+c)^(1/2)/b^2+3/8*d*cos(b*x+a)^
2*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\pi}\,d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{32b^{5/2}}+\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}-\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) + (3*d*Sqrt[c + d*x]*
Cos[a + b*x]^2)/(8*b^2) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/2)) + (3*d^(3/2)*S
qrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2
*b*c)/d])/(32*b^(5/2)) + ((c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \frac{3d\sqrt{c+dx}\,\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{1}{2}\int (c+$$

$$= \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\,\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+b}{2b}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\,\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(}{}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\,\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(}{}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\,\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(}{}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\,\cos^2(a+bx)}{8b^2} - \frac{3d^{3/2}\sqrt{\pi}\,\cos(2}{}$$

Mathematica [A]
time = 1.06, size = 175, normalized size = 0.86

$$\frac{\sqrt{\frac{b}{d}}\left(-15d^2\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+15d^2\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(15d^2\cos(2(a+bx))+4b(c+dx)(4b(c+dx)+5d\sin(2(a+bx))))\right)}{160b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqr
t[c + d*x])/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x]
)/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(15*d^2*Cos[2*
(a + b*x)] + 4*b*(c + d*x)*(4*b*(c + d*x) + 5*d*Sin[2*(a + b*x)]))))/(160*b
^3)
```

Maple [A]
time = 0.06, size = 197, normalized size = 0.97

method	result

derivativedivides	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{5}+\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\text{Fresn}\cdots\right.}{\cdots}\right.}{4b\cdots}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{5}+\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\text{Fresn}\cdots\right.}{\cdots}\right.}{4b\cdots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/10*(d*x+c)^(5/2)+1/8/b*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)
/d)-3/8/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*
d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 274, normalized size = 1.35

$$\frac{\sqrt{2}\left(\frac{128\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d}+160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+120\sqrt{2}\sqrt{dx+c}\,bd\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-15\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)-15\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)}{1280b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1280*sqrt(2)*(128*sqrt(2)*(d*x + c)^(5/2)*b^3/d + 160*sqrt(2)*(d*x + c)^(
3/2)*b^2*sin(2*((d*x + c)*b - b*c + a*d)/d) + 120*sqrt(2)*sqrt(d*x + c)*b*d
*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*((I + 1
)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1
/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(-2*I*b/d)))/b^3
```

Fricas [A]
time = 0.43, size = 195, normalized size = 0.96

$$-\frac{15\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\operatorname{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\pi d^3\sqrt{\frac{b}{\pi d}}\operatorname{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(16b^3d^2x^2+32b^3cdx+16b^3c^2+30bd^2\cos(bx+a)^2-15bd^2+40(b^2d^2x+b^2cd)\cos(bx+a)\sin(bx+a)\right)\sqrt{dx+c}}{160b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/160*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 + 30*b*d^2*cos(b*x + a)^2 - 15*b*d^2 + 40*(b^2*d^2*x + b^2*c
*d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.62, size = 817, normalized size = 4.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/960*(240*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + s
qrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x + c))
*c^2 - d^2*(64*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)
*c^2)/d^2 - 15*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqr
t(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a
*d)/d)/b^2)/d^2 - 15*(sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-4*I*(d*x + c)^(3/2)*b*d + 8
*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c
+ I*a*d)/d)/b^2)/d^2) - 40*(3*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*s
```

```
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*(d*x + c)^(3/2) - 48*sqrt(d*x + c)*c
+ 6*I*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt
(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.50 $\int \sqrt{c+dx} \cos^2(a+bx)\,dx$

Optimal. Leaf size=158

$$\frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right.}{8b^{3/2}}$$

```
[Out] 1/3*(d*x+c)^(3/2)/d-1/8*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(2*b
*x+2*a)*(d*x+c)^(1/2)/b
```

Rubi [A]

time = 0.18, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(3*d) - (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]
*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d
])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{c+dx}\cos^2(a+bx)\,dx &= \int \left(\frac{1}{2}\sqrt{c+dx}+\frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)\right)dx \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{1}{2}\int \sqrt{c+dx}\cos(2a+2bx)\,dx \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b} \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}-\frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b}- \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)dx\right.}{4b} \\
&= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}\,C\left(\frac{2\sqrt{b}}{\sqrt{}}\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.35, size = 146, normalized size = 0.92

$$\frac{1}{48}\sqrt{c+dx}\left(\frac{16(c+dx)}{d}-\frac{3i\sqrt{2}\,e^{2i\left(a-\frac{bc}{d}\right)}\mathrm{Gamma}\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{b\sqrt{-\frac{ib(c+dx)}{d}}}+\frac{3i\sqrt{2}\,e^{-2i\left(a-\frac{bc}{d}\right)}\mathrm{Gamma}\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*((16*(c + d*x))/d - ((3*I)*Sqrt[2]*E^((2*I)*(a - (b*c)/d))*G
amma[3/2, ((-2*I)*b*(c + d*x))/d])/(b*Sqrt[((-I)*b*(c + d*x))/d]) + ((3*I)*
Sqrt[2]*Gamma[3/2, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*Sqrt[
(I*b*(c + d*x))/d])))/48
```

Maple [A]
time = 0.06, size = 150, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2bc}{d}\right)\mathrm{FresnelC}\left(\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2bc}{d}\right)\mathrm{FresnelC}\left(\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/6*(d*x+c)^(3/2)+1/8/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/
d)-1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 229, normalized size = 1.45

$$\frac{\sqrt{2}\left(\frac{32\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2}{d}+24\sqrt{2}\sqrt{dx+c}\,b\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)-3\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)}{192\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2/d + 24*sqrt(2)*sqrt(d*x + c)*
b*sin(2*((d*x + c)*b - b*c + a*d)/d) - 3*((I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d
^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4
)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 3*(-(I - 1)*4^(
1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqr
t(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*
b/d)))/b^2
```

Fricas [A]
time = 0.44, size = 148, normalized size = 0.94

$$-\frac{3\,\pi d^{2} \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2\,(bc-ad)}{d}\right) \mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) + 3\,\pi d^{2} \sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2\,(bc-ad)}{d}\right) - 4\,(2\,b^{2}dx + 3\,bd\cos\left(bx+a\right)\sin\left(bx+a\right) + 2\,b^{2}c)\sqrt{dx+c}}{24\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c)*sqrt(d*x + c))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\cos^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.52, size = 434, normalized size = 2.75

$$-\frac{12\left(\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)} + \frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)} - 4\,\sqrt{dx+c}\right)c - \frac{3\,\sqrt{\pi}\,(4\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b} - \frac{3\,\sqrt{\pi}\,(4\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b} - 16\,(dx+c)^{\frac{3}{2}} + 48\,\sqrt{dx+c}\,c - \frac{6i\,\sqrt{dx+c}\,de^{\left(-\frac{2\,(i\,(dx+c)b-i\,bc+i\,ad)}{d}\right)}}{b} + \frac{6i\,\sqrt{dx+c}\,de^{\left(-\frac{2\,(-i\,(dx+c)b+i\,bc-i\,ad)}{d}\right)}}{b}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/48*(12*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqr
t(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I
```

```
*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x + c))*c
 - 3*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) - 3*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
+ 1)*b) - 16*(d*x + c)^(3/2) + 48*sqrt(d*x + c)*c - 6*I*sqrt(d*x + c)*d*e^(
-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-2*(-I*(d*
x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.51 $\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

```
[Out] 1/2*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^
(1/2)/b^(1/2)/d^(1/2)-1/2*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(1/2)/d^(1/2)+(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] Sqrt[c + d*x]/d + (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*FresnelS[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*
Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{1}{2\sqrt{c+dx}}+\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)dx\\ &= \frac{\sqrt{c+dx}}{d}+\frac{1}{2}\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx\\ &= \frac{\sqrt{c+dx}}{d}+\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx-\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}\right.}{\sqrt{c+}}\\ &= \frac{\sqrt{c+dx}}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Sub}}{}\\ &= \frac{\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin(2}{2\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 145, normalized size = 1.12

$$\frac{8\left(\frac{c}{d}+x\right)-\frac{i\sqrt{2}\,e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{i\sqrt{2}\,e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},\frac{2ib(c+dx)}{d}\right)}{b}}{8\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] (8*(c/d + x) - (I*Sqrt[2]*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[1/2, ((-2*I)*b*(c + d*x))/d])/b + (I*Sqrt[2]*Sqrt[(I*b*(c + d*x))/d
]*Gamma[1/2, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))))/(8*Sqrt[c
 + d*x])
```

Maple [A]
time = 0.06, size = 108, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\sqrt{dx+c}+\frac{\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$	108
default	$\frac{\sqrt{dx+c}+\frac{\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2*(d*x+c)^(1/2)+1/4*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*Fresnel
C(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/P
i^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 187, normalized size = 1.44

$$-\frac{\sqrt{2}\left(\left(\left(i-1\right)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+\left(i+1\right)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2i\,b}{d}}\right)+\left(-\left(i+1\right)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(i-1\right)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2i\,b}{d}}\right)-\frac{8\sqrt{2}\sqrt{dx+c}\,b}{d}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*(((I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)
/d) + (I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*co
s(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - 8*sqrt(2)*sqrt(d*x + c)*b/d)
/b
```

Fricas [A]

time = 0.39, size = 114, normalized size = 0.88

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+2\sqrt{dx+c}\,b}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 2*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/sqrt(c + d*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 165, normalized size = 1.27

$$-\frac{\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-4\sqrt{dx+c}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(pi)
*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\cos\left(a+b\,x\right)}^2}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^(1/2), x)
```

3.52 $\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}-\frac{2\sqrt{b}\,\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{d^{3/2}}-\frac{2\sqrt{b}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)\sin\left(2a\right.}{d^{3/2}}$$

```
[Out] -2*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*b^(1
/2)*Pi^(1/2)/d^(3/2)-2*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*s
in(2*a-2*b*c/d)*b^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b*x+a)^2/d/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3394, 12, 3387, 3386, 3432, 3385, 3433}

$$-\frac{2\sqrt{\pi}\,\sqrt{b}\,\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{\pi}\,\sqrt{b}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{d^{3/2}}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^2)/(d*Sqrt[c + d*x]) - (2*Sqrt[b]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/d^(3/2) - (2*
Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin
[2*a - (2*b*c)/d])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} + \frac{(4b)\int -\frac{\sin(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{(2b)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{\left(2b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{d} - \frac{\left(2b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}\right.}{\sqrt{c+}}}{d}$$

$$= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{\left(4b\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{(4b\sin}{}$$

$$= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\,\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\,\sqrt{\pi}\,C\left(\frac{2\sqrt{b}}{\sqrt{}}\right.}{}$$

Mathematica [A]
time = 0.41, size = 133, normalized size = 0.99

$$\frac{2\left(-\frac{\cos^2(a+bx)}{\sqrt{c+dx}}-\sqrt{\frac{b}{d}}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{\frac{b}{d}}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-(Cos[a + b*x]^2/Sqrt[c + d*x]) - Sqrt[b/d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/
d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[b/d]*Sqrt[Pi]*Fres
nelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d]))/d
```

Maple [A]
time = 0.09, size = 146, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{1}{\sqrt{dx+c}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx}\cdots}{\sqrt{\pi}\cdots}\right.\cdots\right.}{d\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{-\frac{1}{\sqrt{dx+c}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx}\cdots}{\sqrt{\pi}\cdots}\right.\cdots\right.}{d\sqrt{\frac{b}{d}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/2/(d*x+c)^(1/2)-1/2/(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-
b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.63, size = 135, normalized size = 1.00

$$\frac{\sqrt{2}\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{2i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{2i(dx+c)b}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\sqrt{\frac{(dx+c)b}{d}}-8}{8\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) + (I - 1)*sq
rt(2)*gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + ((I - 1)*sqr
t(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*(d*x
 + c)*b/d))*sin(-2*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) - 8)/(sqrt(d*x + c)*
d)
```

Fricas [A]
time = 0.38, size = 136, normalized size = 1.01

$$-\frac{2\left((\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\sqrt{dx+c}\cos\left(bx+a\right)^2\right)}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt
(d*x + c)*sqrt(b/(pi*d))) + (pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(2*sq
rt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + sqrt(d*x + c)*cos(b*x +
 a)^2)/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2\left(a+bx\right)}{\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\cos\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2/(c + d*x)^(3/2), x)
```

3.53 $\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{8b^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{3d^{5/2}}+\frac{8b^{3/2}\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)\sin\left(2a\right.}{3d^{5/2}}$$

```
[Out] -2/3*cos(b*x+a)^2/d/(d*x+c)^(3/2)-8/3*b^(3/2)*cos(2*a-2*b*c/d)*FresnelC(2*b
^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(5/2)+8/3*b^(3/2)*Fresnel
S(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(5/
2)+8/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$-\frac{8\sqrt{\pi}\,b^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{3d^{5/2}}+\frac{8\sqrt{\pi}\,b^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{3d^{5/2}}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x]^2)/(3*d*(c + d*x)^(3/2)) - (8*b^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(3*d^(5/2
)) + (8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])]*Sin[2*a - (2*b*c)/d])/(3*d^(5/2)) + (8*b*Cos[a + b*x]*Sin[a + b*x])/(3
*d^2*Sqrt[c + d*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} + \frac{(8b^2)\int \frac{1}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{(16b^2)\int \frac{\cos^2(}{\sqrt{c-}}}{3d^2}$$

$$= \frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{(16b^2)\int\left(\frac{1}{2\sqrt{c+dx}}\right.}{3d^2}$$

$$= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{(8b^2)\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(8b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(16b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int\cos\left(\frac{2b}{}\right.\right.}{3d^3}$$

$$= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} + \frac{8b^{3/2}\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\,\sqrt{}}{\sqrt{d}}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.93, size = 181, normalized size = 1.06

$$\frac{e^{-\frac{2i(bc+ad)}{d}}\left(-2\sqrt{2}\,de^{4ia}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)-2\sqrt{2}\,de^{\frac{4ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{2ib(c+dx)}{d}\right)+2e^{\frac{2i(bc+ad)}{d}}\left(-d\cos^2(a+bx)+2b(c+dx)\sin(2(a+bx))\right)\right)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*d*E^((4*I)*a)*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*b
*(c + d*x))/d] - 2*Sqrt[2]*d*E^(((4*I)*b*c)/d)*((I*b*(c + d*x))/d)^(3/2)*Ga
mma[1/2, ((2*I)*b*(c + d*x))/d] + 2*E^(((2*I)*(b*c + a*d))/d)*(-(d*Cos[a +
b*x]^2) + 2*b*(c + d*x)*Sin[2*(a + b*x)]))/(3*d^2*E^(((2*I)*(b*c + a*d))/d)
*(c + d*x)^(3/2))
```

Maple [A]
time = 0.09, size = 189, normalized size = 1.11

method	result

derivativedivides	$\frac{-\frac{1}{3(dx+c)^{\frac{3}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$
default	$\frac{-\frac{1}{3(dx+c)^{\frac{3}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2da-2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/6/(d*x+c)^(3/2)-1/6/(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-
2/3*b/d*(-1/(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(
b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.64, size = 136, normalized size = 0.80

$$\frac{3\sqrt{2}\left(\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{2i(dx+c)b}{d}\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{2i(dx+c)b}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}-4}{12(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I + 1)
*sqrt(2)*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I + 1)
*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-3/2, -2*I*
(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) - 4)/((d*x + c
)^(3/2)*d)
```

Fricas [A]
time = 0.41, size = 206, normalized size = 1.21

$$-\frac{2\left(4\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+\left(d\cos(bx+a)^2-4(bdx+bc)\cos(bx+a)\sin(bx+a)\right)\sqrt{dx+c}\right)}{3\left(d^4x^2+2cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-2*(b*c
 - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 4*(pi*b*d^2*x^2 +
2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-2*(b*c - a*d)/d) + (d*cos(b*x + a)^2 - 4*(b*d*x + b*c)*cos(b*
x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^(5/2), x)
```

3.54 $\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}} \, dx$

Optimal. Leaf size=216

$$-\frac{16b^2}{15d^3\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}}+\frac{32b^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}}+\frac{32b^{5/2}\sqrt{}}{}$$

```
[Out] -2/5*cos(b*x+a)^2/d/(d*x+c)^(5/2)+8/15*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^
(3/2)+32/15*b^(5/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/d^(7/2)+32/15*b^(5/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(7/2)-16/15*b^2/d^3/(d*x+c)
^(1/2)+32/15*b^2*cos(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3395, 32, 3394, 12, 3387, 3386, 3432, 3385, 3433}

$$\frac{32\sqrt{\pi}b^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{15d^{7/2}}+\frac{32\sqrt{\pi}b^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}}+\frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^2)/(5*d*(c + d*x)^(5/2))
 + (32*b^2*Cos[a + b*x]^2)/(15*d^3*Sqrt[c + d*x]) + (32*b^(5/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(15*d^(7/2)) + (32*b^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(15*d^(7/2)) + (8*b*Cos[a + b*x]*Sin[
a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} - \frac{(16b^2)\int \frac{\cos^2(a+bx}{(c+dx)^{3/2}}}{15d^2}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} -$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} +$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} +$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} +$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{32b^{5/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\Big(}{15d^{7/2}}$$

Mathematica [A]
time = 0.83, size = 244, normalized size = 1.13

$$\frac{-3d^2+16b^2c^2\cos(2(a+bx))-3d^2\cos(2(a+bx))+32b^2cdx\cos(2(a+bx))+16b^2d^2x^2\cos(2(a+bx))+32b\left(\frac{b}{d}\right)^{3/2}d\sqrt{\pi}\,(c+dx)^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+32b\left(\frac{b}{d}\right)^{3/2}d\sqrt{\pi}\,(c+dx)^{5/2}\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+4bcd\sin(2(a+bx))+4bd^2x\sin(2(a+bx))}{15d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-3*d^2 + 16*b^2*c^2*Cos[2*(a + b*x)] - 3*d^2*Cos[2*(a + b*x)] + 32*b^2*c*d
*x*Cos[2*(a + b*x)] + 16*b^2*d^2*x^2*Cos[2*(a + b*x)] + 32*b*(b/d)^(3/2)*d*
Sqrt[Pi]*(c + d*x)^(5/2)*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
+ d*x])/Sqrt[Pi]] + 32*b*(b/d)^(3/2)*d*Sqrt[Pi]*(c + d*x)^(5/2)*FresnelC[(2
*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 4*b*c*d*Sin[2*(a
 + b*x)] + 4*b*d^2*x*Sin[2*(a + b*x)])/(15*d^3*(c + d*x)^(5/2))
```

Maple [A]
time = 0.09, size = 230, normalized size = 1.06

method	result

derivativedivides	$\frac{-\frac{1}{5(dx+c)^{\frac{5}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\right.\right.}{}\right.}{}\right.}{5d}}{d}$
default	$\frac{-\frac{1}{5(dx+c)^{\frac{5}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\right.\right.}{}\right.}{}\right.}{5d}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/10/(d*x+c)^(5/2)-1/10/(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d
)-2/5*b/d*(-1/3/(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+4/3*b/d*(-1/
(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-2*b/d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.65, size = 136, normalized size = 0.63

$$\frac{5\sqrt{2}\left(\left(\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{2i\,(dx+c)b}{d}\right)-\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{2i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+\left(-\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{2i\,(dx+c)b}{d}\right)+\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{2i\,(dx+c)b}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}-2}{10\,(dx+c)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/10*(5*sqrt(2)*(((I + 1)*sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) - (I - 1)*
sqrt(2)*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I - 1)*
sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) + (I + 1)*sqrt(2)*gamma(-5/2, -2*I*(
d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) - 2)/((d*x + c)
^(5/2)*d)
```

Fricas [**A**]
time = 0.44, size = 323, normalized size = 1.50

$$\frac{2\left(16\left(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+16\left(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-\left(16\,b^2d^2x^2+32\,b^2cdx+16\,b^2c^2-3\,d^2\right)\cos\left(bx+a\right)^2-4\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}\right)}{15\left(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c
^3)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b
/(pi*d))) + 16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi
*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2
*(b*c - a*d)/d) - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - (16*b^2*d^2*x
^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 3*d^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d
)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*
d^4*x + c^3*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+bx\right)}{\left(c+dx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(7/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2}{{\left(c+d\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(c + d*x)^(7/2),x)
[Out] int(cos(a + b*x)^2/(c + d*x)^(7/2), x)
```

3.55 $\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx$

Optimal. Leaf size=247

$$-\frac{16b^2}{105d^3(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}}+\frac{128b^{7/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{105d^{9/2}}$$

[Out] -16/105*b^2/d^3/(d*x+c)^(3/2)-2/7*cos(b*x+a)^2/d/(d*x+c)^(7/2)+32/105*b^2*cos(b*x+a)^2/d^3/(d*x+c)^(3/2)+8/35*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(5/2)+128/105*b^(7/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(9/2)-128/105*b^(7/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(9/2)-128/105*b^3*cos(b*x+a)*sin(b*x+a)/d^4/(d*x+c)^(1/2)

Rubi [A]
time = 0.27, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$,
Rules used = {3395, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$\frac{128\sqrt{\pi}\,b^{7/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{105d^{9/2}}-\frac{128\sqrt{\pi}\,b^{7/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{105d^{9/2}}-\frac{128b^3\sin(a+bx)\cos(a+bx)}{105d^4\sqrt{c+dx}}+\frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}}+\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^2/(c + d*x)^(9/2),x]

[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cos[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) + (32*b^2*Cos[a + b*x]^2)/(105*d^3*(c + d*x)^(3/2)) + (128*b^(7/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(105*d^(9/2)) - (128*b^(7/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(105*d^(9/2)) + (8*b*Cos[a + b*x]*Sin[a + b*x])/(35*d^2*(c + d*x)^(5/2)) - (128*b^3*Cos[a + b*x]*Sin[a + b*x])/(105*d^4*Sqrt[c + d*x])

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{(8b^2)\int \frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} - \frac{(16b^2)\int \frac{\cos^2(a+bx}{(c+dx)^{5/2}}}{35d^2}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a}{35d}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{128b^{7/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)}{105d^{9/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.54, size = 237, normalized size = 0.96

$$\frac{2\left(-8b^2d(c+dx)^2-15d^3\cos^2(a+bx)+16b^2d(c+dx)^2\cos^2(a+bx)+16\sqrt{2}\,b^2de^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^2\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)+16\sqrt{2}\,b^2de^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^2\left(\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{2ib(c+dx)}{d}\right)+6bd^2(c+dx)\sin(2(a+bx))-32b^3(c+dx)^3\sin(2(a+bx))\right)}{105d^4(c+dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (2*(-8*b^2*d*(c + d*x)^2 - 15*d^3*Cos[a + b*x]^2 + 16*b^2*d*(c + d*x)^2*Cos
[a + b*x]^2 + 16*Sqrt[2]*b^2*d*E^((2*I)*(a - (b*c)/d))*(c + d*x)^2*(((-I)*b
*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*b*(c + d*x))/d] + (16*Sqrt[2]*b^2*d
*(c + d*x)^2*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d])/E
^((2*I)*(a - (b*c)/d)) + 6*b*d^2*(c + d*x)*Sin[2*(a + b*x)] - 32*b^3*(c + d
*x)^3*Sin[2*(a + b*x)]))/(105*d^4*(c + d*x)^(7/2))
```

Maple [A]
time = 0.09, size = 273, normalized size = 1.11

method	result

derivativedivides	$\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{7(dx+c)^{\frac{7}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx}{d}\right.}{\sqrt{(}}\right.}{}\right.}{}\right.}{}}{d}$

default	$\dfrac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{7(dx+c)^{\frac{7}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+\cdots}{d}\right.}{\sqrt{d\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/14/(d*x+c)^(7/2)-1/14/(d*x+c)^(7/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d
)-2/7*b/d*(-1/5/(d*x+c)^(5/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+4/5*b/d*(-1/
3/(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-4/3*b/d*(-1/(d*x+c)^(1/2)*
sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c
)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*
FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.66, size = 136, normalized size = 0.55

$$-\frac{7\sqrt{2}\left(\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{7}{2},\frac{2i(dx+c)b}{d}\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{7}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{7}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{7}{2},-\frac{2i(dx+c)b}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}+1}{7(dx+c)^{\frac{7}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*(7*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I + 1)
*sqrt(2)*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I + 1)
*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-7/2, -2*I*
(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(7/2) + 1)/((d*x + c
)^(7/2)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(195) = 390$.
time = 0.45, size = 417, normalized size = 1.69

$$\frac{2 \left(64 \left(\pi b^3 d^4 x^4 + 4 \pi b^3 c d^3 x^3 + 6 \pi b^3 c^2 d^2 x^2 + 4 \pi b^3 c^3 d x + \pi b^3 c^4\right) \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2 (bc-ad)}{d}\right) \mathrm{C}\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 64 \left(\pi b^3 d^4 x^4 + 4 \pi b^3 c d^3 x^3 + 6 \pi b^3 c^2 d^2 x^2 + 4 \pi b^3 c^3 d x + \pi b^3 c^4\right) \sqrt{\frac{b}{\pi d}} \mathrm{S}\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2 (bc-ad)}{d}\right) - \left(8 b^2 d^3 x^2 + 16 b^2 c d^2 x + 8 b^2 c^2 d - \left(16 b^2 d^3 x^2 + 32 b^2 c d^2 x + 16 b^2 c^2 d - 15 d^3\right) \cos\left(bx+a\right)^2 + 4 \left(16 b^3 d^3 x^3 + 48 b^3 c d^2 x^2 + 16 b^3 c^3 - 3 b c d^2 + 3 \left(16 b^3 c^2 d - b d^3\right) x\right) \cos\left(bx+a\right) \sin\left(bx+a\right)\right) \sqrt{dx+c}\right)}{105 \left(d^8 x^4 + 4 c d^7 x^3 + 6 c^2 d^6 x^2 + 4 c^3 d^5 x + c^4 d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4*p
i*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_co
s(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3
 + 6*pi*b^3*c^2*d^2*x^2 + 4*pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*fre
snel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - (8*b^2*d^3
*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - (16*b^2*d^3*x^2 + 32*b^2*c*d^2*x + 16
*b^2*c^2*d - 15*d^3)*cos(b*x + a)^2 + 4*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2
+ 16*b^3*c^3 - 3*b*c*d^2 + 3*(16*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)*sin(b*x
 + a))*sqrt(d*x + c))/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x
+ c^4*d^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^2}{(c+d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(c + d*x)^(9/2),x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^(9/2), x)
```

3.56 $\int (c+dx)^{5/2} \cos^3(a+bx)\,dx$

Optimal. Leaf size=410

$$\frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2}+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+$$

```
[Out] 5/3*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2+5/18*d*(d*x+c)^(3/2)*cos(b*x+a)^3/b^2+2/
3*(d*x+c)^(5/2)*sin(b*x+a)/b+1/3*(d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)/b+5/
864*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+4
5/32*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+45/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-45/16*d^
2*sin(b*x+a)*(d*x+c)^(1/2)/b^3-5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.76, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 3377, 3387, 3386, 3432, 3385, 3433, 3393}

$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3}-\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2}+\frac{2(c+dx)^{5/2}\sin(a+bx)}{3b}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(3*b^2) + (5*d*(c + d*x)^(3/2)*Cos[a + b
*x]^3)/(18*b^2) + (45*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*C
os[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi/2]
*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16
*b^(7/2)) - (45*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + (2*(c + d*x)^(5/
2)*Sin[a + b*x])/(3*b) + ((c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b
) - (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2}{3}\int (c$$

$$= \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{5/2}\cos^2(a}{3}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{2(c+dx)^{5/2}\sin(}{3b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} - \frac{45d^2\sqrt{c+dx}\ \sin}{16b^3}$$

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$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\ \cos\Big(}{}$$

Mathematica [A]
time = 1.99, size = 542, normalized size = 1.32

$$\frac{1620b^2cd\sqrt{c+dx}\cos(a+bx)+1620b^2d^2x\sqrt{c+dx}\cos(a+bx)+60b^2cd\sqrt{c+dx}\cos(3(a+bx))+60b^2d^2x\sqrt{c+dx}\cos(3(a+bx))+1215\sqrt{\frac{b}{d}}d^3\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+5\sqrt{\frac{b}{d}}d^3\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+5\sqrt{\frac{b}{d}}d^3\sqrt{6\pi}\,\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)+1215\sqrt{\frac{b}{d}}d^3\sqrt{2\pi}\,\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right)+648b^3c^2\sqrt{c+dx}\sin(a+bx)-2430bd^2\sqrt{c+dx}\sin(a+bx)+1296b^3cdx\sqrt{c+dx}\sin(a+bx)+648b^3d^2x^2\sqrt{c+dx}\sin(a+bx)+72b^3c^2\sqrt{c+dx}\sin(3(a+bx))-30bd^2\sqrt{c+dx}\sin(3(a+bx))+144b^3cdx\sqrt{c+dx}\sin(3(a+bx))+72b^3d^2x^2\sqrt{c+dx}\sin(3(a+bx))}{864b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3,x]
```

```
[Out] (1620*b^2*c*d*Sqrt[c + d*x]*Cos[a + b*x] + 1620*b^2*d^2*x*Sqrt[c + d*x]*Cos
[a + b*x] + 60*b^2*c*d*Sqrt[c + d*x]*Cos[3*(a + b*x)] + 60*b^2*d^2*x*Sqrt[c
 + d*x]*Cos[3*(a + b*x)] + 1215*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[a - (b*c)/d]*F
resnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 5*Sqrt[b/d]*d^3*Sqrt[6*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + 5*Sqrt[b/
d]*d^3*Sqrt[6*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3
*b*c)/d] + 1215*Sqrt[b/d]*d^3*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*Sin[a - (b*c)/d] + 648*b^3*c^2*Sqrt[c + d*x]*Sin[a + b*x] - 2430
*b*d^2*Sqrt[c + d*x]*Sin[a + b*x] + 1296*b^3*c*d*x*Sqrt[c + d*x]*Sin[a + b*
x] + 648*b^3*d^2*x^2*Sqrt[c + d*x]*Sin[a + b*x] + 72*b^3*c^2*Sqrt[c + d*x]*
Sin[3*(a + b*x)] - 30*b*d^2*Sqrt[c + d*x]*Sin[3*(a + b*x)] + 144*b^3*c*d*x*
```

```
Sqrt[c + d*x]*Sin[3*(a + b*x)] + 72*b^3*d^2*x^2*Sqrt[c + d*x]*Sin[3*(a + b*
x)])/(864*b^4)
```

Maple [A]

time = 0.07, size = 474, normalized size = 1.16

method	result
derivativedivides	$\frac{3d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{4b}-\frac{15d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots$
default	$\frac{3d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{4b}-\frac{15d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(5/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-15/8/b*d*(-1/2/b*
d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/
2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/
24/b*d*(d*x+c)^(5/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-5/24/b*d*(-1/6/b*d*(d
*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)
```

```
*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 547, normalized size = 1.33

$$\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)+6480\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-5\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{3i\,b}{d}}\right)-1215\left(-(i+1)\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{i\,b}{d}}\right)-1215\left((i-1)\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{i\,b}{d}}\right)-5\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3i\,b}{d}}\right)+24\left(\frac{12\,(dx+c)^{\frac{5}{2}}b^4}{d}-5\sqrt{dx+c}\,b^2d\right)\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)+648\left(\frac{4\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{dx+c}\,b^2d\right)\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)\right)d}{3456\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) + 6480*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/
4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(3*I*b/d)) - 1215*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 1215*((I - 1)*sqrt(2)*s
qrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi
)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)
) - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c -
 a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x + c)^(5/2)*b^
4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d) + 648*(4*(d
*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*b - b*c + a*d)
/d))*d/b^5
```

Fricas [A]
time = 0.42, size = 368, normalized size = 0.90

$$\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+1215\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+1215\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+24\left(10\left(b^2d^2x+b^2cd\right)\cos(bx+a)^3+60\left(b^2d^2x+b^2cd\right)\cos(bx+a)+\left(24\,b^3d^2x^2+48\,b^3cdx+24\,b^3c^2-100\,bd^2+\left(12\,b^3d^2x^2+24\,b^3cdx+12\,b^3c^2-5\,bd^2\right)\cos(bx+a)^2\right)\sin(bx+a)\right)\sqrt{dx+c}}{864\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*
sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
 + b^2*c*d)*cos(b*x + a)^3 + 60*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) + (24*b^
3*d^2*x^2 + 48*b^3*c*d*x + 24*b^3*c^2 - 100*b*d^2 + (12*b^3*d^2*x^2 + 24*b^
```

```
3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c)
)/b^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] Result contains complex when optimal does not.
time = 0.87, size = 2469, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/1728*(72*(9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(27*(sqrt(2)*sqrt(pi)*(4*b^2
*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqr
t(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(6)*
sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x
+ c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^2)/d^2 + 27*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2
)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
I*b*c + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^
2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(-2
*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3
```

```
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*(81*(sqrt(2)*sqrt(pi)*(
8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*
I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/
2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b
^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^3) - 6*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3
/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*
sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^(-3*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b^3)/d^3 + 81*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d -
 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d
+ 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c
)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b
^3)/d^3 + (sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I
*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*
(-12*I*(d*x + c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x
 + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 5*I
*sqrt(d*x + c)*d^3)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) - 36
*(27*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*sqrt(2)*sqrt(pi)*(2*b*c - I*
d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*
sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) - 54*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b +
 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 54*I*sqrt
(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*
e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.57 $\int (c+dx)^{3/2} \cos^3(a+bx)\,dx$

Optimal. Leaf size=354

$$\frac{d\sqrt{c+dx}\,\cos(a+bx)}{b^2}+\frac{d\sqrt{c+dx}\,\cos^3(a+bx)}{6b^2}-\frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-$$

```
[Out] 2/3*(d*x+c)^(3/2)*sin(b*x+a)/b+1/3*(d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)/b-
1/144*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)+1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)
-9/16*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+9/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+d*cos(b*
x+a)*(d*x+c)^(1/2)/b^2+1/6*d*cos(b*x+a)^3*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.64, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 3377, 3387, 3386, 3432, 3385, 3433, 3393}

$$-\frac{9\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{9\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{d\sqrt{c+dx}\,\cos^3(a+bx)}{6b^2}+\frac{d\sqrt{c+dx}\,\cos(a+bx)}{b^2}+\frac{2(c+dx)^{3/2}\sin(a+bx)}{3b}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3,x]
```

```
[Out] (d*Sqrt[c + d*x]*Cos[a + b*x])/b^2 + (d*Sqrt[c + d*x]*Cos[a + b*x]^3)/(6*b^
2) - (9*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/
d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) + (d^
(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(24*b^(5/2)) + (9*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (2*(c + d*x
)^(3/2)*Sin[a + b*x])/(3*b) + ((c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x])
/(3*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \frac{d\sqrt{c+dx}\,\cos^3(a+bx)}{6b^2} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2}{3}\int (c+$$

$$= \frac{d\sqrt{c+dx}\,\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos^2(a+}{3b}$$

$$= \frac{d\sqrt{c+dx}\,\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\,\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+b}{3b}$$

$$= \frac{d\sqrt{c+dx}\,\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\,\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+b}{3b}$$

$$= \frac{d\sqrt{c+dx}\,\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\,\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+b}{3b}$$

$$= \frac{d\sqrt{c+dx}\,\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\,\cos^3(a+bx)}{6b^2} - \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right.}{}$$

Mathematica [**A**]
time = 1.03, size = 390, normalized size = 1.10

$$\frac{162\sqrt{\frac{b}{d}}\,d\sqrt{c+dx}\,\cos(a+bx)+6\sqrt{\frac{b}{d}}\,d\sqrt{c+dx}\,\cos(3(a+bx))-81d\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-d\sqrt{6\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+d\sqrt{6\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)+81d\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right)+108bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\sin(a+bx)+108b\sqrt{\frac{b}{d}}\,dx\sqrt{c+dx}\,\sin(a+bx)+12bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\sin(3(a+bx))+12b\sqrt{\frac{b}{d}}\,dx\sqrt{c+dx}\,\sin(3(a+bx))}{144b^2\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3,x]
```

```
[Out] (162*Sqrt[b/d]*d*Sqrt[c + d*x]*Cos[a + b*x] + 6*Sqrt[b/d]*d*Sqrt[c + d*x]*C
os[3*(a + b*x)] - 81*d*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]] - d*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]
*Sqrt[6/Pi]*Sqrt[c + d*x]] + d*Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqr
t[c + d*x]]*Sin[3*a - (3*b*c)/d] + 81*d*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] + 108*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Sin
[a + b*x] + 108*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Sin[a + b*x] + 12*b*c*Sqrt[b/
d]*Sqrt[c + d*x]*Sin[3*(a + b*x)] + 12*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Sin[3*
(a + b*x)])/(144*b^2*Sqrt[b/d])
```

Maple [**A**]
time = 0.06, size = 386, normalized size = 1.09

method	result
derivativedivides	$\frac{3d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{4b} - \frac{9d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\ldots}{\sqrt{?}\ldots}\right.\right.}{\ldots}\right.}{4b}$
default	$\frac{3d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{4b} - \frac{9d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\ldots}{\sqrt{?}\ldots}\right.\right.}{\ldots}\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-9/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d))+1/24/b*d*(d*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.58, size = 497, normalized size = 1.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d + 432*(d
*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b^2
*cos(3*((d*x + c)*b - b*c + a*d)/d) + 648*sqrt(d*x + c)*b^2*cos(((d*x + c)*
b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)
*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 81*(-(I - 1)*sqr
```

```
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqr
t(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d
)) - 81*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) +
 (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^
(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2
)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^4
```

Fricas [**A**]
time = 0.41, size = 299, normalized size = 0.84

$$-\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+81\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-81\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-24\,\left(bd\cos\left(bx+a\right)^3+6\,bd\cos\left(bx+a\right)+2\,\left(2\,b^2dx+2\,b^2c+\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{144\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 81*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 +
 6*b*d*cos(b*x + a) + 2*(2*b^2*d*x + 2*b^2*c + (b^2*d*x + b^2*c)*cos(b*x +
a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*cos(a + b*x)**3, x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.72, size = 1541, normalized size = 4.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/288*(12*(9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
```

```
2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(27*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 +
 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x
+ c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(p
i)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b^2) + 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b
*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^
2 + 27*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
- 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
+ I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*e
rf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(-2*I*(d*
x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - 4*(27*sqrt(2)*sqrt(pi)*(2*b*c + I
*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sq
rt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 27*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/
2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 54*I*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b + 54*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b +
 I*b*c - I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c -
I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.58 $\int \sqrt{c+dx}\,\cos^3(a+bx)\,dx$

Optimal. Leaf size=304

$$-\frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\sqrt{d}\ \ldots$$

```
[Out] -1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-
3/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d
^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-3/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+3/4*sin(b*x
+a)*(d*x+c)^(1/2)/b+1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.31, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{3\sqrt{c+dx}\sin(a+bx)}{4b}+\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt[d
]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a -
 (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (3*Sqrt[c + d*x
]*Sin[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{c+dx}\, \cos^3(a+bx)\, dx &= \int \left(\frac{3}{4}\sqrt{c+dx}\, \cos(a+bx) + \frac{1}{4}\sqrt{c+dx}\, \cos(3a+3bx)\right) dx \\
&= \frac{1}{4}\int \sqrt{c+dx}\, \cos(3a+3bx)\, dx + \frac{3}{4}\int \sqrt{c+dx}\, \cos(a+bx)\, dx \\
&= \frac{3\sqrt{c+dx}\, \sin(a+bx)}{4b} + \frac{\sqrt{c+dx}\, \sin(3a+3bx)}{12b} - \frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\, dx}{24b} - \frac{(3d)\ \cdots}{\cdots} \\
&= \frac{3\sqrt{c+dx}\, \sin(a+bx)}{4b} + \frac{\sqrt{c+dx}\, \sin(3a+3bx)}{12b} - \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(3\cdots\right)}{\sqrt{c\cdots}}}{24b} \cdots \\
&= \frac{3\sqrt{c+dx}\, \sin(a+bx)}{4b} + \frac{\sqrt{c+dx}\, \sin(3a+3bx)}{12b} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int \text{s}\cdots\right.}{1\cdots} \\
&= -\frac{3\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}\,\sqrt{\frac{\pi}{6}}\,\cos\left(3a-\frac{3bc}{d}\right)\cdots}{12\cdots}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 254, normalized size = 0.84

$$
\frac{ie^{-\frac{3i(bc+ad)}{d}}\sqrt{c+dx}\left(-27e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+27e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(-e^{6ia}\sqrt{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)\right)\right)}{72b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3,x]
```

```
[Out] ((I/72)*Sqrt[c + d*x]*(-27*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d
]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 27*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((
-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(-(E^((6*I)*a)*
Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d]) + E^(((6*I)*b*c
)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(
((3*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Maple [A]
time = 0.06, size = 294, normalized size = 0.97

method	result

derivativedivides	$\frac{\frac{3d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{4b}-\frac{3d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\sqrt{2}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{\frac{3d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{4b}-\frac{3d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\sqrt{2}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d))+1/24/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d
)-1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 424, normalized size = 1.39

$$\frac{\left(\frac{24\sqrt{dx+c}\,b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{216\sqrt{dx+c}\,b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right)-27\left((i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-27\left(-(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3ib}{d}}\right)\right)d}{288\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d + 216*sqrt
(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) - 27*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)
/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(I*b/d)) - 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)
*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c
 - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt
(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt
(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b
/d))*d/b^3
```

Fricas [A]
time = 0.42, size = 245, normalized size = 0.81

$$-\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-24\,\left(b\cos\left(bx+a\right)^{2}+2\,b\right)\sqrt{dx+c}\,\sin\left(bx+a\right)}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b
*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 + 2*b)*sqrt(
d*x + c)*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\cos^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.57, size = 844, normalized size = 2.78

$$\frac{\frac{27\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{\sqrt{6}\,\sqrt{\pi}\,(6\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{27\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{\sqrt{6}\,\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}-6\left(\frac{9\,\sqrt{2}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}+\frac{\sqrt{6}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}+\frac{9\,\sqrt{2}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}+\frac{\sqrt{6}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}\right)c-\frac{54\,i\,\sqrt{dx+c}\,d\,e^{\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{d}\right)}}{b}+\frac{6\,i\,\sqrt{dx+c}\,d\,e^{\left(-\frac{3\,(i\,(dx+c)b-i\,bc+i\,ad)}{d}\right)}}{b}+\frac{54\,i\,\sqrt{dx+c}\,d\,e^{\left(\frac{-i\,(dx+c)b+i\,bc-i\,ad}{d}\right)}}{b}-\frac{6\,i\,\sqrt{dx+c}\,d\,e^{\left(-\frac{3\,(-i\,(dx+c)b+i\,bc-i\,ad)}{d}\right)}}{b}}{144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/144*(27*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*sqrt(2)*sqrt(pi)*(2*b*c
 - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqr
t(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) - 6*(9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
```

```
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c - 54*I*sqrt(d*x + c)*d*e^((I*(d*x
 + c)*b - I*b*c + I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b -
I*b*c + I*a*d)/d)/b + 54*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a
*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)
/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.59 $\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=257

$$\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\cdots$$

```
[Out] 1/12*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*6^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-1/12*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)+3
/4*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^
(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-3/4*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(
Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(2*Sqrt[b]*Sqrt[d]) - (3*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}}+\frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx \\
&= \frac{1}{4}\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx+\frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx \\
&= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx+\frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx- \\
&= \frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{2d}+\frac{\left(3\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\right.\right.}{2d} \\
&= \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c-}}{\sqrt{d}}\right.}{2\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 236, normalized size = 0.92

$$\frac{ie^{-\frac{3i(bc+ad)}{d}}\left(-9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+9e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(-e^{6ia}\sqrt{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{3ib(c+dx)}{d}\right)\right)\right)}{24b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] ((I/24)*(-9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2,
 ((-I)*b*(c + d*x))/d] + 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))
/d]*Gamma[1/2, (I*b*(c + d*x))/d] + Sqrt[3]*(-(E^((6*I)*a)*Sqrt[((-I)*b*(c
+ d*x))/d]*Gamma[1/2, ((-3*I)*b*(c + d*x))/d]) + E^(((6*I)*b*c)/d)*Sqrt[(I*
b*(c + d*x))/d]*Gamma[1/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(((3*I)*(b*c + a*
d))/d)*Sqrt[c + d*x])
```

Maple [A]
time = 0.06, size = 212, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}}+\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\left(c\right.}{d}$
default	$\frac{\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}}+\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\left(c\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/8*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))+1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 377, normalized size = 1.47

$$-\frac{\left(\left(\frac{(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)}{d}+\frac{(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{3i\,b}{d}}\right)-9\left(-\frac{(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)}{d}-\frac{(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{i\,b}{d}}\right)-9\left(\frac{(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)}{d}+\frac{(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{i\,b}{d}}\right)+\left(-\frac{(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)}{d}-\frac{(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3i\,b}{d}}\right)\right)d}{48\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*
d)/d)/d + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c -
a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(pi)*
b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I + 1
)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c
 - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b
*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^2
```

Fricas [A]
time = 0.39, size = 213, normalized size = 0.83

$$\frac{\sqrt{6}\,\pi\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+9\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{6}\,\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(6)*pi*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*s
qrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)
/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqrt(b
/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)
/d) - sqrt(6)*pi*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-3*(b*c - a*d)/d))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{3}\left(a+bx\right)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**3/sqrt(c + d*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.46, size = 330, normalized size = 1.28

$$-\frac{\frac{9\,\sqrt{2}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{6}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{9\,\sqrt{2}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{6}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^3}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3/(c + d*x)^(1/2), x)
```

3.60 $\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx$

<u>**Optimal**</u>. Leaf size=271

$$-\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}}-\frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}}{\sqrt{}}\right.}{d^{3/2}}$$

```
[Out] -3/2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-3/2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-1/2*cos(3*
a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)
*6^(1/2)*Pi^(1/2)/d^(3/2)-1/2*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*sin(3*a-3*b*c/d)*b^(1/2)*6^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b*x+a)^
3/d/(d*x+c)^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.34, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3394, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(d*Sqrt[c + d*x]) - (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a - (b*c)
/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b
]*Sqrt[(3*Pi)/2]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelC[(Sqrt[b]*Sqrt[6
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/d^(3/2) - (3*Sqrt[b]*Sqr
t[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/
d])/d^(3/2)
```

<u>Rule 3385</u>

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

<u>Rule 3386</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} + \frac{(6b)\int \left(-\frac{\sin(a+bx)}{4\sqrt{c+dx}} - \frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d} \\
&= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{2d} - \frac{(3b)\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx}{2d} \\
&= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{2d} - \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right.}{\sqrt{c+dx}}}{2d} \\
&= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{(3b\cos}{} \\
&= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos}{}
\end{aligned}
$$

Mathematica [A]

time = 1.01, size = 299, normalized size = 1.10

$$
-\frac{3\cos(a+bx)+\cos(3(a+bx))+3\sqrt{\frac{b}{d}}\sqrt{2\pi}\sqrt{c+dx}\cos\left(a-\frac{bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+\sqrt{\frac{b}{d}}\sqrt{6\pi}\sqrt{c+dx}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{\frac{b}{d}}\sqrt{6\pi}\sqrt{c+dx}\,\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)+3\sqrt{\frac{b}{d}}\sqrt{2\pi}\sqrt{c+dx}\,\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right)}{2d\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] -1/2*(3*Cos[a + b*x] + Cos[3*(a + b*x)] + 3*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c + d
*x]*Cos[a - (b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + Sqrt[b/
d]*Sqrt[6*Pi]*Sqrt[c + d*x]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]] + Sqrt[b/d]*Sqrt[6*Pi]*Sqrt[c + d*x]*FresnelC[Sqrt[b/d]*
Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 3*Sqrt[b/d]*Sqrt[2*Pi]*Sqr
t[c + d*x]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d])/(
d*Sqrt[c + d*x])
```

Maple [A]

time = 0.06, size = 286, normalized size = 1.06

method	result

derivativedivides	$\frac{-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2\sqrt{dx+c}}-\frac{3b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\cdots\right)\right)}{2d\sqrt{\frac{b}{d}}}\cdots}{\cdots}$
default	$\frac{-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2\sqrt{dx+c}}-\frac{3b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\cdots\right)\right)}{2d\sqrt{\frac{b}{d}}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/4/(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/4*b/d*2^(1/2)*Pi^(
1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d))-1/4/(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/4*b/d
*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC
(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.70, size = 253, normalized size = 0.93

$$\frac{\sqrt{3}\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{3i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{3i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{3i(dx+c)b}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\sqrt{\frac{(dx+c)b}{d}}-3\left(\left((i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{i(dx+c)b}{d}\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\sqrt{\frac{(dx+c)b}{d}}}{16\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(3)*((-(I + 1)*sqrt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) + (I - 1)*s
qrt(2)*gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I - 1)*sq
rt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -3*I*(d*
x + c)*b/d))*sin(-3*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) - 3*(((I + 1)*sqrt(
2)*gamma(-1/2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*
b/d))*cos(-(b*c - a*d)/d) + (-(I - 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b/d)
+ (I + 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*sqrt(
(d*x + c)*b/d))/(sqrt(d*x + c)*d)
```

Fricas [A]
time = 0.38, size = 265, normalized size = 0.98

$$-\frac{\sqrt{6}\,(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{6}\,(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+4\sqrt{dx+c}\cos(bx+a)^3}{2\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")

[Out] -1/2*(sqrt(6)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(6)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 4*sqrt(d*x + c)*cos(b*x + a)^3)/(d^2*x + c*d)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3/(d*x+c)**(3/2),x)

[Out] Integral(cos(a + b*x)**3/(c + d*x)**(3/2), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(cos(b*x + a)^3/(d*x + c)^(3/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^3}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)^3/(c + d*x)^(3/2),x)

[Out] int(cos(a + b*x)^3/(c + d*x)^(3/2), x)

3.61 $\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}-\frac{b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{b^{3/2}\sqrt{6\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}}{d^{5/2}}$$

```
[Out] -2/3*cos(b*x+a)^3/d/(d*x+c)^(3/2)-b^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+b^(3/2)*Fresn
elS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi
^(1/2)/d^(5/2)-b^(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/d^(5/2)+b^(3/2)*FresnelS(b^(1/2)*6^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/d^(5
/2)+4*b*cos(b*x+a)^2*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 3387, 3386, 3432, 3385, 3433, 3393}

$$-\frac{\sqrt{2\pi}\,b^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{6\pi}\,b^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{\sqrt{6\pi}\,b^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{\sqrt{2\pi}\,b^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) - (b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(5/2) - (b^(3
/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/d^(5/2) + (b^(3/2)*Sqrt[6*Pi]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/d^(5/2) + (b^(3/2)*Sqrt[2*Pi]*
FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(5
/2) + (4*b*Cos[a + b*x]^2*Sin[a + b*x])/(d^2*Sqrt[c + d*x])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} + \frac{(8b^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(12b^2)\int \frac{\cos^3(a}{\sqrt{c+}}}{d^2}$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} - \frac{(12b^2)\int\left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}} + \frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)}{d^2}$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} - \frac{(3b^2)\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(9b^2)\int \frac{\cos(a+}{\sqrt{c+}}}{d^2}$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{8b^{3/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{}}{}\right.}{}$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{8b^{3/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{}}{}\right.}{}$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{b^{3/2}\sqrt{6\pi}\,\cos\left(3a\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.31, size = 268, normalized size = 0.92

$$\frac{-4d\cos^3(a+bx) - 3de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right) - 3de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right) - 3\sqrt{3}\,de^{3i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{3ib(c+dx)}{d}\right) - 3\sqrt{3}\,de^{-3i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{3ib(c+dx)}{d}\right) + 24b(c+dx)\cos^2(a+bx)\sin(a+bx)}{6d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-4*d*Cos[a + b*x]^3 - 3*d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(3/2)
*Gamma[1/2, ((-I)*b*(c + d*x))/d] - (3*d*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (I*b*(c + d*x))/d])/E^(I*(a - (b*c)/d)) - 3*Sqrt[3]*d*E^((3*I)*(a - (b*c
)/d))*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-3*I)*b*(c + d*x))/d] - (3*
Sqrt[3]*d*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((3*I)*b*(c + d*x))/d])/E^((
3*I)*(a - (b*c)/d)) + 24*b*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(6*d^2*(c
 + d*x)^(3/2))
```

Maple [A]
time = 0.06, size = 368, normalized size = 1.26

method	result
derivativedivides	$\frac{-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2(dx+c)^{\frac{3}{2}}}-\frac{b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da}{}\right.\right.}{d\sqrt{\frac{b}{d}}}\right.}{d}}{}$
default	$\frac{-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{2(dx+c)^{\frac{3}{2}}}-\frac{b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da}{}\right.\right.}{d\sqrt{\frac{b}{d}}}\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/4/(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^
(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/12
/(d*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(1/2)*s
in(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(c
os(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.70, size = 253, normalized size = 0.87

$$-\frac{3\left(\sqrt{3}\left(\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{3i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+\left((i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{3i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{3i(dx+c)b}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}-\left(\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}\right)}{16\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/16*(sqrt(3)*(((I - 1)*sqrt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) - (I + 1)*s
qrt(2)*gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I + 1)*sq
rt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-3/2, -3*I*(d*
```

```
x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) - ((-(I - 1)*sqrt
(2)*gamma(-3/2, I*(d*x + c)*b/d) + (I + 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)
*b/d))*cos(-(b*c - a*d)/d) + (-(I + 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d)
 + (I - 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*((d*
x + c)*b/d)^(3/2))/((d*x + c)^(3/2)*d)
```

Fricas [A]
time = 0.45, size = 367, normalized size = 1.26

$$-\frac{3\sqrt{6}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-3\sqrt{6}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+2\left(d\cos\left(bx+a\right)^3-6\left(bdx+bc\right)\cos\left(bx+a\right)^2\sin\left(bx+a\right)\right)\sqrt{dx+c}}{3\left(d^4x^2+2cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(6)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sq
rt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-(b*c - a
*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi*b*
d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 3*sqrt(6)*(pi*b*d^2*x^2 + 2*
pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqr
t(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 2*(d*cos(b*x + a)^3 - 6*(b*d*x + b*c)*
cos(b*x + a)^2*sin(b*x + a))*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^3}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)^3/(c + d*x)^(5/2), x)
```

3.62 $\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=356

$$-\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}+\frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}}+\frac{2b^{5/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{6b}{}$$

```
[Out] -2/5*cos(b*x+a)^3/d/(d*x+c)^(5/2)+4/5*b*cos(b*x+a)^2*sin(b*x+a)/d^2/(d*x+c)
^(3/2)+2/5*b^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)+2/5*b^(5/2)*FresnelC(b^(1/2)*2^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/d^(7/2)+6/5*
b^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*6^(1/2)*Pi^(1/2)/d^(7/2)+6/5*b^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/d^(7/2)-16/5*b^
2*cos(b*x+a)/d^3/(d*x+c)^(1/2)+24/5*b^2*cos(b*x+a)^3/d^3/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 3378, 3387, 3386, 3432, 3385, 3433, 3394}

$$\frac{6\sqrt{6\pi}\,b^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2\sqrt{2\pi}\,b^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2\sqrt{2\pi}\,b^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{6\sqrt{6\pi}\,b^{5/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}}-\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (-16*b^2*Cos[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^3)/(5*d*(c +
 d*x)^(5/2)) + (24*b^2*Cos[a + b*x]^3)/(5*d^3*Sqrt[c + d*x]) + (2*b^(5/2)*S
qrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*P
i]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(5*d^(7/2)) + (2*b^(5/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(5*d^(7/2)) + (4*b*Cos[a + b*x]^2*Sin[a
+ b*x])/(5*d^2*(c + d*x)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2} - \frac{(12b^2)\int \frac{\cos^3(a+b}{(c+dx)^{3/}}}{5d^2}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{16b^{5/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)S\left(\right.}{5d^{7/2}}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{2b^{5/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)S\left(\right.}{5d^{7/2}}$$

Mathematica [**A**]
time = 4.47, size = 465, normalized size = 1.31

$$\frac{4b^2c^2\cos(a+bx)-3d^2\cos(a+bx)+8b^2cdx\cos(a+bx)+4b^2d^2x^2\cos(a+bx)+12b^2c^2\cos(3(a+bx))-d^2\cos(3(a+bx))+24b^2cdx\cos(3(a+bx))+12b^2d^2x^2\cos(3(a+bx))+4b\left(\frac{b}{d}\right)^{3/2}d\sqrt{2\pi}\,(c+dx)^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+12b\left(\frac{b}{d}\right)^{3/2}d\sqrt{6\pi}\,(c+dx)^{5/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+12b\left(\frac{b}{d}\right)^{3/2}d\sqrt{6\pi}\,(c+dx)^{5/2}\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)+4b\left(\frac{b}{d}\right)^{3/2}d\sqrt{2\pi}\,(c+dx)^{5/2}\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right)+2bcd\sin(a+bx)+2bd^2x\sin(a+bx)+2bcd\sin(3(a+bx))+2bd^2x\sin(3(a+bx))}{10d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (4*b^2*c^2*Cos[a + b*x] - 3*d^2*Cos[a + b*x] + 8*b^2*c*d*x*Cos[a + b*x] + 4
*b^2*d^2*x^2*Cos[a + b*x] + 12*b^2*c^2*Cos[3*(a + b*x)] - d^2*Cos[3*(a + b*
x)] + 24*b^2*c*d*x*Cos[3*(a + b*x)] + 12*b^2*d^2*x^2*Cos[3*(a + b*x)] + 4*b
*(b/d)^(3/2)*d*Sqrt[2*Pi]*(c + d*x)^(5/2)*Cos[a - (b*c)/d]*FresnelS[Sqrt[b/
d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 12*b*(b/d)^(3/2)*d*Sqrt[6*Pi]*(c + d*x)^(5/2
```

```
)*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + 12*b*
(b/d)^(3/2)*d*Sqrt[6*Pi]*(c + d*x)^(5/2)*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt
[c + d*x]]*Sin[3*a - (3*b*c)/d] + 4*b*(b/d)^(3/2)*d*Sqrt[2*Pi]*(c + d*x)^(5
/2)*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] + 2*b*c*d
*Sin[a + b*x] + 2*b*d^2*x*Sin[a + b*x] + 2*b*c*d*Sin[3*(a + b*x)] + 2*b*d^2
*x*Sin[3*(a + b*x)])/(10*d^3*(c + d*x)^(5/2))
```

Maple [A]
time = 0.06, size = 450, normalized size = 1.26

method	result
derivativedivides	$\frac{-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{10(dx+c)^{\frac{5}{2}}}-\frac{3b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{}\right)}{}\right)}{5d}}{}$
default	$\frac{-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{10(dx+c)^{\frac{5}{2}}}-\frac{3b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{}\right)}{}\right)}{5d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/20/(d*x+c)^(5/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x
+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(1/2)*cos(1/d*
```

```
b*(d*x+c)+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*F
resnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*Fr
esnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/20/(d*x+c)^(5/2
)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+c)^(3/2)*sin(3/d*b*(
d*x+c)+3*(a*d-b*c)/d)+2*b/d*(-1/(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)
/d)-b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2
^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*F
resnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

<u>Maxima</u> [**C**] Result contains complex when optimal does not.
time = 0.71, size = 254, normalized size = 0.71

$$-\frac{3 \left(3 \sqrt{3} \left(\left(-(i+1) \sqrt{2} \Gamma\left(-\frac{5}{2}, \frac{3i(dx+c)b}{d}\right) + (i-1) \sqrt{2} \Gamma\left(-\frac{5}{2}, -\frac{3i(dx+c)b}{d}\right)\right) \cos\left(-\frac{3(bc-ad)}{d}\right) + \left((i-1) \sqrt{2} \Gamma\left(-\frac{5}{2}, \frac{3i(dx+c)b}{d}\right) - (i+1) \sqrt{2} \Gamma\left(-\frac{5}{2}, -\frac{3i(dx+c)b}{d}\right)\right) \sin\left(-\frac{3(bc-ad)}{d}\right)\right) \left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} - \left(\left((i+1) \sqrt{2} \Gamma\left(-\frac{5}{2}, \frac{i(dx+c)b}{d}\right) - (i-1) \sqrt{2} \Gamma\left(-\frac{5}{2}, -\frac{i(dx+c)b}{d}\right)\right) \cos\left(-\frac{bc-ad}{d}\right) + \left(-(i-1) \sqrt{2} \Gamma\left(-\frac{5}{2}, \frac{i(dx+c)b}{d}\right) + (i+1) \sqrt{2} \Gamma\left(-\frac{5}{2}, -\frac{i(dx+c)b}{d}\right)\right) \sin\left(-\frac{bc-ad}{d}\right)\right) \left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}\right)}{16 (dx+c)^{\frac{5}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -3/16*(3*sqrt(3)*((-(I + 1)*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) + (I - 1
)*sqrt(2)*gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I - 1)
*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-5/2, -3*I*
(d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) - (((I + 1)*sq
rt(2)*gamma(-5/2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-5/2, -I*(d*x +
c)*b/d))*cos(-(b*c - a*d)/d) + (-(I - 1)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/
d) + (I + 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*((
d*x + c)*b/d)^(5/2))/((d*x + c)^(5/2)*d)
```

<u>Fricas</u> [**A**]
time = 0.44, size = 528, normalized size = 1.48

$$\frac{2 \left(3 \sqrt{6} \left(\pi b^2 d^3 x^3 + 3 \pi b^2 c d^2 x^2 + 3 \pi b^2 c^2 d x + \pi b^2 c^3\right) \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) \mathrm{S}\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + \sqrt{2} \left(\pi b^2 d^3 x^3 + 3 \pi b^2 c d^2 x^2 + 3 \pi b^2 c^2 d x + \pi b^2 c^3\right) \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) \mathrm{S}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + \sqrt{2} \left(\pi b^2 d^3 x^3 + 3 \pi b^2 c d^2 x^2 + 3 \pi b^2 c^2 d x + \pi b^2 c^3\right) \sqrt{\frac{b}{\pi d}} \mathrm{C}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{bc-ad}{d}\right) + 3 \sqrt{6} \left(\pi b^2 d^3 x^3 + 3 \pi b^2 c d^2 x^2 + 3 \pi b^2 c^2 d x + \pi b^2 c^3\right) \sqrt{\frac{b}{\pi d}} \mathrm{C}\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{3(bc-ad)}{d}\right) + \left(\left(12 b^2 d^2 x^2 + 24 b^2 c d x + 12 b^2 c^2 - d^2\right) \cos(bx+a)^3 + 2 \left(b d^2 x + b c d\right) \cos(bx+a)^2 \sin(bx+a) - 8 \left(b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2\right) \cos(bx+a)\right) \sqrt{dx+c}\right)}{5 \left(d^6 x^3 + 3 c d^5 x^2 + 3 c^2 d^4 x + c^3 d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(6)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi
*b^2*c^3)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi
*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(s
qrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c
*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 3*sqrt(6)*(pi*b^2*d^3
*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*f
resnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + ((
12*b^2*d^2*x^2 + 24*b^2*c*d*x + 12*b^2*c^2 - d^2)*cos(b*x + a)^3 + 2*(b*d^2
*x + b*c*d)*cos(b*x + a)^2*sin(b*x + a) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
```

```
2*c^2)*cos(b*x + a))*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
c^3*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(7/2),x)
[Out] Integral(cos(a + b*x)**3/(c + d*x)**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^3/(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^3}{(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/(c + d*x)^(7/2),x)
[Out] int(cos(a + b*x)^3/(c + d*x)^(7/2), x)
```

3.63 $\int x^{3/2} \cos(x)\, dx$

Optimal. Leaf size=49

$$\frac{3}{2}\sqrt{x}\,\cos(x) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x}\right) + x^{3/2}\sin(x)$$

```
[Out] x^(3/2)*sin(x)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+3/2*
x^(1/2)*cos(x)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3385, 3433}

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x}\right) + x^{3/2}\sin(x) + \frac{3}{2}\sqrt{x}\,\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Cos[x],x]
```

```
[Out] (3*Sqrt[x]*Cos[x])/2 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[x]])/2 + x^(3
/2)*Sin[x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}\cos(x)\,dx &= x^{3/2}\sin(x) - \frac{3}{2}\int \sqrt{x}\,\sin(x)\,dx \\
&= \frac{3}{2}\sqrt{x}\,\cos(x) + x^{3/2}\sin(x) - \frac{3}{4}\int \frac{\cos(x)}{\sqrt{x}}\,dx \\
&= \frac{3}{2}\sqrt{x}\,\cos(x) + x^{3/2}\sin(x) - \frac{3}{2}\mathrm{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{x}\right) \\
&= \frac{3}{2}\sqrt{x}\,\cos(x) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x}\right) + x^{3/2}\sin(x)
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.01, size = 55, normalized size = 1.12

$$\frac{\sqrt{x}\,\mathrm{Gamma}\left(\frac{5}{2},-ix\right)}{2\sqrt{-ix}}+\frac{\sqrt{x}\,\mathrm{Gamma}\left(\frac{5}{2},ix\right)}{2\sqrt{ix}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Cos[x],x]
```

```
[Out] (Sqrt[x]*Gamma[5/2, (-I)*x])/(2*Sqrt[(-I)*x]) + (Sqrt[x]*Gamma[5/2, I*x])/(
2*Sqrt[I*x])
```

Maple [A]
time = 0.04, size = 34, normalized size = 0.69

method	result	size
derivativedivides	$x^{\frac{3}{2}}\sin\left(x\right)-\frac{3\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{4}+\frac{3\sqrt{x}\,\cos(x)}{2}$	34
default	$x^{\frac{3}{2}}\sin\left(x\right)-\frac{3\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{4}+\frac{3\sqrt{x}\,\cos(x)}{2}$	34
meijerg	$2\sqrt{2}\,\sqrt{\pi}\left(\frac{3\sqrt{x}\,\sqrt{2}\,\cos(x)}{8\sqrt{\pi}}+\frac{x^{\frac{3}{2}}\sqrt{2}\,\sin(x)}{4\sqrt{\pi}}-\frac{3\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)}{8}\right)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] x^(3/2)*sin(x)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+3/2*
x^(1/2)*cos(x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 74, normalized size = 1.51

$$x^{\frac{3}{2}}\sin\left(x\right) - \frac{3}{32}\sqrt{\pi}\left(-\left(i-1\right)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - \left(i+1\right)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) + \left(i+1\right)\sqrt{2}\,\mathrm{erf}\left(\sqrt{-i}\sqrt{x}\right) - \left(i-1\right)\sqrt{2}\,\mathrm{erf}\left(\left(-1\right)^{\frac{1}{4}}\sqrt{x}\right)\right) + \frac{3}{2}\sqrt{x}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="maxima")
```

```
[Out] x^(3/2)*sin(x) - 3/32*sqrt(pi)*(-(I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*
sqrt(x)) - (I + 1)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (I + 1)*sqr
t(2)*erf(sqrt(-I)*sqrt(x)) - (I - 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x))) + 3/2
*sqrt(x)*cos(x)
```

Fricas [A]
time = 0.38, size = 35, normalized size = 0.71

$$-\frac{3}{4}\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right) + \frac{1}{2}\left(2\,x\sin\left(x\right) + 3\,\cos\left(x\right)\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="fricas")
```

```
[Out] -3/4*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x)/sqrt(pi)) + 1/2*(2*x*sin(
x) + 3*cos(x))*sqrt(x)
```

Sympy [A]
time = 3.17, size = 83, normalized size = 1.69

$$\frac{5x^{\frac{3}{2}}\sin\left(x\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{15\sqrt{x}\cos\left(x\right)\Gamma\left(\frac{5}{4}\right)}{8\Gamma\left(\frac{9}{4}\right)} - \frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{16\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*cos(x),x)
```

```
[Out] 5*x**(3/2)*sin(x)*gamma(5/4)/(4*gamma(9/4)) + 15*sqrt(x)*cos(x)*gamma(5/4)/
(8*gamma(9/4)) - 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*sqrt(x)/sqrt(pi))*gam
ma(5/4)/(16*gamma(9/4))
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 69, normalized size = 1.41

$$\left(\frac{3}{16}i + \frac{3}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - \left(\frac{3}{16}i - \frac{3}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - \frac{1}{4}\left(2i\,x^{\frac{3}{2}} - 3\sqrt{x}\right)e^{\left(i\,x\right)} - \frac{1}{4}\left(-2i\,x^{\frac{3}{2}} - 3\sqrt{x}\right)e^{\left(-i\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="giac")
```

```
[Out] (3/16*I + 3/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (3/16
*I - 3/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x)) - 1/4*(2*I*
x^(3/2) - 3*sqrt(x))*e^(I*x) - 1/4*(-2*I*x^(3/2) - 3*sqrt(x))*e^(-I*x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{3/2}\,\cos\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(x),x)
```

```
[Out] int(x^(3/2)*cos(x), x)
```

3.64 $\int \sqrt{x} \cos(x) \, dx$

Optimal. Leaf size=36

$$-\sqrt{\frac{\pi}{2}}\, S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x}\right)+\sqrt{x}\,\sin(x)$$

```
[Out] -1/2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+sin(x)*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3386, 3432}

$$\sqrt{x}\,\sin(x)-\sqrt{\frac{\pi}{2}}\, S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Cos[x],x]
```

```
[Out] -(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]) + Sqrt[x]*Sin[x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{x}\cos(x)\,dx = \sqrt{x}\sin(x) - \frac{1}{2}\int \frac{\sin(x)}{\sqrt{x}}\,dx$$
$$= \sqrt{x}\sin(x) - \text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{x}\right)$$
$$= -\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) + \sqrt{x}\sin(x)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.00, size = 48, normalized size = 1.33

$$\frac{\sqrt{-ix}\,\text{Gamma}\left(\frac{3}{2}, -ix\right) + \sqrt{ix}\,\text{Gamma}\left(\frac{3}{2}, ix\right)}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Cos[x],x]
[Out] (Sqrt[(-I)*x]*Gamma[3/2, (-I)*x] + Sqrt[I*x]*Gamma[3/2, I*x])/(2*Sqrt[x])
```

Maple [A]
time = 0.03, size = 27, normalized size = 0.75

method	result	size
derivativedivides	$-\frac{\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{2}+\sin\left(x\right)\sqrt{x}$	27
default	$-\frac{\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{2}+\sin\left(x\right)\sqrt{x}$	27
meijerg	$\sqrt{2}\,\sqrt{\pi}\left(\frac{\sqrt{x}\,\sqrt{2}\,\sin(x)}{2\sqrt{\pi}}-\frac{\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)}{2}\right)$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(x),x,method=_RETURNVERBOSE)
[Out] -1/2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+sin(x)*x^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 67, normalized size = 1.86

$$-\frac{1}{16}\sqrt{\pi}\left((i+1)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\,\sqrt{x}\right)+(i-1)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\,\sqrt{x}\right)-(i-1)\sqrt{2}\,\mathrm{erf}\left(\sqrt{-i}\,\sqrt{x}\right)+(i+1)\sqrt{2}\,\mathrm{erf}\left((-1)^{\frac{1}{4}}\sqrt{x}\right)\right)+\sqrt{x}\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*((I + 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*sqrt(x)) + (I - 1
)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (I - 1)*sqrt(2)*erf(sqrt(-I)
*sqrt(x)) + (I + 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x))) + sqrt(x)*sin(x)
```

Fricas [A]
time = 0.40, size = 26, normalized size = 0.72

$$-\frac{1}{2}\sqrt{2}\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)+\sqrt{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(pi)*fresnel_sin(sqrt(2)*sqrt(x)/sqrt(pi)) + sqrt(x)*sin(x
)
```

Sympy [A]
time = 0.51, size = 61, normalized size = 1.69

$$\frac{3\sqrt{x}\sin\left(x\right)\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{3\sqrt{2}\sqrt{\pi}\,S\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*cos(x),x)
```

```
[Out] 3*sqrt(x)*sin(x)*gamma(3/4)/(4*gamma(7/4)) - 3*sqrt(2)*sqrt(pi)*fresnels(sq
rt(2)*sqrt(x)/sqrt(pi))*gamma(3/4)/(8*gamma(7/4))
```

Giac [C] Result contains complex when optimal does not.
time = 0.56, size = 53, normalized size = 1.47

$$-\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)+\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-\frac{1}{2}i\sqrt{x}\,e^{(ix)}+\frac{1}{2}i\sqrt{x}\,e^{(-ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="giac")
```

```
[Out] -(1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (1/8*I
 + 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x)) - 1/2*I*sqrt(x
)*e^(I*x) + 1/2*I*sqrt(x)*e^(-I*x)
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.72

$$\sqrt{x}\ \sin(x) - \frac{\sqrt{2}\ \sqrt{\pi}\ \mathrm{S}\left(\frac{\sqrt{2}\ \sqrt{x}}{\sqrt{\pi}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(x),x)
[Out] x^(1/2)*sin(x) - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*x^(1/2))/pi^(1/2)))/2
```

3.65 $\int \frac{\cos(x)}{\sqrt{x}} \, dx$

Optimal. Leaf size=24

$$\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

```
[Out] FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3385, 3433}

$$\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[x],x]
```

```
[Out] Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[x]]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{\sqrt{x}} \, dx &= 2\text{Subst}\left(\int \cos\left(x^2\right) \, dx, x, \sqrt{x}\right)\\ &= \sqrt{2\pi}\, C\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.01, size = 51, normalized size = 2.12

$$-\frac{i\left(\sqrt{-ix}\,\text{Gamma}\left(\frac{1}{2},-ix\right)-\sqrt{ix}\,\text{Gamma}\left(\frac{1}{2},ix\right)\right)}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[x],x]
```

```
[Out] ((-1/2*I)*(Sqrt[(-I)*x]*Gamma[1/2, (-I)*x] - Sqrt[I*x]*Gamma[1/2, I*x]))/Sq
rt[x]
```

Maple [A]
time = 0.03, size = 19, normalized size = 0.79

method	result	size
derivativedivides	$\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}$	19
default	$\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}$	19
meijerg	$\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 60, normalized size = 2.50

$$-\frac{1}{8}\sqrt{\pi}\left((i-1)\sqrt{2}\,\text{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\,\sqrt{x}\right)+(i+1)\sqrt{2}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\,\sqrt{x}\right)-(i+1)\sqrt{2}\,\text{erf}\left(\sqrt{-i}\,\sqrt{x}\right)+(i-1)\sqrt{2}\,\text{erf}\left((-1)^{\frac{1}{4}}\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*((I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*sqrt(x)) + (I + 1)
*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (I + 1)*sqrt(2)*erf(sqrt(-I)*
sqrt(x)) + (I - 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x)))
```

Fricas [A]
time = 0.39, size = 18, normalized size = 0.75

$$\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x)/sqrt(pi))
```

Sympy [A]
time = 0.46, size = 37, normalized size = 1.54

$$\frac{\sqrt{2}\,\sqrt{\pi}\,C\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*sqrt(x)/sqrt(pi))*gamma(1/4)/(4*gamma(5/4
))
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 35, normalized size = 1.46

$$-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\,\sqrt{x}\right)+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="giac")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (1/4*I
 - 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x))
```

Mupad [B]
time = 0.03, size = 18, normalized size = 0.75

$$\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x^(1/2),x)
```

```
[Out] 2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*x^(1/2))/pi^(1/2))
```

3.66 $\int \frac{\cos(x)}{x^{3/2}}\, dx$

Optimal. Leaf size=35

$$-\frac{2\cos(x)}{\sqrt{x}} - 2\sqrt{2\pi}\, S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x}\right)$$

```
[Out] -2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)-2*cos(x)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3386, 3432}

$$-2\sqrt{2\pi}\, S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x}\right) - \frac{2\cos(x)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/x^(3/2),x]
```

```
[Out] (-2*Cos[x])/Sqrt[x] - 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(x)}{x^{3/2}}\,dx &= -\frac{2\cos(x)}{\sqrt{x}} - 2\int \frac{\sin(x)}{\sqrt{x}}\,dx \\
&= -\frac{2\cos(x)}{\sqrt{x}} - 4\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{x}\right) \\
&= -\frac{2\cos(x)}{\sqrt{x}} - 2\sqrt{2\pi}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 63, normalized size = 1.80

$$
\frac{-e^{-ix}(1+e^{2ix}) + \sqrt{-ix}\,\text{Gamma}\left(\frac{1}{2}, -ix\right) + \sqrt{ix}\,\text{Gamma}\left(\frac{1}{2}, ix\right)}{\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/x^(3/2),x]
```

```
[Out] (-((1 + E^((2*I)*x))/E^(I*x)) + Sqrt[(-I)*x]*Gamma[1/2, (-I)*x] + Sqrt[I*x]
*Gamma[1/2, I*x])/Sqrt[x]
```

Maple [A]
time = 0.04, size = 28, normalized size = 0.80

method	result	size
derivativedivides	$-2\,\text{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi} - \frac{2\cos(x)}{\sqrt{x}}$	28
default	$-2\,\text{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi} - \frac{2\cos(x)}{\sqrt{x}}$	28
meijerg	$\frac{\sqrt{2}\,\sqrt{\pi}\left(-\frac{4\sqrt{2}\,\cos(x)}{\sqrt{\pi}\,\sqrt{x}} - 8\,\text{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\right)}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)-2*cos(x)/x^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 21, normalized size = 0.60

$$
-\left(\frac{1}{4}i + \frac{1}{4}\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2}, i\,x\right) + \left(\frac{1}{4}i - \frac{1}{4}\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2}, -i\,x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="maxima")
[Out] -(1/4*I + 1/4)*sqrt(2)*gamma(-1/2, I*x) + (1/4*I - 1/4)*sqrt(2)*gamma(-1/2,
 -I*x)
```

Fricas [A]
time = 0.41, size = 31, normalized size = 0.89

$$-\frac{2\left(\sqrt{2}\,\sqrt{\pi}\,x\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)+\sqrt{x}\,\cos\left(x\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="fricas")
[Out] -2*(sqrt(2)*sqrt(pi)*x*fresnel_sin(sqrt(2)*sqrt(x)/sqrt(pi)) + sqrt(x)*cos(
x))/x
```

Sympy [A]
time = 1.01, size = 61, normalized size = 1.74

$$\frac{\sqrt{2}\,\sqrt{\pi}\,S\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)}+\frac{\cos\left(x\right)\Gamma\left(-\frac{1}{4}\right)}{2\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x**(3/2),x)
[Out] sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*sqrt(x)/sqrt(pi))*gamma(-1/4)/(2*gamma(3/
4)) + cos(x)*gamma(-1/4)/(2*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="giac")
[Out] integrate(cos(x)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(x\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x^(3/2),x)
[Out] int(cos(x)/x^(3/2), x)
```

3.67 $\int (c+dx)^{4/3}\cos(a+bx)\,dx$

Optimal. Leaf size=183

$$\frac{4d\sqrt[3]{c+dx}\,\cos(a+bx)}{3b^2}+\frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}-\frac{2id^2e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gam}}{9b^3(c+dx)^{2/3}}$$

```
[Out] 4/3*d*(d*x+c)^(1/3)*cos(b*x+a)/b^2+2/9*I*d^2*exp(I*(a-b*c/d))*(-I*b*(d*x+c)
/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b^3/(d*x+c)^(2/3)-2/9*I*d^2*(I*b*(d*x+c
)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b^3/exp(I*(a-b*c/d))/(d*x+c)^(2/3)+(d*x
+c)^(4/3)*sin(b*x+a)/b
```

Rubi [A]
time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3377, 3388, 2212}

$$\frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}-\frac{2id^2e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}+\frac{4d\sqrt[3]{c+dx}\,\cos(a+bx)}{3b^2}+\frac{(c+dx)^{4/3}\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(4/3)*Cos[a + b*x],x]
```

```
[Out] (4*d*(c + d*x)^(1/3)*Cos[a + b*x])/(3*b^2) + (((2*I)/9)*d^2*E^(I*(a - (b*c)
/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(b^3*(c
 + d*x)^(2/3)) - (((2*I)/9)*d^2*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(
c + d*x))/d])/(b^3*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3)) + ((c + d*x)^(4/3)*
Sin[a + b*x])/b
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^{4/3}\cos(a+bx)\,dx &= \frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{(4d)\int \sqrt[3]{c+dx}\,\sin(a+bx)\,dx}{3b}\\
&= \frac{4d\sqrt[3]{c+dx}\,\cos(a+bx)}{3b^2}+\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{(4d^2)\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx}{9b^2}\\
&= \frac{4d\sqrt[3]{c+dx}\,\cos(a+bx)}{3b^2}+\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{(2d^2)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx}{9b^2}-\cdots\\
&= \frac{4d\sqrt[3]{c+dx}\,\cos(a+bx)}{3b^2}+\frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}-\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 122, normalized size = 0.67

$$\frac{de^{-\frac{i(bc+ad)}{d}}\sqrt[3]{c+dx}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{7}{3},-\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{-\frac{ib(c+dx)}{d}}}+\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{7}{3},\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(4/3)*Cos[a + b*x],x]
[Out] (d*(c + d*x)^(1/3)*((E^((2*I)*a)*Gamma[7/3, ((-I)*b*(c + d*x))/d])/(((-I)*b
*(c + d*x))/d)^(1/3) + (E^(((2*I)*b*c)/d)*Gamma[7/3, (I*b*(c + d*x))/d])/((
I*b*(c + d*x))/d)^(1/3)))/(2*b^2*E^((I*(b*c + a*d))/d))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{4}{3}}\cos(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(4/3)*cos(b*x+a),x)
[Out] int((d*x+c)^(4/3)*cos(b*x+a),x)
```

Maxima [A]
time = 0.36, size = 235, normalized size = 1.28

$$\frac{9\,(dx+c)^{\frac{4}{3}}b\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+12\,(dx+c)^{\frac{1}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},\frac{i\,(dx+c)b}{d}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-\frac{i\,(dx+c)b}{d}\right)\right)d^2\cos\left(-\frac{bc-ad}{d}\right)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},\frac{i\,(dx+c)b}{d}\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-\frac{i\,(dx+c)b}{d}\right)\right)d^2\sin\left(-\frac{bc-ad}{d}\right)\right)(dx+c)^{\frac{1}{3}}}{9\,b^2\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/9*(9*(d*x + c)^(4/3)*b*((d*x + c)*b/d)^(1/3)*d*sin(((d*x + c)*b - b*c + a
*d)/d) + 12*(d*x + c)^(1/3)*((d*x + c)*b/d)^(1/3)*d^2*cos(((d*x + c)*b - b*
c + a*d)/d) + (((sqrt(3) - I)*gamma(1/3, I*(d*x + c)*b/d) + (sqrt(3) + I)*g
amma(1/3, -I*(d*x + c)*b/d))*d^2*cos(-(b*c - a*d)/d) + ((-I*sqrt(3) - 1)*ga
mma(1/3, I*(d*x + c)*b/d) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x + c)*b/d))*d
^2*sin(-(b*c - a*d)/d))*(d*x + c)^(1/3))/(b^2*((d*x + c)*b/d)^(1/3)*d)
```

Fricas [A]
time = 0.13, size = 132, normalized size = 0.72

$$\frac{-2i\,d^2\left(\frac{i\,b}{d}\right)^{\frac{2}{3}}e^{\left(\frac{i\,bc-i\,ad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{i\,bdx+i\,bc}{d}\right)+2i\,d^2\left(-\frac{i\,b}{d}\right)^{\frac{2}{3}}e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{-i\,bdx-i\,bc}{d}\right)+3\,(4\,bd\cos\left(bx+a\right)+3\,(b^2dx+b^2c)\sin\left(bx+a\right))(dx+c)^{\frac{1}{3}}}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/9*(-2*I*d^2*(I*b/d)^(2/3)*e^((I*b*c - I*a*d)/d)*gamma(1/3, (I*b*d*x + I*b
*c)/d) + 2*I*d^2*(-I*b/d)^(2/3)*e^((-I*b*c + I*a*d)/d)*gamma(1/3, (-I*b*d*x
 - I*b*c)/d) + 3*(4*b*d*cos(b*x + a) + 3*(b^2*d*x + b^2*c)*sin(b*x + a))*(d
*x + c)^(1/3))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{4}{3}}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(4/3)*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(4/3)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(4/3)*cos(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a + b\,x)\,\left(c + d\,x\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^(4/3),x)
```

```
[Out] int(cos(a + b*x)*(c + d*x)^(4/3), x)
```

3.68 $\int (c+dx)^{2/3}\cos(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}}+\frac{(c+dx)^{2/3}\,\text{s}}{b}$$

```
[Out] 1/3*d*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/b^2
/(d*x+c)^(1/3)+1/3*d*(I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,I*b*(d*x+c)/d)/b^2/exp
(I*(a-b*c/d))/(d*x+c)^(1/3)+(d*x+c)^(2/3)*sin(b*x+a)/b
```

Rubi [A]
time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3377, 3389, 2212}

$$\frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}}+\frac{(c+dx)^{2/3}\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(2/3)*Cos[a + b*x],x]
```

```
[Out] (d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c +
 d*x))/d])/(3*b^2*(c + d*x)^(1/3)) + (d*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3
, (I*b*(c + d*x))/d])/(3*b^2*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3)) + ((c + d
*x)^(2/3)*Sin[a + b*x])/b
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^{2/3}\cos(a+bx)\,dx &= \frac{(c+dx)^{2/3}\sin(a+bx)}{b} - \frac{(2d)\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}\,dx}{3b} \\
&= \frac{(c+dx)^{2/3}\sin(a+bx)}{b} - \frac{(id)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{3b} + \frac{(id)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{3b} \\
&= \frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(}{}\right.}{3b^2\sqrt[3]{c+dx}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 124, normalized size = 0.82

$$
-\frac{ie^{-\frac{i(bc+ad)}{d}}(c+dx)^{2/3}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{5}{3},-\frac{ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{2/3}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{5}{3},\frac{ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{2/3}}\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(2/3)*Cos[a + b*x],x]
[Out] ((-1/2*I)*(c + d*x)^(2/3)*((E^((2*I)*a)*Gamma[5/3, ((-I)*b*(c + d*x))/d])/(
((-I)*b*(c + d*x))/d)^(2/3) - (E^(((2*I)*b*c)/d)*Gamma[5/3, (I*b*(c + d*x))
/d])/((I*b*(c + d*x))/d)^(2/3)))/(b*E^((I*(b*c + a*d))/d))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$
\int (dx+c)^{\frac{2}{3}}\cos(bx+a)\;dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(2/3)*cos(b*x+a),x)
[Out] int((d*x+c)^(2/3)*cos(b*x+a),x)
```

Maxima [A]
time = 0.36, size = 186, normalized size = 1.22

$$
\frac{6(dx+c)^{\frac{2}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{2}{3}}d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right) + \left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},\frac{i(dx+c)b}{d}\right) + \left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)d\cos\left(-\frac{bc-ad}{d}\right) + \left(\left(-i\sqrt{3}+1\right)\Gamma\left(\frac{2}{3},\frac{i(dx+c)b}{d}\right) + \left(i\sqrt{3}+1\right)\Gamma\left(\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)d\sin\left(-\frac{bc-ad}{d}\right)\right)(dx+c)^{\frac{2}{3}}}{6b\left(\frac{(dx+c)b}{d}\right)^{\frac{2}{3}}d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(d*x + c)^(2/3)*((d*x + c)*b/d)^(2/3)*d*sin(((d*x + c)*b - b*c + a*d
)/d) + (((sqrt(3) + I)*gamma(2/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(2/
3, -I*(d*x + c)*b/d))*d*cos(-(b*c - a*d)/d) + ((-I*sqrt(3) + 1)*gamma(2/3,
I*(d*x + c)*b/d) + (I*sqrt(3) + 1)*gamma(2/3, -I*(d*x + c)*b/d))*d*sin(-(b*
c - a*d)/d))*(d*x + c)^(2/3))/(b*((d*x + c)*b/d)^(2/3)*d)
```

Fricas [A]
time = 0.10, size = 102, normalized size = 0.67

$$\frac{d\left(\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)+d\left(-\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{-ibdx-ibc}{d}\right)+3\left(dx+c\right)^{\frac{2}{3}}b\sin\left(bx+a\right)}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(d*(I*b/d)^(1/3)*e^((I*b*c - I*a*d)/d)*gamma(2/3, (I*b*d*x + I*b*c)/d)
+ d*(-I*b/d)^(1/3)*e^((-I*b*c + I*a*d)/d)*gamma(2/3, (-I*b*d*x - I*b*c)/d)
+ 3*(d*x + c)^(2/3)*b*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c+dx\right)^{\frac{2}{3}}\cos\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(2/3)*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(2/3)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(2/3)*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^(2/3),x)
[Out] int(cos(a + b*x)*(c + d*x)^(2/3), x)
```

3.69 $\int \sqrt[3]{c+dx} \cos(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\operatorname{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\operatorname{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{\sqrt[3]{c+dx}\,\sin(a+bx)}{b}$$

```
[Out] 1/6*d*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b^2
/(d*x+c)^(2/3)+1/6*d*(I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b^2/exp
(I*(a-b*c/d))/(d*x+c)^(2/3)+(d*x+c)^(1/3)*sin(b*x+a)/b
```

Rubi [A]
time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3377, 3389, 2212}

$$\frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\operatorname{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\operatorname{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{\sqrt[3]{c+dx}\,\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)*Cos[a + b*x],x]
```

```
[Out] (d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c +
 d*x))/d])/(6*b^2*(c + d*x)^(2/3)) + (d*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3
, (I*b*(c + d*x))/d])/(6*b^2*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3)) + ((c + d
*x)^(1/3)*Sin[a + b*x])/b
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{c+dx}\,\cos(a+bx)\,dx &= \frac{\sqrt[3]{c+dx}\,\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}\,dx}{3b}\\ &= \frac{\sqrt[3]{c+dx}\,\sin(a+bx)}{b}-\frac{(id)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx}{6b}+\frac{(id)\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx}{6b}\\ &= \frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+d}{d}\right.}{6b^2(c+dx)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 124, normalized size = 0.82

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt[3]{c+dx}\left(\frac{e^{2ia}\mathrm{Gamma}\left(\frac{4}{3},-\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{4}{3},\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{\frac{ib(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*(c + d*x)^(1/3)*((E^((2*I)*a)*Gamma[4/3, ((-I)*b*(c + d*x))/d])/(
((-I)*b*(c + d*x))/d)^(1/3) - (E^(((2*I)*b*c)/d)*Gamma[4/3, (I*b*(c + d*x))
/d])/((I*b*(c + d*x))/d)^(1/3)))/(b*E^((I*(b*c + a*d))/d))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{1}{3}}\cos(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)*cos(b*x+a),x)
```

```
[Out] int((d*x+c)^(1/3)*cos(b*x+a),x)
```

Maxima [A]
time = 0.35, size = 186, normalized size = 1.22

$$\frac{12\,(dx+c)^{\frac{1}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left(\left(\left(i\sqrt{3}+1\right)\Gamma\left(\frac{1}{3},\frac{i\,(dx+c)b}{d}\right)+\left(-i\sqrt{3}+1\right)\Gamma\left(\frac{1}{3},-\frac{i\,(dx+c)b}{d}\right)\right)d\cos\left(-\frac{bc-ad}{d}\right)+\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},\frac{i\,(dx+c)b}{d}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-\frac{i\,(dx+c)b}{d}\right)\right)d\sin\left(-\frac{bc-ad}{d}\right)\right)(dx+c)^{\frac{1}{3}}}{12\,b\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)^(1/3)*((d*x + c)*b/d)^(1/3)*d*sin(((d*x + c)*b - b*c + a
*d)/d) + (((I*sqrt(3) + 1)*gamma(1/3, I*(d*x + c)*b/d) + (-I*sqrt(3) + 1)*g
amma(1/3, -I*(d*x + c)*b/d))*d*cos(-(b*c - a*d)/d) + ((sqrt(3) - I)*gamma(1
/3, I*(d*x + c)*b/d) + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c)*b/d))*d*sin(-(
b*c - a*d)/d))*(d*x + c)^(1/3))/(b*((d*x + c)*b/d)^(1/3)*d)
```

Fricas [A]
time = 0.11, size = 102, normalized size = 0.67

$$\frac{d\left(\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{ibdx+ibc}{d}\right)+d\left(-\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{-ibdx-ibc}{d}\right)+6\left(dx+c\right)^{\frac{1}{3}}b\sin\left(bx+a\right)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(d*(I*b/d)^(2/3)*e^((I*b*c - I*a*d)/d)*gamma(1/3, (I*b*d*x + I*b*c)/d)
+ d*(-I*b/d)^(2/3)*e^((-I*b*c + I*a*d)/d)*gamma(1/3, (-I*b*d*x - I*b*c)/d)
+ 6*(d*x + c)^(1/3)*b*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt[3]{c+dx}\,\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(1/3)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^(1/3),x)
[Out] int(cos(a + b*x)*(c + d*x)^(1/3), x)
```

3.70 $\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=135

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

```
[Out] -1/2*I*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/b/
(d*x+c)^(1/3)+1/2*I*(I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,I*b*(d*x+c)/d)/b/exp(I*
(a-b*c/d))/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3388, 2212}

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] ((-1/2*I)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)
*b*(c + d*x))/d])/(b*(c + d*x)^(1/3)) + ((I/2)*((I*b*(c + d*x))/d)^(1/3)*Ga
mma[2/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \frac{1}{2}\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx + \frac{1}{2}\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx$$
$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

Mathematica [A]
time = 0.04, size = 124, normalized size = 0.92

$$\frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c +
d*x))/d]) + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c
+ d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)}{\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(1/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(1/3),x)
```

Maxima [A]
time = 0.36, size = 137, normalized size = 1.01

$$\frac{(dx+c)^{\frac{2}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},\frac{i(dx+c)b}{d}\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},\frac{i(dx+c)b}{d}\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)}{4\left(\frac{(dx+c)b}{d}\right)^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(d*x + c)^(2/3)*(((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c)*b/d) + (-I*sqr
t(3) - 1)*gamma(2/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((sqrt(3) + I
```

```
)*gamma(2/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)*b/d))
*sin(-(b*c - a*d)/d))/(((d*x + c)*b/d)^(2/3)*d)
```

Fricas [A]
time = 0.12, size = 86, normalized size = 0.64

$$\frac{i\left(\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)-i\left(-\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{-ibdx-ibc}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] 1/2*(I*(I*b/d)^(1/3)*e^((I*b*c - I*a*d)/d)*gamma(2/3, (I*b*d*x + I*b*c)/d)
- I*(-I*b/d)^(1/3)*e^((-I*b*c + I*a*d)/d)*gamma(2/3, (-I*b*d*x - I*b*c)/d))
/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(a+bx\right)}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(1/3),x)
[Out] Integral(cos(a + b*x)/(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(1/3),x)
[Out] int(cos(a + b*x)/(c + d*x)^(1/3), x)
```

3.71 $\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=135

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

```
[Out] -1/2*I*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b/
(d*x+c)^(2/3)+1/2*I*(I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b/exp(I*
(a-b*c/d))/(d*x+c)^(2/3)
```

Rubi [A]
time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3388, 2212}

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] ((-1/2*I)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)
*b*(c + d*x))/d])/(b*(c + d*x)^(2/3)) + ((I/2)*((I*b*(c + d*x))/d)^(2/3)*Ga
mma[1/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \frac{1}{2}\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx + \frac{1}{2}\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx$$
$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

Mathematica [A]
time = 0.04, size = 124, normalized size = 0.92

$$\frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\operatorname{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\operatorname{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c +
d*x))/d]) + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c
+ d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(2/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(2/3),x)
```

Maxima [A]
time = 0.34, size = 138, normalized size = 1.02

$$-\frac{(dx+c)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},\frac{i(dx+c)b}{d}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\left(i\sqrt{3}+1\right)\Gamma\left(\frac{1}{3},\frac{i(dx+c)b}{d}\right)+\left(-i\sqrt{3}+1\right)\Gamma\left(\frac{1}{3},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)}{4\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(d*x + c)^(1/3)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)*b/d) + (sqrt(3)
 + I)*gamma(1/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((I*sqrt(3) + 1)*
```

```
gamma(1/3, I*(d*x + c)*b/d) + (-I*sqrt(3) + 1)*gamma(1/3, -I*(d*x + c)*b/d)
)*sin(-(b*c - a*d)/d))/(((d*x + c)*b/d)^(1/3)*d)
```

Fricas [A]
time = 0.10, size = 86, normalized size = 0.64

$$\frac{i\left(\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{ibdx+ibc}{d}\right)-i\left(-\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{-ibdx-ibc}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(I*(I*b/d)^(2/3)*e^((I*b*c - I*a*d)/d)*gamma(1/3, (I*b*d*x + I*b*c)/d)
- I*(-I*b/d)^(2/3)*e^((-I*b*c + I*a*d)/d)*gamma(1/3, (-I*b*d*x - I*b*c)/d))
/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(a+bx\right)}{\left(c+dx\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(2/3),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(2/3), x)
```

3.72 $\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=151

$$-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}+\frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}+\frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}$$

```
[Out] -3*cos(b*x+a)/d/(d*x+c)^(1/3)+3/2*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*G
AMMA(2/3,-I*b*(d*x+c)/d)/d/(d*x+c)^(1/3)+3/2*(I*b*(d*x+c)/d)^(1/3)*GAMMA(2/
3,I*b*(d*x+c)/d)/d/exp(I*(a-b*c/d))/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3378, 3389, 2212}

$$\frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}+\frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(4/3),x]
```

```
[Out] (-3*Cos[a + b*x])/(d*(c + d*x)^(1/3)) + (3*E^(I*(a - (b*c)/d))*(((-I)*b*(c
+ d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(2*d*(c + d*x)^(1/3)) +
(3*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x))/d])/(2*d*E^(I*(a -
(b*c)/d))*(c + d*x)^(1/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx &= -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} - \frac{(3b)\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}\,dx}{d} \\ &= -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} - \frac{(3ib)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{2d} + \frac{(3ib)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{2d} \\ &= -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}}{2d\sqrt[3]{c+dx}}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 121, normalized size = 0.80

$$-\frac{e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(-\frac{1}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(-\frac{1}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(4/3),x]
[Out] -1/2*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[-1/3, ((-I)*b*(c + d*x
))/d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[-1/3, (I*b*(c + d
*x))/d])/(d*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(4/3),x)
[Out] int(cos(b*x+a)/(d*x+c)^(4/3),x)
```

Maxima [**A**]
time = 0.37, size = 138, normalized size = 0.91

$$-\frac{\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{1}{3},\frac{i(dx+c)b}{d}\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{1}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},\frac{i(dx+c)b}{d}\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}}{4(dx+c)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/4*(((sqrt(3) + I)*gamma(-1/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(-1/
3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((I*sqrt(3) - 1)*gamma(-1/3, I*
(d*x + c)*b/d) + (-I*sqrt(3) - 1)*gamma(-1/3, -I*(d*x + c)*b/d))*sin(-(b*c
- a*d)/d))*((d*x + c)*b/d)^(1/3)/((d*x + c)^(1/3)*d)
```

Fricas [A]
time = 0.11, size = 117, normalized size = 0.77

$$\frac{3\left((dx+c)\left(\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)+(dx+c)\left(-\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{-ibdx-ibc}{d}\right)-2\,(dx+c)^{\frac{2}{3}}\cos\left(bx+a\right)\right)}{2\,(d^2x+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/2*((d*x + c)*(I*b/d)^(1/3)*e^((I*b*c - I*a*d)/d)*gamma(2/3, (I*b*d*x + I*
b*c)/d) + (d*x + c)*(-I*b/d)^(1/3)*e^((-I*b*c + I*a*d)/d)*gamma(2/3, (-I*b*
d*x - I*b*c)/d) - 2*(d*x + c)^(2/3)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+bx\right)}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(4/3),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(4/3), x)
```

3.73 $\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx$

Optimal. Leaf size=153

$$-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}+\frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}+\frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}$$

```
[Out] -3/2*cos(b*x+a)/d/(d*x+c)^(2/3)+3/4*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3)
*GAMMA(1/3,-I*b*(d*x+c)/d)/d/(d*x+c)^(2/3)+3/4*(I*b*(d*x+c)/d)^(2/3)*GAMMA(
1/3,I*b*(d*x+c)/d)/d/exp(I*(a-b*c/d))/(d*x+c)^(2/3)
```

Rubi [A]
time = 0.10, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3378, 3389, 2212}

$$\frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}+\frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(5/3),x]
```

```
[Out] (-3*Cos[a + b*x])/(2*d*(c + d*x)^(2/3)) + (3*E^(I*(a - (b*c)/d))*(((-I)*b*(
c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(4*d*(c + d*x)^(2/3))
+ (3*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c + d*x))/d])/(4*d*E^(I*(a
- (b*c)/d))*(c + d*x)^(2/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx &= -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} - \frac{(3b)\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}\,dx}{2d} \\
&= -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} - \frac{(3ib)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx}{4d} + \frac{(3ib)\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx}{4d} \\
&= -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma}{4d(c+dx)^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 121, normalized size = 0.79

$$
-\frac{e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(-\frac{2}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\mathrm{Gamma}\left(-\frac{2}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d(c+dx)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(5/3),x]
```

```
[Out] -1/2*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[-2/3, ((-I)*b*(c + d*x
))/d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[-2/3, (I*b*(c + d
*x))/d])/(d*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$
\int \frac{\cos(bx+a)}{(dx+c)^{\frac{5}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(5/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(5/3),x)
```

Maxima [A]
time = 0.37, size = 138, normalized size = 0.90

$$
\frac{\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},\frac{i(dx+c)b}{d}\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3},\frac{i(dx+c)b}{d}\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{2}{3}}}{4(dx+c)^{\frac{2}{3}}d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] 1/4*(((-I*sqrt(3) - 1)*gamma(-2/3, I*(d*x + c)*b/d) + (I*sqrt(3) - 1)*gamma
(-2/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((sqrt(3) - I)*gamma(-2/3,
I*(d*x + c)*b/d) + (sqrt(3) + I)*gamma(-2/3, -I*(d*x + c)*b/d))*sin(-(b*c -
 a*d)/d))*((d*x + c)*b/d)^(2/3)/((d*x + c)^(2/3)*d)
```

Fricas [A]
time = 0.13, size = 117, normalized size = 0.76

$$\frac{3\left((dx+c)\left(\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{ibdx+ibc}{d}\right)+(dx+c)\left(-\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{-ibdx-ibc}{d}\right)-2\,(dx+c)^{\frac{1}{3}}\cos\left(bx+a\right)\right)}{4\,(d^2x+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] 3/4*((d*x + c)*(I*b/d)^(2/3)*e^((I*b*c - I*a*d)/d)*gamma(1/3, (I*b*d*x + I*
b*c)/d) + (d*x + c)*(-I*b/d)^(2/3)*e^((-I*b*c + I*a*d)/d)*gamma(1/3, (-I*b*
d*x - I*b*c)/d) - 2*(d*x + c)^(1/3)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(5/3),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(5/3),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(5/3), x)
```

3.74 $\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx$

Optimal. Leaf size=182

$$-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}+\frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}}-\frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib}{}\right.}{8d^2\sqrt[3]{c+dx}}$$

```
[Out] -3/4*cos(b*x+a)/d/(d*x+c)^(4/3)+9/8*I*b*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(
1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/d^2/(d*x+c)^(1/3)-9/8*I*b*(I*b*(d*x+c)/d)^(1
/3)*GAMMA(2/3,I*b*(d*x+c)/d)/d^2/exp(I*(a-b*c/d))/(d*x+c)^(1/3)+9/4*b*sin(b
*x+a)/d^2/(d*x+c)^(1/3)
```

Rubi [A]

time = 0.13, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3378, 3388, 2212}

$$\frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}}-\frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}}+\frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}}-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(7/3),x]
```

```
[Out] (-3*Cos[a + b*x])/(4*d*(c + d*x)^(4/3)) + (((9*I)/8)*b*E^(I*(a - (b*c)/d))*
(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(d^2*(c + d*
x)^(1/3)) - (((9*I)/8)*b*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x
))/d])/(d^2*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3)) + (9*b*Sin[a + b*x])/(4*d^
2*(c + d*x)^(1/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx &= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} - \frac{(3b)\int \frac{\sin(a+bx)}{(c+dx)^{4/3}}\,dx}{4d} \\
&= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}} - \frac{(9b^2)\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx}{4d^2} \\
&= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}} - \frac{(9b^2)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{8d^2} - \frac{(9b^2)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{8d^2} \\
&= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} - \frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+}{d}}}{8d^2\sqrt[3]{c+}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 125, normalized size = 0.69

$$\frac{ibe^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(-\frac{4}{3},-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(-\frac{4}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d^2\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(7/3),x]
[Out] ((I/2)*b*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[-4/3, ((-I)*b*(c +
 d*x))/d] - E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[-4/3, (I*b*(c
 + d*x))/d]))/(d^2*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)}{\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(7/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(7/3),x)
```

Maxima [A]
time = 0.37, size = 137, normalized size = 0.75

$$-\frac{\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3},\frac{i(dx+c)b}{d}\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{4}{3},\frac{i(dx+c)b}{d}\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{4}{3},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{4}{3}}}{4\,(dx+c)^{\frac{4}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(((I*sqrt(3) - 1)*gamma(-4/3, I*(d*x + c)*b/d) + (-I*sqrt(3) - 1)*gamm
a(-4/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((sqrt(3) + I)*gamma(-4/3,
 I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(-4/3, -I*(d*x + c)*b/d))*sin(-(b*c
- a*d)/d))*((d*x + c)*b/d)^(4/3)/((d*x + c)^(4/3)*d)
```

Fricas [A]
time = 0.10, size = 185, normalized size = 1.02

$$-\frac{3\left(3\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)\left(\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{i\,bc-i\,ad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{i\,bdx+i\,bc}{d}\right)+3\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)\left(-\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{-i\,bdx-i\,bc}{d}\right)+2\,(dx+c)^{\frac{2}{3}}\left(d\cos(bx+a)-3\,(bdx+bc)\sin(bx+a)\right)\right)}{8\,(d^4x^2+2\,cd^3x+c^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/8*(3*(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)*(I*b/d)^(1/3)*e^((I*b*c - I*a
*d)/d)*gamma(2/3, (I*b*d*x + I*b*c)/d) + 3*(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*
b*c^2)*(-I*b/d)^(1/3)*e^((-I*b*c + I*a*d)/d)*gamma(2/3, (-I*b*d*x - I*b*c)/
d) + 2*(d*x + c)^(2/3)*(d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(b*x + a)))/(d^
4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(a+bx)}{(c+dx)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(7/3),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(7/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*x + c)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{7/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x)^(7/3),x)
[Out] int(cos(a + b*x)/(c + d*x)^(7/3), x)
```

3.75 $\int x\sqrt{\cos(a+bx)}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(x\sqrt{\cos(a+bx)},x\right)$$

```
[Out] Unintegrable(x*cos(b*x+a)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\sqrt{\cos(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] Defer[Int][x*Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos(a+bx)}\,dx$$

Mathematica [A]
time = 107.45, size = 0, normalized size = 0.00

$$\int x\sqrt{\cos(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] Integrate[x*Sqrt[Cos[a + b*x]], x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(\sqrt{\cos}\,(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\cos{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(cos(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int x\,\sqrt{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(1/2),x)
```

```
[Out] int(x*cos(a + b*x)^(1/2), x)
```

3.76 $\int \sqrt{\cos(a+bx)}\,dx$

Optimal. Leaf size=16

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2719\}$

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(42) = 84$.
time = 0.05, size = 133, normalized size = 8.31

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,\mathrm{EllipticE}\left(\cos\left(\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,b}$
risch	$-\frac{i\sqrt{2}\sqrt{\left(e^{2i(bx+a)}+1\right)e^{-i(bx+a)}}}{b}-\frac{i\left(-\frac{2\left(e^{2i(bx+a)}+1\right)}{\sqrt{\left(e^{2i(bx+a)}+1\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(\right.}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),
2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1
/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 57, normalized size = 3.56

$$\frac{i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) +
 I*sin(b*x + a))) - I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cos (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/2),x)
[Out] Integral(sqrt(cos(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cos(b*x + a)), x)
```

Mupad [**B**]
time = 0.18, size = 15, normalized size = 0.94

$$\frac{2\,\mathrm{E}\left(\frac{a}{2} + \frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(1/2),x)
[Out] (2*ellipticE(a/2 + (b*x)/2, 2))/b
```

3.77 $\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{\sqrt{\cos(a+bx)}}{x},x\right)$$

[Out] Unintegrable(cos(b*x+a)^(1/2)/x,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[Cos[a + b*x]]/x,x]

[Out] Defer[Int][Sqrt[Cos[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx=\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$$

Mathematica [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[Cos[a + b*x]]/x,x]

[Out] Integrate[Sqrt[Cos[a + b*x]]/x, x]

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/2)/x,x)
```

```
[Out] int(cos(b*x+a)^(1/2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*x + a))/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos{\left(a + bx \right)}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(cos(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*x + a))/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sqrt{\cos\left(a+b\,x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(1/2)/x,x)
```

```
[Out] int(cos(a + b*x)^(1/2)/x, x)
```

3.78 $\int x\cos^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}+\frac{1}{3}\text{Int}\left(\frac{x}{\sqrt{\cos(a+bx)}},x\right)$$

```
[Out] 4/9*cos(b*x+a)^(3/2)/b^2+2/3*x*sin(b*x+a)*cos(b*x+a)^(1/2)/b+1/3*Unintegrable(x/cos(b*x+a)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b) + Defer[Int][x/Sqrt[Cos[a + b*x]], x]/3
```

Rubi steps

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx=\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}+\frac{1}{3}\int\frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Mathematica [A]
time = 1.20, size = 0, normalized size = 0.00

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Cos[a + b*x]^(3/2),x]
```

```
[Out] Integrate[x*Cos[a + b*x]^(3/2), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(\cos^{\frac{3}{2}}(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cos(a + b*x)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\cos\left(a+b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(3/2),x)
```

```
[Out] int(x*cos(a + b*x)^(3/2), x)
```

3.79 $\int \cos^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}$$

```
[Out] 2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))/b+2/3*sin(b*x+a)*cos(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2715, 2720}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(
3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.86

$$\frac{2\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\sin(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[a + b*x]))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(62) = 124$.
time = 0.00, size = 179, normalized size = 4.26

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b*x
+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(sin
(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2
*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/
sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 68, normalized size = 1.62

$$\frac{2\sqrt{\cos(bx+a)}\sin(bx+a)-i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(cos(b*x + a))*sin(b*x + a) - I*sqrt(2)*weierstrassPInverse(-4,
0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, co
s(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

Mupad [B]
time = 0.19, size = 35, normalized size = 0.83

$$\frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{3\,b}+\frac{2\,\sqrt{\cos\left(a+b\,x\right)}\,\sin\left(a+b\,x\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(3/2),x)
```

```
[Out] (2*ellipticF(a/2 + (b*x)/2, 2))/(3*b) + (2*cos(a + b*x)^(1/2)*sin(a + b*x))
/(3*b)
```

3.80 $\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\cos^{\frac{3}{2}}(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(cos(b*x+a)^(3/2)/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[a + b*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][Cos[a + b*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx$$

Mathematica [A]
time = 9.49, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[a + b*x]^(3/2)/x,x]
```

```
[Out] Integrate[Cos[a + b*x]^(3/2)/x, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(bx+a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2)/x,x)
```

```
[Out] int(cos(b*x+a)^(3/2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2)/x,x)
```

```
[Out] Integral(cos(a + b*x)**(3/2)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a+b\,x)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(3/2)/x,x)
```

```
[Out] int(cos(a + b*x)^(3/2)/x, x)
```

3.81 $\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx$

Optimal. Leaf size=42

$$\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}$$

```
[Out] 4/9*cos(b*x+a)^(3/2)/b^2+2/3*x*sin(b*x+a)*cos(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {3391}

$$\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[-1/3*x/Sqrt[Cos[a + b*x]] + x*Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b
)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx\right)+\int x\cos^{\frac{3}{2}}(a+bx)\,dx\\ &= \frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 40, normalized size = 0.95

$$\frac{\sqrt{\cos(a+bx)}\left(\frac{8\cos(a+bx)}{3b}+4x\sin(a+bx)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/3*x/Sqrt[Cos[a + b*x]] + x*Cos[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[a + b*x]]*((8*Cos[a + b*x])/(3*b) + 4*x*Sin[a + b*x]))/(6*b)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int x\left(\cos^{\frac{3}{2}}\left(bx+a\right)\right)-\frac{x}{3\sqrt{\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2) - 1/3*x/sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{x}{\sqrt{\cos\left(a+bx\right)}}\right)dx+\int 3x\cos^{\frac{3}{2}}\left(a+bx\right)dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2)-1/3*x/cos(b*x+a)**(1/2),x)
```

```
[Out] (Integral(-x/sqrt(cos(a + b*x)), x) + Integral(3*x*cos(a + b*x)**(3/2), x))
/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2) - 1/3*x/sqrt(cos(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\cos\left(a+b\,x\right)^{3/2}-\frac{x}{3\,\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(3/2) - x/(3*cos(a + b*x)^(1/2)),x)
```

```
[Out] int(x*cos(a + b*x)^(3/2) - x/(3*cos(a + b*x)^(1/2)), x)
```

3.82 $\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} dx$

Optimal. Leaf size=63

$$-\frac{\cos^{\frac{3}{2}}(x)}{2x^2}+\frac{3\sqrt{\cos(x)}\sin(x)}{4x}+\frac{3}{8}\text{Int}\left(\frac{1}{x\sqrt{\cos(x)}},x\right)-\frac{9}{8}\text{Int}\left(\frac{\cos^{\frac{3}{2}}(x)}{x},x\right)$$

```
[Out] -1/2*cos(x)^(3/2)/x^2+3/4*sin(x)*cos(x)^(1/2)/x-9/8*Unintegrable(cos(x)^(3/
2)/x,x)+3/8*Unintegrable(1/x/cos(x)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[x]^(3/2)/x^3,x]
```

```
[Out] -1/2*Cos[x]^(3/2)/x^2 + (3*Sqrt[Cos[x]]*Sin[x])/(4*x) + (3*Defer[Int][1/(x*
Sqrt[Cos[x]]), x])/8 - (9*Defer[Int][Cos[x]^(3/2)/x, x])/8
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} dx = -\frac{\cos^{\frac{3}{2}}(x)}{2x^2}+\frac{3\sqrt{\cos(x)}\sin(x)}{4x}+\frac{3}{8}\int \frac{1}{x\sqrt{\cos(x)}} dx-\frac{9}{8}\int \frac{\cos^{\frac{3}{2}}(x)}{x} dx$$

Mathematica [A]
time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[x]^(3/2)/x^3,x]
```

```
[Out] Integrate[Cos[x]^(3/2)/x^3, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(3/2)/x^3,x)
```

```
[Out] int(cos(x)^(3/2)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^(3/2)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**(3/2)/x**3,x)
```

```
[Out] Integral(cos(x)**(3/2)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(cos(x)^(3/2)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(x)^{3/2}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(3/2)/x^3,x)
```

```
[Out] int(cos(x)^(3/2)/x^3, x)
```

3.83 $\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x}{\sqrt{\cos(a+bx)}},x\right)$$

```
[Out] Unintegrable(x/cos(b*x+a)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Sqrt[Cos[a + b*x]],x]
[Out] Defer[Int][x/Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/Sqrt[Cos[a + b*x]],x]
[Out] Integrate[x/Sqrt[Cos[a + b*x]], x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(b*x+a)^(1/2),x)
```

```
[Out] int(x/cos(b*x+a)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\cos (a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(cos(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(cos(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(a + b*x)^(1/2),x)
```

```
[Out] int(x/cos(a + b*x)^(1/2), x)
```

3.84 $\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=16

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b
*x),2^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2720}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.00, size = 18, normalized size = 1.12

method	result	size
default	$\frac{2\,\mathrm{am}^{-1}\left(\frac{bx}{2}+\frac{a}{2}\middle\vert\sqrt{2}\right)}{b}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*InverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 51, normalized size = 3.19

$$\frac{-i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*s
qrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

Mupad [B]
time = 0.20, size = 15, normalized size = 0.94

$$\frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(1/2),x)
```

```
[Out] (2*ellipticF(a/2 + (b*x)/2, 2))/b
```

3.85 $\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x\sqrt{\cos(a+bx)}},x\right)$$

```
[Out] Unintegrable(1/x/cos(b*x+a)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[Cos[a + b*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[Cos[a + b*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[Cos[a + b*x]]),x]
[Out] Integrate[1/(x*Sqrt[Cos[a + b*x]]), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cos(b*x+a)^(1/2),x)
```

```
[Out] int(1/x/cos(b*x+a)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(cos(b*x + a))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\cos\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(cos(a + b*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(cos(b*x + a))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\,\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*x)^(1/2)),x)
```

```
[Out] int(1/(x*cos(a + b*x)^(1/2)), x)
```

3.86 $\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=55

$$\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\text{Int}\left(x\sqrt{\cos(a+bx)},x\right)$$

```
[Out] 2*x*sin(b*x+a)/b/cos(b*x+a)^(1/2)+4*cos(b*x+a)^(1/2)/b^2-Unintegrable(x*cos
(b*x+a)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]]) - De
fer[Int][x*Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx=\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int x\sqrt{\cos(a+bx)}\,dx$$

Mathematica [A]
time = 1.61, size = 0, normalized size = 0.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/Cos[a + b*x]^(3/2),x]
```

```
[Out] Integrate[x/Cos[a + b*x]^(3/2), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(b*x+a)^(3/2),x)
```

```
[Out] int(x/cos(b*x+a)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/cos(b*x + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(x/cos(a + b*x)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/cos(b*x + a)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\cos\left(a+b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(a + b*x)^(3/2),x)
```

```
[Out] int(x/cos(a + b*x)^(3/2), x)
```

3.87 $\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=38

$$-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}+\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] -2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*
b*x),2^(1/2))/b+2*sin(b*x+a)/b/cos(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2719}

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}\,dx\\ &= -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}+\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.00

$$-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}+\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-3/2),x]
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(62) = 124.
time = 0.00, size = 182, normalized size = 4.79

method	result
default	$-\frac{2\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin^2\left(\frac{bx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*(-2*cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1
/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(
cos(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)
^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 93, normalized size = 2.45

$$\frac{-i\sqrt{2}\cos(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+i\sqrt{2}\cos(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))+2\sqrt{\cos(bx+a)}\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*cos(b*x + a)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*sqrt(co
s(b*x + a))*sin(b*x + a))/(b*cos(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

Mupad [**B**]
time = 0.44, size = 42, normalized size = 1.11

$$\frac{2\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(a+b\,x\right)^2\right)}{b\,\sqrt{\cos\left(a+b\,x\right)}\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(3/2),x)
```

```
[Out] (2*sin(a + b*x)*hypergeom([-1/4, 1/2], 3/4, cos(a + b*x)^2))/(b*cos(a + b*x
)^(1/2)*(sin(a + b*x)^2)^(1/2))
```

3.88 $\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x\cos^{\frac{3}{2}}(a+bx)},x\right)$$

```
[Out] Unintegrable(1/x/cos(b*x+a)^(3/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Cos[a + b*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*Cos[a + b*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Mathematica [A]
time = 8.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Cos[a + b*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*Cos[a + b*x]^(3/2)), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cos(b*x+a)^(3/2),x)
```

```
[Out] int(1/x/cos(b*x+a)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cos(b*x + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(1/(x*cos(a + b*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*cos(b*x + a)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\cos\left(a+b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*x)^(3/2)),x)
[Out] int(1/(x*cos(a + b*x)^(3/2)), x)
```

3.89 $\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right)\,dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] 2*x*sin(b*x+a)/b/cos(b*x+a)^(1/2)+4*cos(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3396}

$$\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cos[a + b*x]^(3/2) + x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right)dx &= \int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx+\int x\sqrt{\cos(a+bx)}\,dx\\ &= \frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 33, normalized size = 0.87

$$\frac{2(2\cos(a+bx)+bx\sin(a+bx))}{b^2\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cos[a + b*x]^(3/2) + x*Sqrt[Cos[a + b*x]],x]
[Out] (2*(2*Cos[a + b*x] + b*x*Sin[a + b*x]))/(b^2*Sqrt[Cos[a + b*x]])
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x}{\cos \left(bx+a\right)^{\frac{3}{2}}}+x\left(\sqrt{\cos}\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x)
[Out] int(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(cos(b*x + a)) + x/cos(b*x + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\cos^{2}\left(a+bx\right)+1\right)}{\cos^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)**(3/2)+x*cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*(cos(a + b*x)**2 + 1)/cos(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a)) + x/cos(b*x + a)^(3/2), x)
```

Mupad [B]
time = 0.66, size = 51, normalized size = 1.34

$$\frac{2\,\sqrt{\cos\left(a+b\,x\right)}\,\left(2\,\cos\left(2\,a+2\,b\,x\right)+b\,x\,\sin\left(2\,a+2\,b\,x\right)+2\right)}{b^2\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(1/2) + x/cos(a + b*x)^(3/2),x)
```

```
[Out] (2*cos(a + b*x)^(1/2)*(2*cos(2*a + 2*b*x) + b*x*sin(2*a + 2*b*x) + 2))/(b^2
*(cos(2*a + 2*b*x) + 1))
```

3.90 $\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)}+x\sqrt{\cos(x)}\right)\,dx$

Optimal. Leaf size=20

$$4\sqrt{\cos(x)}+\frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

```
[Out] 2*x*sin(x)/cos(x)^(1/2)+4*cos(x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3396}

$$4\sqrt{\cos(x)}+\frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cos[x]^(3/2) + x*Sqrt[Cos[x]],x]
```

```
[Out] 4*Sqrt[Cos[x]] + (2*x*Sin[x])/Sqrt[Cos[x]]
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)}+x\sqrt{\cos(x)}\right)\,dx &= \int \frac{x}{\cos^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\cos(x)}\,dx\\
&= 4\sqrt{\cos(x)}+\frac{2x\sin(x)}{\sqrt{\cos(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 17, normalized size = 0.85

$$\frac{2(2\cos(x)+x\sin(x))}{\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cos[x]^(3/2) + x*Sqrt[Cos[x]],x]
```

```
[Out] (2*(2*Cos[x] + x*Sin[x]))/Sqrt[Cos[x]]
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x}{\cos \left(x\right)^{\frac{3}{2}}}+x\left(\sqrt{\cos}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^(3/2)+x*cos(x)^(1/2),x)
```

```
[Out] int(x/cos(x)^(3/2)+x*cos(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cos(x)) + x/cos(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\cos^{2}\left(x\right)+1\right)}{\cos^{\frac{3}{2}}\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)**(3/2)+x*cos(x)**(1/2),x)
```

```
[Out] Integral(x*(cos(x)**2 + 1)/cos(x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(cos(x)) + x/cos(x)^(3/2), x)
```

Mupad [B]
time = 0.33, size = 15, normalized size = 0.75

$$\frac{4\,\cos\left(x\right)+2\,x\,\sin\left(x\right)}{\sqrt{\cos\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^(1/2) + x/cos(x)^(3/2),x)
[Out] (4*cos(x) + 2*x*sin(x))/cos(x)^(1/2)
```

3.91 $\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right) dx$

Optimal. Leaf size=24

$$-\frac{4}{3\sqrt{\cos(x)}}+\frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}$$

```
[Out] 2/3*x*sin(x)/cos(x)^(3/2)-4/3/cos(x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3396}

$$\frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cos[x]^(5/2) - x/(3*Sqrt[Cos[x]]),x]
```

```
[Out] -4/(3*Sqrt[Cos[x]]) + (2*x*Sin[x])/(3*Cos[x]^(3/2))
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right) dx &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\cos(x)}}\,dx\right)+\int \frac{x}{\cos^{\frac{5}{2}}(x)}\,dx\\ &= -\frac{4}{3\sqrt{\cos(x)}}+\frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 17, normalized size = 0.71

$$-\frac{8-4x\tan(x)}{6\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cos[x]^(5/2) - x/(3*Sqrt[Cos[x]]),x]
```

```
[Out] -1/6*(8 - 4*x*Tan[x])/Sqrt[Cos[x]]
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x}{\cos\left(x\right)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x)
```

```
[Out] int(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(cos(x)) + x/cos(x)^(5/2), x)
```

Fricas [A]
time = 0.35, size = 15, normalized size = 0.62

$$\frac{2\,\left(x\sin\left(x\right)-2\,\cos\left(x\right)\right)}{3\,\cos\left(x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x*sin(x) - 2*cos(x))/cos(x)^(3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3x}{\cos^{\frac{5}{2}}\left(x\right)}\right)dx+\int\frac{x}{\sqrt{\cos\left(x\right)}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)**(5/2)-1/3*x/cos(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x/cos(x)**(5/2), x) + Integral(x/sqrt(cos(x)), x))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="giac")
[Out] integrate(-1/3*x/sqrt(cos(x)) + x/cos(x)^(5/2), x)
```

Mupad [B]
time = 0.14, size = 16, normalized size = 0.67

$$-\frac{4\cos(x)-2x\sin(x)}{3\cos(x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^(5/2) - x/(3*cos(x)^(1/2)),x)
[Out] -(4*cos(x) - 2*x*sin(x))/(3*cos(x)^(3/2))
```

3.92 $\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cos(x)} \right) dx$

Optimal. Leaf size=47

$$-\frac{4}{15\cos^{\frac{3}{2}}(x)} + \frac{12\sqrt{\cos(x)}}{5} + \frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)} + \frac{6x\sin(x)}{5\sqrt{\cos(x)}}$$

```
[Out] -4/15/cos(x)^(3/2)+2/5*x*sin(x)/cos(x)^(5/2)+6/5*x*sin(x)/cos(x)^(1/2)+12/5
*cos(x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3396}

$$-\frac{4}{15\cos^{\frac{3}{2}}(x)} + \frac{12\sqrt{\cos(x)}}{5} + \frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)} + \frac{6x\sin(x)}{5\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cos[x]^(7/2) + (3*x*Sqrt[Cos[x]])/5,x]
```

```
[Out] -4/(15*Cos[x]^(3/2)) + (12*Sqrt[Cos[x]])/5 + (2*x*Sin[x])/(5*Cos[x]^(5/2))
+ (6*x*Sin[x])/(5*Sqrt[Cos[x]])
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cos(x)} \right) dx &= \frac{3}{5}\int x\sqrt{\cos(x)}\, dx + \int \frac{x}{\cos^{\frac{7}{2}}(x)}\, dx \\
&= -\frac{4}{15\cos^{\frac{3}{2}}(x)} + \frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)} + \frac{3}{5}\int \frac{x}{\cos^{\frac{3}{2}}(x)}\, dx + \frac{3}{5}\int x\sqrt{\cos(x)}\, dx \\
&= -\frac{4}{15\cos^{\frac{3}{2}}(x)} + \frac{12\sqrt{\cos(x)}}{5} + \frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)} + \frac{6x\sin(x)}{5\sqrt{\cos(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 33, normalized size = 0.70

$$\frac{46\cos(x)+18\cos(3x)+21x\sin(x)+9x\sin(3x)}{30\cos^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cos[x]^(7/2) + (3*x*Sqrt[Cos[x]])/5,x]
```

```
[Out] (46*Cos[x] + 18*Cos[3*x] + 21*x*Sin[x] + 9*x*Sin[3*x])/(30*Cos[x]^(5/2))
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x}{\cos\left(x\right)^{\frac{7}{2}}}+\frac{3x\left(\sqrt{\cos}\left(x\right)\right)}{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x)
```

```
[Out] int(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(cos(x)) + x/cos(x)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)**(7/2)+3/5*x*cos(x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(cos(x)) + x/cos(x)^(7/2), x)
```

Mupad [B]
time = 0.53, size = 31, normalized size = 0.66

$$\frac{36\cos(x)^3+18\,x\,\sin(x)\,\cos(x)^2-4\,\cos(x)+6\,x\,\sin(x)}{15\cos(x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x*cos(x)^(1/2))/5 + x/cos(x)^(7/2),x)
```

```
[Out] (36*cos(x)^3 - 4*cos(x) + 6*x*sin(x) + 18*x*cos(x)^2*sin(x))/(15*cos(x)^(5/
2))
```

3.93 $\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)}+x^2\sqrt{\cos(x)}\right)\,dx$

Optimal. Leaf size=32

$$8x\sqrt{\cos(x)}-16E\left(\frac{x}{2}\middle|2\right)+\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}$$

```
[Out] -16*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/2))+2*x^2*sin
(x)/cos(x)^(1/2)+8*x*cos(x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3397, 2719}

$$\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}+8x\sqrt{\cos(x)}-16E\left(\frac{x}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Cos[x]^(3/2) + x^2*Sqrt[Cos[x]],x]
```

```
[Out] 8*x*Sqrt[Cos[x]] - 16*EllipticE[x/2, 2] + (2*x^2*Sin[x])/Sqrt[Cos[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3397

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1)
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[d^2*m*((m - 1)/(b^2*f^2*(n + 1)*(n + 2))), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[d*m*(c + d*x)^(m - 1)*((b*
Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rubi steps

$$\int\left(\frac{x^2}{\cos^{\frac{3}{2}}(x)}+x^2\sqrt{\cos(x)}\right)dx=\int\frac{x^2}{\cos^{\frac{3}{2}}(x)}\,dx+\int x^2\sqrt{\cos(x)}\,dx$$
$$=8x\sqrt{\cos(x)}+\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}-8\int\sqrt{\cos(x)}\,dx$$
$$=8x\sqrt{\cos(x)}-16E\left(\frac{x}{2}\middle|2\right)+\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}$$

Mathematica [A]
time = 0.09, size = 29, normalized size = 0.91

$$2\left(-8E\left(\frac{x}{2}\middle|2\right)+\frac{x(4\cos(x)+x\sin(x))}{\sqrt{\cos(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Cos[x]^(3/2) + x^2*Sqrt[Cos[x]],x]
[Out] 2*(-8*EllipticE[x/2, 2] + (x*(4*Cos[x] + x*Sin[x]))/Sqrt[Cos[x]])
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\cos(x)^{\frac{3}{2}}}+x^2\left(\sqrt{\cos}(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x)
[Out] int(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*sqrt(cos(x)) + x^2/cos(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(\cos^2(x)+1)}{\cos^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/cos(x)**(3/2)+x**2*cos(x)**(1/2),x)
```

```
[Out] Integral(x**2*(cos(x)**2 + 1)/cos(x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(cos(x)) + x^2/cos(x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\sqrt{\cos(x)}+\frac{x^2}{{\cos(x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^(1/2) + x^2/cos(x)^(3/2),x)
```

```
[Out] int(x^2*cos(x)^(1/2) + x^2/cos(x)^(3/2), x)
```

3.94 $\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)\,dx$

Optimal. Leaf size=24

$$\frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}$$

```
[Out] 4/9/sec(x)^(3/2)+2/3*x*sin(x)/sec(x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4272, 4274}

$$\frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sec[x]^(3/2) - (x*Sqrt[Sec[x]])/3,x]
```

```
[Out] 4/(9*Sec[x]^(3/2)) + (2*x*Sin[x])/(3*Sqrt[Sec[x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)\,dx &= -\left(\frac{1}{3}\int x\sqrt{\sec(x)}\,dx\right)+\int \frac{x}{\sec^{\frac{3}{2}}(x)}\,dx\\ &= \frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}+\frac{1}{3}\int x\sqrt{\sec(x)}\,dx-\frac{1}{3}\Big(\sqrt{\cos(x)}\,\sqrt{\sec(x)}\\ &= \frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 17, normalized size = 0.71

$$\frac{2(2+3x\tan(x))}{9\sec^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sec[x]^(3/2) - (x*Sqrt[Sec[x]])/3,x]
```

```
[Out] (2*(2 + 3*x*Tan[x]))/(9*Sec[x]^(3/2))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x}{\sec(x)^{\frac{3}{2}}}-\frac{x\left(\sqrt{\sec}\,(x)\right)}{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x)
```

```
[Out] int(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x*sqrt(sec(x)) + x/sec(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3x}{\sec^{\frac{3}{2}}(x)}\right)dx+\int x\sqrt{\sec(x)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)**(3/2)-1/3*x*sec(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x/sec(x)**(3/2), x) + Integral(x*sqrt(sec(x)), x))/3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x*sqrt(sec(x)) + x/sec(x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x\sqrt{\frac{1}{\cos(x)}}}{3} - \frac{x}{\left(\frac{1}{\cos(x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cos(x))^(3/2) - (x*(1/cos(x))^(1/2))/3,x)
```

```
[Out] -int((x*(1/cos(x))^(1/2))/3 - x/(1/cos(x))^(3/2), x)
```

3.95 $\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)\,dx$

Optimal. Leaf size=24

$$\frac{4}{25\sec^{\frac{5}{2}}(x)}+\frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}$$

```
[Out] 4/25/sec(x)^(5/2)+2/5*x*sin(x)/sec(x)^(3/2)
```

Rubi [A]

time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4272, 4274}

$$\frac{4}{25\sec^{\frac{5}{2}}(x)}+\frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sec[x]^(5/2) - (3*x)/(5*Sqrt[Sec[x]]),x]
```

```
[Out] 4/(25*Sec[x]^(5/2)) + (2*x*Sin[x])/(5*Sec[x]^(3/2))
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)dx &= -\left(\frac{3}{5}\int \frac{x}{\sqrt{\sec(x)}}\,dx\right)+\int \frac{x}{\sec^{\frac{5}{2}}(x)}\,dx\\ &= \frac{4}{25\sec^{\frac{5}{2}}(x)}+\frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}+\frac{3}{5}\int \frac{x}{\sqrt{\sec(x)}}\,dx-\frac{1}{5}\left(3\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int\\ &= \frac{4}{25\sec^{\frac{5}{2}}(x)}+\frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 17, normalized size = 0.71

$$\frac{2(2+5x\tan(x))}{25\sec^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sec[x]^(5/2) - (3*x)/(5*Sqrt[Sec[x]]),x]
[Out] (2*(2 + 5*x*Tan[x]))/(25*Sec[x]^(5/2))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x}{\sec(x)^{\frac{5}{2}}}-\frac{3x}{5\sqrt{\sec(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x)
[Out] int(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-3/5*x/sqrt(sec(x)) + x/sec(x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{5x}{\sec^{\frac{5}{2}}(x)}\right)dx+\int\frac{3x}{\sqrt{\sec(x)}}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)**(5/2)-3/5*x/sec(x)**(1/2),x)
```

```
[Out] -(Integral(-5*x/sec(x)**(5/2), x) + Integral(3*x/sqrt(sec(x)), x))/5
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-3/5*x/sqrt(sec(x)) + x/sec(x)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{3\,x}{5\,\sqrt{\frac{1}{\cos\left(x\right)}}}-\frac{x}{{\left(\frac{1}{\cos\left(x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cos(x))^(5/2) - (3*x)/(5*(1/cos(x))^(1/2)),x)
```

```
[Out] -int((3*x)/(5*(1/cos(x))^(1/2)) - x/(1/cos(x))^(5/2), x)
```

3.96 $\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\sec(x)}\right)\,dx$

Optimal. Leaf size=47

$$\frac{4}{49\sec^{\frac{7}{2}}(x)}+\frac{20}{63\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{10x\sin(x)}{21\sqrt{\sec(x)}}$$

```
[Out] 4/49/sec(x)^(7/2)+20/63/sec(x)^(3/2)+2/7*x*sin(x)/sec(x)^(5/2)+10/21*x*sin(
x)/sec(x)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4272, 4274}

$$\frac{20}{63\sec^{\frac{3}{2}}(x)}+\frac{4}{49\sec^{\frac{7}{2}}(x)}+\frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{10x\sin(x)}{21\sqrt{\sec(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sec[x]^(7/2) - (5*x*Sqrt[Sec[x]])/21,x]
```

```
[Out] 4/(49*Sec[x]^(7/2)) + 20/(63*Sec[x]^(3/2)) + (2*x*Sin[x])/(7*Sec[x]^(5/2))
+ (10*x*Sin[x])/(21*Sqrt[Sec[x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\sec(x)}\right)dx = -\left(\frac{5}{21}\int x\sqrt{\sec(x)}\,dx\right) + \int \frac{x}{\sec^{\frac{7}{2}}(x)}\,dx$$
$$= \frac{4}{49\sec^{\frac{7}{2}}(x)} + \frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)} + \frac{5}{7}\int \frac{x}{\sec^{\frac{3}{2}}(x)}\,dx - \frac{1}{21}\left(5\sqrt{\cos(x)}\sqrt{\sec(x)}\right)$$
$$= \frac{4}{49\sec^{\frac{7}{2}}(x)} + \frac{20}{63\sec^{\frac{3}{2}}(x)} + \frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)} + \frac{10x\sin(x)}{21\sqrt{\sec(x)}} + \frac{5}{21}\int x\sqrt{\sec(x)}$$
$$= \frac{4}{49\sec^{\frac{7}{2}}(x)} + \frac{20}{63\sec^{\frac{3}{2}}(x)} + \frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)} + \frac{10x\sin(x)}{21\sqrt{\sec(x)}}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.96

$$\sqrt{\sec(x)}\left(\frac{167}{882} + \frac{88}{441}\cos(2x) + \frac{1}{98}\cos(4x) + \frac{13}{42}x\sin(2x) + \frac{1}{28}x\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sec[x]^(7/2) - (5*x*Sqrt[Sec[x]])/21,x]
[Out] Sqrt[Sec[x]]*(167/882 + (88*Cos[2*x])/441 + Cos[4*x]/98 + (13*x*Sin[2*x])/4
2 + (x*Sin[4*x])/28)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x}{\sec(x)^{\frac{7}{2}}} - \frac{5x\left(\sqrt{\sec}(x)\right)}{21}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x)
[Out] int(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-5/21*x*sqrt(sec(x)) + x/sec(x)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{21x}{\sec^{\frac{7}{2}}(x)}\right) dx + \int 5x\sqrt{\sec(x)}\, dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)**(7/2)-5/21*x*sec(x)**(1/2),x)
[Out] -(Integral(-21*x/sec(x)**(7/2), x) + Integral(5*x*sqrt(sec(x)), x))/21
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="giac")
[Out] integrate(-5/21*x*sqrt(sec(x)) + x/sec(x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{5\,x\,\sqrt{\frac{1}{\cos(x)}}}{21} - \frac{x}{\left(\frac{1}{\cos(x)}\right)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cos(x))^(7/2) - (5*x*(1/cos(x))^(1/2))/21,x)
[Out] -int((5*x*(1/cos(x))^(1/2))/21 - x/(1/cos(x))^(7/2), x)
```

3.97 $\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\sec(x)}\right)\,dx$

Optimal. Leaf size=62

$$\frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16}{27}\sqrt{\cos(x)}\,F\left(\frac{x}{2}\middle|2\right)\sqrt{\sec(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}+\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}$$

```
[Out] 8/9*x/sec(x)^(3/2)-16/27*sin(x)/sec(x)^(1/2)+2/3*x^2*sin(x)/sec(x)^(1/2)-16
/27*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))*cos(x)^(1
/2)*sec(x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4273, 4274, 3854, 3856, 2720}

$$\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}+\frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}-\frac{16}{27}\sqrt{\cos(x)}\sqrt{\sec(x)}\,F\left(\frac{x}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sec[x]^(3/2) - (x^2*Sqrt[Sec[x]])/3,x]
```

```
[Out] (8*x)/(9*Sec[x]^(3/2)) - (16*Sqrt[Cos[x]]*EllipticF[x/2, 2]*Sqrt[Sec[x]])/2
7 - (16*Sin[x])/(27*Sqrt[Sec[x]]) + (2*x^2*Sin[x])/(3*Sqrt[Sec[x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4273

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[(c + d*x)^m*Cos[e + f*x]*((b*Csc[e + f*x])^(n + 1)/(b*f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\sec(x)}\right)dx = -\left(\frac{1}{3}\int x^2\sqrt{\sec(x)}\,dx\right)+\int \frac{x^2}{\sec^{\frac{3}{2}}(x)}\,dx$$
$$= \frac{8x}{9\sec^{\frac{3}{2}}(x)}+\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}+\frac{1}{3}\int x^2\sqrt{\sec(x)}\,dx-\frac{8}{9}\int \frac{1}{\sec^{\frac{3}{2}}(x)}\,dx-\frac{1}{3}$$
$$= \frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}+\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}-\frac{8}{27}\int \sqrt{\sec(x)}\,dx$$
$$= \frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}+\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}-\frac{1}{27}\left(8\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int$$
$$= \frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16}{27}\sqrt{\cos(x)}\,F\left(\frac{x}{2}\middle|2\right)\sqrt{\sec(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}+\frac{2x^2\sin}{3\sqrt{\sec}}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.82

$$\frac{1}{27}\sqrt{\sec(x)}\left(12x+12x\cos(2x)-16\sqrt{\cos(x)}\,F\left(\frac{x}{2}\middle|2\right)-8\sin(2x)+9x^2\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sec[x]^(3/2) - (x^2*Sqrt[Sec[x]])/3,x]
```

```
[Out] (Sqrt[Sec[x]]*(12*x + 12*x*Cos[2*x] - 16*Sqrt[Cos[x]]*EllipticF[x/2, 2] - 8
*Sin[2*x] + 9*x^2*Sin[2*x]))/27
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sec(x)^{\frac{3}{2}}}-\frac{x^2\left(\sqrt{\sec}\,(x)\right)}{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x)
```

```
[Out] int(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x^2*sqrt(sec(x)) + x^2/sec(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3x^2}{\sec^{\frac{3}{2}}(x)}\right)dx+\int x^2\sqrt{\sec(x)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sec(x)**(3/2)-1/3*x**2*sec(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x**2/sec(x)**(3/2), x) + Integral(x**2*sqrt(sec(x)), x))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x^2*sqrt(sec(x)) + x^2/sec(x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2\sqrt{\frac{1}{\cos(x)}}}{3} - \frac{x^2}{\left(\frac{1}{\cos(x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/cos(x))^(3/2) - (x^2*(1/cos(x))^(1/2))/3,x)
```

```
[Out] -int((x^2*(1/cos(x))^(1/2))/3 - x^2/(1/cos(x))^(3/2), x)
```

3.98 $\int (c+dx)^m (b\cos(e+fx))^n\,dx$

Optimal. Leaf size=21

$$\text{Int}\left((c+dx)^m (b\cos(e+fx))^n,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (b\cos(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(b*Cos[e + f*x])^n,x]
[Out] Defer[Int][(c + d*x)^m*(b*Cos[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m (b\cos(e+fx))^n\,dx = \int (c+dx)^m (b\cos(e+fx))^n\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int (c+dx)^m (b\cos(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(b*Cos[e + f*x])^n,x]
[Out] Integrate[(c + d*x)^m*(b*Cos[e + f*x])^n, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(b\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(e+fx))^n (c+dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*cos(f*x+e))**n,x)
[Out] Integral((b*cos(e + f*x))**n*(c + d*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (b\,\cos(e+f\,x))^n\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e + f*x))^n*(c + d*x)^m,x)
[Out] int((b*cos(e + f*x))^n*(c + d*x)^m, x)
```

3.99 $\int (c+dx)^m \cos^3(a+bx)\,dx$

Optimal. Leaf size=275

$$-\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamm}}{8b}$$

```
[Out] -3/8*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)+3/8*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)-1/8*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(
d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/8*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(
d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.21, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3388, 2212}

$$-\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}+\frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3,x]
```

```
[Out] (((-3*I)/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))
/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (((3*I)/8)*(c + d*x)^m*Gamma[1 + m, (I*
b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - ((I/8)*3^(
-1 - m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x
))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1
 + m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d
)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\cos^3(a+bx)\,dx &= \int \left(\frac{3}{4}(c+dx)^m\cos(a+bx)+\frac{1}{4}(c+dx)^m\cos(3a+3bx)\right)\,dx\\ &= \frac{1}{4}\int (c+dx)^m\cos(3a+3bx)\,dx+\frac{3}{4}\int (c+dx)^m\cos(a+bx)\,dx\\ &= \frac{1}{8}\int e^{-i(3a+3bx)}(c+dx)^m\,dx+\frac{1}{8}\int e^{i(3a+3bx)}(c+dx)^m\,dx+\frac{3}{8}\int e^{-i(a+bx)}(c\\ &= -\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+\cdots}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 253, normalized size = 0.92

$$\frac{i3^{-1-m}e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{2+m}e^{2i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)+3^{2+m}e^{2ia+\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,\frac{ib(c+dx)}{d}\right)-e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,\frac{3ib(c+dx)}{d}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3,x]
```

```
[Out] ((I/8)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((2*I)*(2*a + (b*c)/d))*((I*b*
(c + d*x))/d)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]) + 3^(2 + m)*E^((2*I)*a
+ ((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, (I*b*(c + d*x))/d] -
 E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E
^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/
d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(\cos^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3, x)
```

Fricas [A]
time = 0.11, size = 188, normalized size = 0.68

$$\frac{9i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)-i\,e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)-9i\,e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-i\,bdx-i\,bc}{d}\right)+i\,e^{\left(-\frac{dm\log\left(\frac{3i\,b}{d}\right)-3i\,bc+3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(-i\,bdx-i\,bc)}{d}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(9*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I
*b*c)/d) - I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -3
*(I*b*d*x + I*b*c)/d) - 9*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(
m + 1, (-I*b*d*x - I*b*c)/d) + I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)
/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos^3{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (a+b x)^3 (c+d x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^3*(c + d*x)^m, x)
```

3.100 $\int (c+dx)^m \cos^2(a+bx)\,dx$

Optimal. Leaf size=162

$$\frac{(c+dx)^{1+m}}{2d(1+m)}-\frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{i2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+</p>$$

```
[Out] 1/2*(d*x+c)^(1+m)/d/(1+m)-I*2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m
,-2*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+I*2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*
I*b*(d*x+c)/d)/b/exp(2*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3388, 2212}

$$-\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
+ d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)
 + (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos^2(a+bx)\,dx &= \int \left(\frac{1}{2}(c+dx)^m + \frac{1}{2}(c+dx)^m \cos(2a+2bx)\right) dx\\ &= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{2}\int (c+dx)^m \cos(2a+2bx)\,dx\\ &= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{4}\int e^{-i(2a+2bx)}(c+dx)^m\,dx + \frac{1}{4}\int e^{i(2a+2bx)}(c+dx)^m\,dx\\ &= \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx}{d}\right.}{b}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 150, normalized size = 0.93

$$\frac{1}{8}(c+dx)^m\left(\frac{4c+4dx}{d+dm} - \frac{i2^{-m}e^{2i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{i2^{-m}e^{-2i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2,x]
[Out] ((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) - (I*E^((2*I)*(a - (b*c)/d))*Gamma[1
+ m, ((-2*I)*b*(c + d*x))/d])/(2^m*b*(((-I)*b*(c + d*x))/d)^m) + (I*Gamma[1
 + m, ((2*I)*b*(c + d*x))/d])/(2^m*b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m)))/8
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*b*x + 2*a), x) + e^(m*log(d*x +
c) + log(d*x + c)))/(d*m + d)
```

Fricas [A]
time = 0.12, size = 136, normalized size = 0.84

$$\frac{(-i\,dm - i\,d)e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,bdx+i\,bc)}{d}\right)+(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(\frac{2i\,b}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,bdx-i\,bc)}{d}\right)+4\,(bdx+bc)(dx+c)^m}{8\,(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((-I*d*m - I*d)*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m
+ 1, -2*(I*b*d*x + I*b*c)/d) + (I*d*m + I*d)*e^(-(d*m*log(2*I*b/d) - 2*I*b*
c + 2*I*a*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) + 4*(b*d*x + b*c)*(d*
x + c)^m)/(b*d*m + b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^2*(c + d*x)^m, x)
```

3.101 $\int (c+dx)^m \cos(a+bx)\,dx$

Optimal. Leaf size=131

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}$$

```
[Out] -1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)+1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)
```

Rubi [A]
time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3388, 2212}

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d
])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c +
 d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (c+dx)^m \cos(a+bx)\,dx = \frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx + \frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx$$
$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)}{}$$

Mathematica [A]
time = 0.03, size = 122, normalized size = 0.93

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}(c+dx)^m\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,\frac{ib(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x],x]
[Out] ((-1/2*I)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((
-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d])
/((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a), x)
```

Fricas [A]
time = 0.10, size = 96, normalized size = 0.73

$$\frac{i\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)-i\,e^{\left(-\frac{dm\log\left(-\frac{i\,b}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-i\,bdx-i\,bc}{d}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*
c)/d) - I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x -
 I*b*c)/d))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos \left(a+b\,x\right)\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*(c + d*x)^m, x)
```

3.102 $\int (c+dx)^m \sec(a+bx)\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left((c+dx)^m \sec(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*sec(b*x+a),x)
```

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\,dx$$

Mathematica [A]

time = 5.27, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sec[a + b*x], x]
```

Maple [A]

time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \sec (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a),x)
[Out] Integral((c + d*x)**m*sec(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{(c+d\,x)^m}{\cos (a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/cos(a + b*x),x)
[Out] int((c + d*x)^m/cos(a + b*x), x)
```

3.103 $\int (c+dx)^m \sec^2(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left((c+dx)^m \sec^2(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*sec(b*x+a)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sec^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\,dx$$

Mathematica [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\sec^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*sec(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/cos(a + b*x)^2,x)
[Out] int((c + d*x)^m/cos(a + b*x)^2, x)
```

3.104 $\int x^{3+m} \cos(a+bx)\,dx$

Optimal. Leaf size=75

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(4+m,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(4+m,ibx)}{2b^4}$$

```
[Out] -1/2*exp(I*a)*x^m*GAMMA(4+m,-I*b*x)/b^4/((-I*b*x)^m)-1/2*x^m*GAMMA(4+m,I*b*
x)/b^4/exp(I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+4,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+4,ibx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Cos[a + b*x],x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - (x^m*Gamma[4
 + m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^{3+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{3+m}\,dx\\ &= -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.00

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(4+m,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(4+m,ibx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Cos[a + b*x],x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - (x^m*Gamma[4
 + m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 455, normalized size = 6.07

method	result
meijerg	$\frac{2^{3+m}(b^2)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{-4-m}x^{3+m}b^3(b^2)^{\frac{m}{2}}\left(\frac{8}{3}+\frac{2m}{3}\right)\sin(bx)}{\sqrt{\pi}\,(4+m)}-\frac{2^{-3-m}x^{1+m}b(b^2)^{\frac{m}{2}}\left(-m^2-7m-12\right)(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,(4+m)}+\frac{2^{-3-m}x^{2+m}\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(3+m)/b^4*(b^2)^(-1/2*m)*Pi^(1/2)*(3*2^(-4-m)/Pi^(1/2)/(4+m)*x^(3+m)*b^3*
(b^2)^(1/2*m)*(8/3+2/3*m)*sin(b*x)-2^(-3-m)/Pi^(1/2)/(4+m)*x^(1+m)*b*(b^2)^
(1/2*m)*(-m^2-7*m-12)*(cos(b*x)*x*b-sin(b*x))+2^(-3-m)/Pi^(1/2)/(4+m)*x^(2+
m)*b^2*(b^2)^(1/2*m)*(-m^3-8*m^2-19*m-12)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2
,b*x)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2+m)*(1+m)*(3+m
)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)-2^
(3+m)*b^(-4-m)*Pi^(1/2)*(2^(-3-m)/Pi^(1/2)/(5+m)*x^(2+m)*b^(2+m)*(m^2+7*m+1
0)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(cos(b*x)*x*b-sin(b*x))-2^(-3
-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*m*(3+m)*(2+m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/
2,b*x)*sin(b*x)+2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(3+m)*(2+m)*(b*x)^(-5/2-m
)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 3)*cos(b*x + a), x)
```

Fricas [A]
time = 0.10, size = 54, normalized size = 0.72

$$\frac{i\,e^{\left(-(m+3)\log(i\,b)-i\,a\right)}\Gamma(m+4,i\,bx)-i\,e^{\left(-(m+3)\log(-i\,b)+i\,a\right)}\Gamma(m+4,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 3)*log(I*b) - I*a)*gamma(m + 4, I*b*x) - I*e^(-(m + 3)*log(
-I*b) + I*a)*gamma(m + 4, -I*b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m + 3)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+3}\,\cos\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 3)*cos(a + b*x),x)
```

```
[Out] int(x^(m + 3)*cos(a + b*x), x)
```

3.105 $\int x^{2+m}\cos(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(3+m,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(3+m,ibx)}{2b^3}$$

```
[Out] 1/2*I*exp(I*a)*x^m*GAMMA(3+m,-I*b*x)/b^3/((-I*b*x)^m)-1/2*I*x^m*GAMMA(3+m,I
*b*x)/b^3/exp(I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3388, 2212}

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+3,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+3,ibx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\cos(a+bx)\,dx&=\frac{1}{2}\int e^{-i(a+bx)}x^{2+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{2+m}\,dx\\&=\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 1.00

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(3+m,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(3+m,ibx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 354, normalized size = 4.48

method	result
meijerg	$\frac{2^{2+m}\left(b^2\right)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{-3-m}x^{2+m}\left(b^2\right)^{\frac{3}{2}+\frac{m}{2}}\left(2+\frac{2m}{3}\right)\sin(bx)}{\sqrt{\pi}\,(3+m)b}-\frac{2^{-2-m}x^{2+m}\left(b^2\right)^{\frac{3}{2}+\frac{m}{2}}(2+m)m(bx)^{-\frac{3}{2}-m}\text{LommelS1}\left(m+\frac{1}{2},\frac{3}{2},bx\right)\sin}{\sqrt{\pi}\,b}\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(2+m)/b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-3-m)/Pi^(1/2)/(3+m)*x^(2+m)*
(b^2)^(3/2+1/2*m)*(2+2/3*m)/b*sin(b*x)-2^(-2-m)/Pi^(1/2)*x^(2+m)*(b^2)^(3/2
+1/2*m)/b*(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(-2-m)/
Pi^(1/2)*x^(2+m)*(b^2)^(3/2+1/2*m)/b*(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin
(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)-2^(2+m)*b^(-3-m)*Pi^(1/2)*(-2^(-2-m)
/Pi^(1/2)*x^(1+m)*b^(1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-2-m)/Pi^(1/2)/(4+m)*x
^(2+m)*b^(2+m)*(m^2+5*m+4)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+
2^(-2-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(2+m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-
sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)*cos(b*x + a), x)
```

Fricas [A]
time = 0.11, size = 54, normalized size = 0.68

$$\frac{i\,e^{\left(-(m+2)\log(i\,b)-i\,a\right)}\Gamma(m+3,i\,bx)-i\,e^{\left(-(m+2)\log(-i\,b)+i\,a\right)}\Gamma(m+3,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 2)*log(I*b) - I*a)*gamma(m + 3, I*b*x) - I*e^(-(m + 2)*log(
-I*b) + I*a)*gamma(m + 3, -I*b*x))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m + 2)*cos(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*cos(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+2}\,\cos\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)*cos(a + b*x),x)
```

```
[Out] int(x^(m + 2)*cos(a + b*x), x)
```

3.106 $\int x^{1+m}\cos(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(2+m,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(2+m,ibx)}{2b^2}$$

```
[Out] 1/2*exp(I*a)*x^m*GAMMA(2+m,-I*b*x)/b^2/((-I*b*x)^m)+1/2*x^m*GAMMA(2+m,I*b*x
)/b^2/exp(I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+2,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+2,ibx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Cos[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{1+m}\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^{1+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{1+m}\,dx\\ &= \frac{e^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.00

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(2+m,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(2+m,ibx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Cos[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 291, normalized size = 3.88

method	result
meijerg	$\frac{2^{1+m}\left(b^2\right)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{2^{-1-m}x^{1+m}b\left(b^2\right)^{\frac{m}{2}}\sin(bx)}{\sqrt{\pi}\,(2+m)}+\frac{3\,2^{-2-m}x^{2+m}b^2\left(b^2\right)^{\frac{m}{2}}\left(\frac{2}{3}+\frac{2m}{3}\right)(bx)^{-\frac{3}{2}-m}\text{LommelS1}\left(m+\frac{3}{2},\frac{3}{2},bx\right)\sin(bx)}{\sqrt{\pi}\,(2+m)}+\frac{2^{-1-}\cdots}{}\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1+m)/b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)/(2+m)*x^(1+m)*b*(b^2
)^(1/2*m)*sin(b*x)+3*2^(-2-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2/3
+2/3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+2^(-1-m)/Pi^(1/2)*x
^(2+m)*b^2*(b^2)^(1/2*m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lomme
lS1(m+1/2,1/2,b*x))*cos(a)-2^(1+m)*b^(-2-m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)*x^(
2+m)*b^(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-1-m)/Pi^
(1/2)*x^(2+m)*b^(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2
,1/2,b*x))*sin(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)*cos(b*x + a), x)
```

Fricas [A]
time = 0.09, size = 54, normalized size = 0.72

$$\frac{i\,e^{(-(m+1)\log(i\,b)-i\,a)}\Gamma(m+2,i\,bx)-i\,e^{(-(m+1)\log(-i\,b)+i\,a)}\Gamma(m+2,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 1)*log(I*b) - I*a)*gamma(m + 2, I*b*x) - I*e^(-(m + 1)*log(
-I*b) + I*a)*gamma(m + 2, -I*b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m + 1)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+1}\,\cos\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)*cos(a + b*x),x)
```

```
[Out] int(x^(m + 1)*cos(a + b*x), x)
```

3.107 $\int x^m \cos(a+bx)\,dx$

Optimal. Leaf size=79

$$-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(1+m,-ibx)}{2b}+\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(1+m,ibx)}{2b}$$

```
[Out] -1/2*I*exp(I*a)*x^m*GAMMA(1+m,-I*b*x)/b/((-I*b*x)^m)+1/2*I*x^m*GAMMA(1+m,I*
b*x)/b/exp(I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3388, 2212}

$$\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+1,ibx)}{2b}-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+1,-ibx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^m
*Gamma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^m\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^m\,dx+\frac{1}{2}\int e^{i(a+bx)}x^m\,dx\\ &= -\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b}+\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 1.00

$$-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(1+m,-ibx)}{2b}+\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(1+m,ibx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^m
*Gamma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 379, normalized size = 4.80

method	result
meijerg	$2^m(b^2)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{-1-m}(b^2)^{\frac{1}{2}+\frac{m}{2}}x^m(6+2m)\sin(bx)}{\sqrt{\pi}\,(1+m)(9+3m)b}+\frac{(b^2)^{\frac{1}{2}+\frac{m}{2}}x^m2^{-m}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,(1+m)b}+\frac{2^{-m}x^{2+m}(b^2)^{\frac{1}{2}+\frac{m}{2}}b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^m*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-1-m)/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m
)*x^m*(6+2*m)/(9+3*m)/b*sin(b*x)+1/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m)*x^m*2^(
-m)/b*(cos(b*x)*x*b-sin(b*x))+2^(-m)/Pi^(1/2)/(1+m)*x^(2+m)*(b^2)^(1/2+1/2*
m)*b*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-m)/Pi^(1/2)/(1+m
)*x^(2+m)*(b^2)^(1/2+1/2*m)*b*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lommel
S1(m+3/2,1/2,b*x))*cos(a)-2^m*b^(-1-m)*Pi^(1/2)*(1/Pi^(1/2)/(2+m)*x^(1+m)*b
^(1+m)*2^(-m)*sin(b*x)-2^(-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)*(b*x)^(-3/2-m)
*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-3*2^(-1-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)
*(4/3+2/3*m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x)
)*sin(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cos(b*x + a), x)
```

Fricas [A]
time = 0.10, size = 50, normalized size = 0.63

$$\frac{i\,e^{(-m\log(i\,b)-i\,a)}\Gamma(m+1,i\,bx)-i\,e^{(-m\log(-i\,b)+i\,a)}\Gamma(m+1,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) - I*e^(-m*log(-I*b) + I*a)
*gamma(m + 1, -I*b*x))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(b*x+a),x)
```

```
[Out] Integral(x**m*cos(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cos(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\, \cos(a + b\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*x),x)
```

```
[Out] int(x^m*cos(a + b*x), x)
```

3.108 $\int x^{-1+m}\cos(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m,-ibx)-\frac{1}{2}e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m,ibx)$$

```
[Out] -1/2*exp(I*a)*x^m*GAMMA(m,-I*b*x)/((-I*b*x)^m)-1/2*x^m*GAMMA(m,I*b*x)/exp(I
*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$-\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m,-ibx)-\frac{1}{2}e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Cos[a + b*x],x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - (x^m*Gamma[m, I*b*x])/
(2*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^{-1+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{-1+m}\,dx\\ &= -\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx)-\frac{1}{2}e^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.95

$$\frac{1}{2}e^{-ia}x^m\left(-e^{2ia}(-ibx)^{-m}\mathrm{Gamma}(m,-ibx)-(ibx)^{-m}\mathrm{Gamma}(m,ibx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Cos[a + b*x],x]
```

```
[Out] (x^m*(-((E^((2*I)*a)*Gamma[m, (-I)*b*x])/((-I)*b*x)^m) - Gamma[m, I*b*x]/(I
*b*x)^m))/(2*E^(I*a))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 427, normalized size = 6.57

method	result
meijerg	$2^{-1+m}(b^2)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{3x^{-1+m}2^{-m}(b^2)^{\frac{m}{2}}(2x^2b^2+2m+4)\sin(bx)}{\sqrt{\pi}\,m(6+3m)b}+\frac{2^{1-m}x^{-1+m}(b^2)^{\frac{m}{2}}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,mb}-\frac{3x^{2+m}2^{1-}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(-1+m)*(b^2)^(-1/2*m)*Pi^(1/2)*(3/Pi^(1/2)/m*x^(-1+m)*2^(-m)*(b^2)^(1/2*m
)*(2*b^2*x^2+2*m+4)/(6+3*m)/b*sin(b*x)+2^(1-m)/Pi^(1/2)/m*x^(-1+m)*(b^2)^(1
/2*m)/b*(cos(b*x)*x*b-sin(b*x))-3/Pi^(1/2)/m*x^(2+m)*2^(1-m)*(b^2)^(1/2*m)*
b^2/(6+3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-1/Pi^(1/2)/m*x^
(2+m)*2^(1-m)*(b^2)^(1/2*m)*b^2*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lomm
elS1(m+1/2,1/2,b*x))*cos(a)-2^(-1+m)*b^(-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)/(1+m
)*x^m*b^m*sin(b*x)-2^(1-m)/Pi^(1/2)/(1+m)*x^m*b^m/m*(cos(b*x)*x*b-sin(b*x))
-1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2
,b*x)*sin(b*x)+1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)/m*(b*x)^(-5/2-m)*(c
os(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)*cos(b*x + a), x)
```

Fricas [A]
time = 0.11, size = 50, normalized size = 0.77

$$\frac{i\,e^{(-(m-1)\log(i\,b)-i\,a)}\Gamma(m,i\,bx)-i\,e^{(-(m-1)\log(-i\,b)+i\,a)}\Gamma(m,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m - 1)*log(I*b) - I*a)*gamma(m, I*b*x) - I*e^(-(m - 1)*log(-I*b
) + I*a)*gamma(m, -I*b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m - 1)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-1}\,\cos\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*cos(a + b*x),x)
```

```
[Out] int(x^(m - 1)*cos(a + b*x), x)
```

3.109 $\int x^{-2+m}\cos(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\text{Gamma}(-1+m,-ibx)-\frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\text{Gamma}(-1+m,ibx)$$

```
[Out] 1/2*I*b*exp(I*a)*x^m*GAMMA(-1+m,-I*b*x)/((-I*b*x)^m)-1/2*I*b*x^m*GAMMA(-1+m
,I*b*x)/exp(I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$\frac{1}{2}ie^{ia}bx^m(-ibx)^{-m}\text{Gamma}(m-1,-ibx)-\frac{1}{2}ie^{-ia}bx^m(ibx)^{-m}\text{Gamma}(m-1,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*G
amma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\cos(a+bx)\,dx&=\frac{1}{2}\int e^{-i(a+bx)}x^{-2+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{-2+m}\,dx\\&=\frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx)-\frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.00

$$\frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\text{Gamma}(-1+m,-ibx)-\frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\text{Gamma}(-1+m,ibx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*G
amma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 530, normalized size = 7.07

method	result
meijerg	$2^{-2+m}b^2(b^2)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{1-m}x^{-2+m}(b^2)^{-\frac{1}{2}+\frac{m}{2}}(2x^2b^2+2m+2)\sin(bx)}{\sqrt{\pi}\,(-1+m)(3+3m)b}-\frac{2^{2-m}x^{-2+m}(b^2)^{-\frac{1}{2}+\frac{m}{2}}(x^2b^2-m^2-m)(\cos}{\sqrt{\pi}\,(-1+m)b(1+m)m}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(-2+m)*b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(1-m)/Pi^(1/2)/(-1+m)*x^(-2+m
)*(b^2)^(-1/2+1/2*m)*(2*b^2*x^2+2*m+2)/(3+3*m)/b*sin(b*x)-2^(2-m)/Pi^(1/2)/
(-1+m)*x^(-2+m)*(b^2)^(-1/2+1/2*m)/b*(b^2*x^2-m^2-m)/(1+m)/m*(cos(b*x)*x*b-
sin(b*x))-3*2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/(3+3*m)*
(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(2-m)/Pi^(1/2)/(-1+m)*x^(
2+m)*(b^2)^(-1/2+1/2*m)*b^3/(1+m)/m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*
LommelS1(m+3/2,1/2,b*x))*cos(a)-2^(-2+m)*b^(1-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)
/m*x^(-1+m)*b^(-1+m)*(-2*b^2*x^2+2*m^2+2*m-4)/(2+m)/(-1+m)*sin(b*x)-3*2^(2-
m)/Pi^(1/2)/m*x^(-1+m)*b^(-1+m)/(-3+3*m)*(cos(b*x)*x*b-sin(b*x))+2^(2-m)/Pi
^(1/2)/m*x^(2+m)*b^(2+m)/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x
)*sin(b*x)+3*2^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(-3+3*m)*(b*x)^(-5/2-m)*(co
s(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)*cos(b*x + a), x)
```

Fricas [A]
time = 0.12, size = 54, normalized size = 0.72

$$\frac{i\,e^{\left(-(m-2)\log(i\,b)-i\,a\right)}\Gamma(m-1,i\,bx)-i\,e^{\left(-(m-2)\log(-i\,b)+i\,a\right)}\Gamma(m-1,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m - 2)*log(I*b) - I*a)*gamma(m - 1, I*b*x) - I*e^(-(m - 2)*log(
-I*b) + I*a)*gamma(m - 1, -I*b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m - 2)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-2}\,\cos\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)*cos(a + b*x),x)
```

```
[Out] int(x^(m - 2)*cos(a + b*x), x)
```

3.110 $\int x^{-3+m}\cos(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\text{Gamma}(-2+m,-ibx)+\frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\text{Gamma}(-2+m,ibx)$$

```
[Out] 1/2*b^2*exp(I*a)*x^m*GAMMA(-2+m,-I*b*x)/((-I*b*x)^m)+1/2*b^2*x^m*GAMMA(-2+m
,I*b*x)/exp(I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3388, 2212}

$$\frac{1}{2}e^{ia}b^2x^m(-ibx)^{-m}\text{Gamma}(m-2,-ibx)+\frac{1}{2}e^{-ia}b^2x^m(ibx)^{-m}\text{Gamma}(m-2,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Cos[a + b*x],x]
```

```
[Out] (b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamma
[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\cos(a+bx)\,dx&=\frac{1}{2}\int e^{-i(a+bx)}x^{-3+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{-3+m}\,dx\\&=\frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx)+\frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.00

$$\frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\text{Gamma}(-2+m,-ibx)+\frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\text{Gamma}(-2+m,ibx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Cos[a + b*x],x]
```

```
[Out] (b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamma
[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 600, normalized size = 8.00

method	result
meijerg	$2^{-3+m}b^2(b^2)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{2^{2-m}x^{-3+m}(b^2)^{\frac{m}{2}}(-2x^4b^4+2x^2b^2m^2+2x^2b^2m-4x^2b^2+2m^3+2m^2-4m)\sin(bx)}{\sqrt{\pi}\,(-2+m)b^3m(2+m)(-1+m)}-\frac{2^{-m+3}x^{-3+}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(-3+m)*b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-2+m)*x^(-3+m)/b^3*
(b^2)^(1/2*m)*(-2*b^4*x^4+2*b^2*m^2*x^2+2*b^2*m*x^2-4*b^2*x^2+2*m^3+2*m^2-4
*m)/m/(2+m)/(-1+m)*sin(b*x)-2^(-m+3)/Pi^(1/2)/(-2+m)*x^(-3+m)/b^3*(b^2)^(1/
2*m)*(b^2*x^2-m^2+m)/m/(-1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^(1/2)/(-2
+m)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,
3/2,b*x)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(-2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(-1+
m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)-2
^(-3+m)*b^(2-m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-1+m)*x^(-2+m)*b^(-2+m)*(-2*b^2
*x^2+2*m^2-2*m-4)/(1+m)/(-2+m)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(-1+m)*x^(-2+m)*b
^(-2+m)*(b^2*x^2-m^2-m)/(1+m)/(-2+m)/m*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^
(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(-2+m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2
,b*x)*sin(b*x)-2^(-m+3)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(-2+m)/m*(b*x
)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 3)*cos(b*x + a), x)
```

Fricas [A]
time = 0.09, size = 54, normalized size = 0.72

$$\frac{i\,e^{\left(-(m-3)\log(i\,b)-i\,a\right)}\Gamma(m-2,i\,bx)-i\,e^{\left(-(m-3)\log(-i\,b)+i\,a\right)}\Gamma(m-2,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="fricas")
[Out] 1/2*(I*e^(-(m - 3)*log(I*b) - I*a)*gamma(m - 2, I*b*x) - I*e^(-(m - 3)*log(
-I*b) + I*a)*gamma(m - 2, -I*b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*cos(b*x+a),x)
[Out] Integral(x**(m - 3)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 3)*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-3}\,\cos\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)*cos(a + b*x),x)
[Out] int(x^(m - 3)*cos(a + b*x), x)
```

3.111 $\int x^{3+m}\cos^2(a+bx)\,dx$

Optimal. Leaf size=99

$$\frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\text{Gamma}(4+m,-2ibx)}{b^4}-\frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\text{Gamma}(4+m,2ibx)}{b^4}$$

```
[Out] 1/2*x^(4+m)/(4+m)-2^(-6-m)*exp(2*I*a)*x^m*GAMMA(4+m,-2*I*b*x)/b^4/((-I*b*x)
^m)-2^(-6-m)*x^m*GAMMA(4+m,2*I*b*x)/b^4/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$-\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\text{Gamma}(m+4,-2ibx)}{b^4}-\frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\text{Gamma}(m+4,2ibx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*x
])/(b^4*((-I)*b*x)^m) - (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^{3+m}\cos^2(a+bx)\,dx = \int\left(\frac{x^{3+m}}{2}+\frac{1}{2}x^{3+m}\cos(2a+2bx)\right)\,dx$$
$$= \frac{x^{4+m}}{2(4+m)}+\frac{1}{2}\int x^{3+m}\cos(2a+2bx)\,dx$$
$$= \frac{x^{4+m}}{2(4+m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{3+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{3+m}\,dx$$
$$= \frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4}-\frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4$$

Mathematica [A]
time = 0.07, size = 92, normalized size = 0.93

$$\frac{1}{64}x^m\left(\frac{32x^4}{4+m}-\frac{2^{-m}e^{2ia}(-ibx)^{-m}\text{Gamma}(4+m,-2ibx)}{b^4}-\frac{2^{-m}e^{-2ia}(ibx)^{-m}\text{Gamma}(4+m,2ibx)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((32*x^4)/(4 + m) - (E^((2*I)*a)*Gamma[4 + m, (-2*I)*b*x])/(2^m*b^4*((
-I)*b*x)^m) - Gamma[4 + m, (2*I)*b*x]/(2^m*b^4*E^((2*I)*a)*(I*b*x)^m)))/64
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^{3+m}\left(\cos^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(3+m)*cos(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m + 4)*integrate(x^3*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 4*log(x
)))/(m + 4)
```

Fricas [A]
time = 0.13, size = 77, normalized size = 0.78

$$\frac{4\,bxx^{m+3} + (i\,m + 4i)e^{(-(m+3)\log(2i\,b)-2i\,a)}\Gamma(m+4, 2i\,bx) + (-i\,m - 4i)e^{(-(m+3)\log(-2i\,b)+2i\,a)}\Gamma(m+4, -2i\,bx)}{8\,(bm + 4\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 3) + (I*m + 4*I)*e^(-(m + 3)*log(2*I*b) - 2*I*a)*gamma(m
+ 4, 2*I*b*x) + (-I*m - 4*I)*e^(-(m + 3)*log(-2*I*b) + 2*I*a)*gamma(m + 4,
-2*I*b*x))/(b*m + 4*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3}\cos^2{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 3)*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+3}\,{\cos\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 3)*cos(a + b*x)^2,x)
```

```
[Out] int(x^(m + 3)*cos(a + b*x)^2, x)
```

3.112 $\int x^{2+m}\cos^2(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{x^{3+m}}{2(3+m)}+\frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\text{Gamma}(3+m,-2ibx)}{b^3}-\frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\text{Gamma}(3+m,2ibx)}{b^3}$$

```
[Out] 1/2*x^(3+m)/(3+m)+I*2^(-5-m)*exp(2*I*a)*x^m*GAMMA(3+m,-2*I*b*x)/b^3/((-I*b*
x)^m)-I*2^(-5-m)*x^m*GAMMA(3+m,2*I*b*x)/b^3/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3393, 3388, 2212}

$$\frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\text{Gamma}(m+3,-2ibx)}{b^3}-\frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\text{Gamma}(m+3,2ibx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(3 + m)/(2*(3 + m)) + (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*b
*x])/(b^3*((-I)*b*x)^m) - (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3*E
^((2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^{2+m}\cos^2(a+bx)\,dx = \int \left(\frac{x^{2+m}}{2}+\frac{1}{2}x^{2+m}\cos(2a+2bx)\right)\,dx$$
$$= \frac{x^{3+m}}{2(3+m)}+\frac{1}{2}\int x^{2+m}\cos(2a+2bx)\,dx$$
$$= \frac{x^{3+m}}{2(3+m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{2+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{2+m}\,dx$$
$$= \frac{x^{3+m}}{2(3+m)}+\frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3}-\frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma}{b^3}$$

Mathematica [A]
time = 0.07, size = 96, normalized size = 0.93

$$\frac{1}{32}x^m\left(\frac{16x^3}{3+m}+\frac{i2^{-m}e^{2ia}(-ibx)^{-m}\mathrm{Gamma}(3+m,-2ibx)}{b^3}-\frac{i2^{-m}e^{-2ia}(ibx)^{-m}\mathrm{Gamma}(3+m,2ibx)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((16*x^3)/(3 + m) + (I*E^((2*I)*a)*Gamma[3 + m, (-2*I)*b*x])/(2^m*b^3*
((-I)*b*x)^m) - (I*Gamma[3 + m, (2*I)*b*x])/(2^m*b^3*E^((2*I)*a)*(I*b*x)^m)
))/32
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^{2+m}\left(\cos^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(2+m)*cos(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m + 3)*integrate(x^2*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 3*log(x
)))/(m + 3)
```

Fricas [A]
time = 0.11, size = 77, normalized size = 0.75

$$\frac{4\,bxx^{m+2} + (i\,m + 3i)e^{\left(-(m+2)\log(2i\,b)-2i\,a\right)}\Gamma(m+3, 2i\,bx) + (-i\,m - 3i)e^{\left(-(m+2)\log(-2i\,b)+2i\,a\right)}\Gamma(m+3, -2i\,bx)}{8\,(bm + 3\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 2) + (I*m + 3*I)*e^(-(m + 2)*log(2*I*b) - 2*I*a)*gamma(m
+ 3, 2*I*b*x) + (-I*m - 3*I)*e^(-(m + 2)*log(-2*I*b) + 2*I*a)*gamma(m + 3,
-2*I*b*x))/(b*m + 3*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2}\cos^2\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 2)*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+2}\cos\left(a + b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)*cos(a + b*x)^2,x)
```

```
[Out] int(x^(m + 2)*cos(a + b*x)^2, x)
```

3.113 $\int x^{1+m} \cos^2(a+bx)\, dx$

Optimal. Leaf size=97

$$\frac{x^{2+m}}{2(2+m)}+\frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\text{Gamma}(2+m,-2ibx)}{b^2}+\frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\text{Gamma}(2+m,2ibx)}{b^2}$$

```
[Out] 1/2*x^(2+m)/(2+m)+2^(-4-m)*exp(2*I*a)*x^m*GAMMA(2+m,-2*I*b*x)/b^2/((-I*b*x)
^m)+2^(-4-m)*x^m*GAMMA(2+m,2*I*b*x)/b^2/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$\frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\text{Gamma}(m+2,-2ibx)}{b^2}+\frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\text{Gamma}(m+2,2ibx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(2 + m)/(2*(2 + m)) + (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*x
])/(b^2*((-I)*b*x)^m) + (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^{1+m}\cos^2(a+bx)\,dx = \int\left(\frac{x^{1+m}}{2}+\frac{1}{2}x^{1+m}\cos(2a+2bx)\right)\,dx$$
$$= \frac{x^{2+m}}{2(2+m)}+\frac{1}{2}\int x^{1+m}\cos(2a+2bx)\,dx$$
$$= \frac{x^{2+m}}{2(2+m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{1+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{1+m}\,dx$$
$$= \frac{x^{2+m}}{2(2+m)}+\frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2}+\frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2}{b^2}$$

Mathematica [A]
time = 0.07, size = 90, normalized size = 0.93

$$\frac{1}{16}x^m\left(\frac{8x^2}{2+m}+\frac{2^{-m}e^{2ia}(-ibx)^{-m}\mathrm{Gamma}(2+m,-2ibx)}{b^2}+\frac{2^{-m}e^{-2ia}(ibx)^{-m}\mathrm{Gamma}(2+m,2ibx)}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((8*x^2)/(2 + m) + (E^((2*I)*a)*Gamma[2 + m, (-2*I)*b*x])/(2^m*b^2*((-
I)*b*x)^m) + Gamma[2 + m, (2*I)*b*x]/(2^m*b^2*E^((2*I)*a)*(I*b*x)^m)))/16
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^{1+m}\left(\cos^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(1+m)*cos(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m + 2)*integrate(x*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 2*log(x))
)/(m + 2)
```

Fricas [A]
time = 0.11, size = 77, normalized size = 0.79

$$\frac{4\,bxx^{m+1} + (i\,m+2i)e^{\left(-(m+1)\log(2i\,b)-2i\,a\right)}\Gamma(m+2,2i\,bx) + (-i\,m-2i)e^{\left(-(m+1)\log(-2i\,b)+2i\,a\right)}\Gamma(m+2,-2i\,bx)}{8\,(bm+2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 1) + (I*m + 2*I)*e^(-(m + 1)*log(2*I*b) - 2*I*a)*gamma(m
+ 2, 2*I*b*x) + (-I*m - 2*I)*e^(-(m + 1)*log(-2*I*b) + 2*I*a)*gamma(m + 2,
-2*I*b*x))/(b*m + 2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1}\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 1)*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+1}\cos\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)*cos(a + b*x)^2,x)
```

```
[Out] int(x^(m + 1)*cos(a + b*x)^2, x)
```

3.114 $\int x^m \cos^2(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{x^{1+m}}{2(1+m)}-\frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\text{Gamma}(1+m,-2ibx)}{b}+\frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\text{Gamma}(1+m,2ibx)}{b}$$

```
[Out] 1/2*x^(1+m)/(1+m)-I*2^(-3-m)*exp(2*I*a)*x^m*GAMMA(1+m,-2*I*b*x)/b/((-I*b*x)
^m)+I*2^(-3-m)*x^m*GAMMA(1+m,2*I*b*x)/b/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3393, 3388, 2212}

$$-\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\text{Gamma}(m+1,-2ibx)}{b}+\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\text{Gamma}(m+1,2ibx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*x]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*b
*x])/(b*((-I)*b*x)^m) + (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^m \cos^2(a+bx)\,dx &= \int \left(\frac{x^m}{2}+\frac{1}{2}x^m\cos(2a+2bx)\right)\,dx\\ &= \frac{x^{1+m}}{2(1+m)}+\frac{1}{2}\int x^m\cos(2a+2bx)\,dx\\ &= \frac{x^{1+m}}{2(1+m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^m\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^m\,dx\\ &= \frac{x^{1+m}}{2(1+m)}-\frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b}+\frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1}{b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 90, normalized size = 0.87

$$\frac{1}{8}x^m\left(\frac{4x}{1+m}-2^{-m}e^{2ia}x(-ibx)^{-1-m}\text{Gamma}(1+m,-2ibx)-2^{-m}e^{-2ia}x(ibx)^{-1-m}\text{Gamma}(1+m,2ibx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*x]^2,x]
[Out] (x^m*((4*x)/(1 + m) - (E^((2*I)*a)*x*((-I)*b*x)^(-1 - m)*Gamma[1 + m, (-2*I
)*b*x])/2^m - (x*(I*b*x)^(-1 - m)*Gamma[1 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)
)))/8
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^m\left(\cos^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(b*x+a)^2,x)
[Out] int(x^m*cos(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + log(x)))/(m
 + 1)
```

Fricas [A]
time = 0.10, size = 69, normalized size = 0.67

$$\frac{4\,bxx^{m} + (i\,m + i)e^{(-m\log(2i\,b) - 2i\,a)}\Gamma(m+1, 2i\,bx) + (-i\,m - i)e^{(-m\log(-2i\,b) + 2i\,a)}\Gamma(m+1, -2i\,bx)}{8\,(bm+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^m + (I*m + I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
+ (-I*m - I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))/(b*m + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\cos^{2}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**m*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m}\cos\left(a+b\,x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*x)^2,x)
```

```
[Out] int(x^m*cos(a + b*x)^2, x)
```

3.115 $\int x^{-1+m}\cos^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{x^m}{2m}-2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\text{Gamma}(m,-2ibx)-2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\text{Gamma}(m,2ibx)$$

```
[Out] 1/2*x^m/m-2^(-2-m)*exp(2*I*a)*x^m*GAMMA(m,-2*I*b*x)/((-I*b*x)^m)-2^(-2-m)*x
^m*GAMMA(m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3393, 3388, 2212}

$$e^{2ia}\left(-2^{-m-2}\right)x^m(-ibx)^{-m}\text{Gamma}(m,-2ibx)-e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\text{Gamma}(m,2ibx)+\frac{x^m}{2m}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^m/(2*m) - (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
- (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int x^{-1+m} \cos^2(a+bx)\,dx &= \int \left(\frac{x^{-1+m}}{2} + \frac{1}{2} x^{-1+m} \cos(2a+2bx)\right) dx \\
&= \frac{x^m}{2m} + \frac{1}{2} \int x^{-1+m} \cos(2a+2bx)\,dx \\
&= \frac{x^m}{2m} + \frac{1}{4} \int e^{-i(2a+2bx)} x^{-1+m}\,dx + \frac{1}{4} \int e^{i(2a+2bx)} x^{-1+m}\,dx \\
&= \frac{x^m}{2m} - 2^{-2-m} e^{2ia} x^m (-ibx)^{-m} \Gamma(m,-2ibx) - 2^{-2-m} e^{-2ia} x^m (ibx)^{-m} \Gamma(m,2ibx)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.91

$$
\frac{1}{4} x^m \left(\frac{2}{m} - 2^{-m} e^{2ia} (-ibx)^{-m} \text{Gamma}(m,-2ibx) - 2^{-m} e^{-2ia} (ibx)^{-m} \text{Gamma}(m,2ibx)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*(2/m - (E^((2*I)*a)*Gamma[m, (-2*I)*b*x])/(2^m*((-I)*b*x)^m) - Gamma[m
, (2*I)*b*x]/(2^m*E^((2*I)*a)*(I*b*x)^m)))/4
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$
\int x^{-1+m} \left(\cos^2 (bx+a)\right) dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(-1+m)*cos(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(m*integrate(x^m*cos(2*b*x + 2*a)/x, x) + x^m)/m
```

Fricas [A]
time = 0.10, size = 64, normalized size = 0.75

$$\frac{4\,bxx^{m-1} + i\,me^{(-(m-1)\log(2i\,b)-2i\,a)}\Gamma(m, 2i\,bx) - i\,me^{(-(m-1)\log(-2i\,b)+2i\,a)}\Gamma(m, -2i\,bx)}{8\,bm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^(m - 1) + I*m*e^(-(m - 1)*log(2*I*b) - 2*I*a)*gamma(m, 2*I*b*x
) - I*m*e^(-(m - 1)*log(-2*I*b) + 2*I*a)*gamma(m, -2*I*b*x))/(b*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1}\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*cos(b*x+a)**2,x)
[Out] Integral(x**(m - 1)*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^(m - 1)*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-1}\cos\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*cos(a + b*x)^2,x)
[Out] int(x^(m - 1)*cos(a + b*x)^2, x)
```

3.116 $\int x^{-2+m} \cos^2(a+bx)\, dx$

Optimal. Leaf size=101

$$-\frac{x^{-1+m}}{2(1-m)}+i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\text{Gamma}(-1+m,-2ibx)-i2^{-1-m}be^{-2ia}x^m(ibx)^{-m}\text{Gamma}(-1+m,2ibx$$

```
[Out] -1/2*x^(-1+m)/(1-m)+I*2^(-1-m)*b*exp(2*I*a)*x^m*GAMMA(-1+m,-2*I*b*x)/((-I*b
*x)^m)-I*2^(-1-m)*b*x^m*GAMMA(-1+m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\text{Gamma}(m-1,-2ibx)-ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\text{Gamma}(m-1,2ibx)-\frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-1 + m)/(1 - m) + (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-2
*I)*b*x])/((-I)*b*x)^m - (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E^(
(2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\cos^2(a+bx)\,dx &= \int\left(\frac{x^{-2+m}}{2}+\frac{1}{2}x^{-2+m}\cos(2a+2bx)\right)\,dx\\ &= -\frac{x^{-1+m}}{2(1-m)}+\frac{1}{2}\int x^{-2+m}\cos(2a+2bx)\,dx\\ &= -\frac{x^{-1+m}}{2(1-m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{-2+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{-2+m}\,dx\\ &= -\frac{x^{-1+m}}{2(1-m)}+i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx)-i2^{-1-m}be^{-2ia}x^m(ib\end{aligned}$$

Mathematica [A]
time = 0.07, size = 91, normalized size = 0.90

$$\frac{1}{2}x^m\left(\frac{1}{(-1+m)x}+i2^{-m}be^{2ia}(-ibx)^{-m}\mathrm{Gamma}(-1+m,-2ibx)-i2^{-m}be^{-2ia}(ibx)^{-m}\mathrm{Gamma}(-1+m,2ibx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*(1/((-1 + m)*x) + (I*b*E^((2*I)*a)*Gamma[-1 + m, (-2*I)*b*x])/(2^m*((-
I)*b*x)^m) - (I*b*Gamma[-1 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*b*x)^m)))/2
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^{-2+m}\left(\cos^2\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(-2+m)*cos(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m - 1)*x*integrate(x^m*cos(2*b*x + 2*a)/x^2, x) + x^m)/((m - 1)*x)
```

Fricas [A]
time = 0.11, size = 77, normalized size = 0.76

$$\frac{4\,bxx^{m-2}+(i\,m-i)e^{\left(-(m-2)\log(2i\,b)-2i\,a\right)}\Gamma(m-1,2i\,bx)+(-i\,m+i)e^{\left(-(m-2)\log(-2i\,b)+2i\,a\right)}\Gamma(m-1,-2i\,bx)}{8\,(bm-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 2) + (I*m - I)*e^(-(m - 2)*log(2*I*b) - 2*I*a)*gamma(m -
1, 2*I*b*x) + (-I*m + I)*e^(-(m - 2)*log(-2*I*b) + 2*I*a)*gamma(m - 1, -2*I
*b*x))/(b*m - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2}\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 2)*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-2}\,{\cos\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)*cos(a + b*x)^2,x)
```

```
[Out] int(x^(m - 2)*cos(a + b*x)^2, x)
```

3.117 $\int x^{-3+m}\cos^2(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{x^{-2+m}}{2(2-m)}+2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\text{Gamma}(-2+m,-2ibx)+2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\text{Gamma}(-2+m,2ibx)$$

```
[Out] -1/2*x^(-2+m)/(2-m)+b^2*exp(2*I*a)*x^m*GAMMA(-2+m,-2*I*b*x)/(2^m)/((-I*b*x)
^m)+b^2*x^m*GAMMA(-2+m,2*I*b*x)/(2^m)/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\text{Gamma}(m-2,-2ibx)+e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\text{Gamma}(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-2 + m)/(2 - m) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int x^{-3+m} \cos^2(a+bx)\,dx &= \int \left(\frac{x^{-3+m}}{2} + \frac{1}{2}x^{-3+m}\cos(2a+2bx)\right)dx \\
&= -\frac{x^{-2+m}}{2(2-m)} + \frac{1}{2}\int x^{-3+m}\cos(2a+2bx)\,dx \\
&= -\frac{x^{-2+m}}{2(2-m)} + \frac{1}{4}\int e^{-i(2a+2bx)}x^{-3+m}\,dx + \frac{1}{4}\int e^{i(2a+2bx)}x^{-3+m}\,dx \\
&= -\frac{x^{-2+m}}{2(2-m)} + 2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx) + 2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\mathrm{I}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 1.00

$$
-\frac{x^{-2+m}}{2(2-m)} + 2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\mathrm{Gamma}(-2+m,-2ibx) + 2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\mathrm{Gamma}(-2+m,2ibx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Cos[a + b*x]^2,x]
[Out] -1/2*x^(-2 + m)/(2 - m) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$
\int x^{-3+m}\left(\cos^2(bx+a)\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*cos(b*x+a)^2,x)
[Out] int(x^(-3+m)*cos(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((m - 2)*x^2*integrate(x^m*cos(2*b*x + 2*a)/x^3, x) + x^m)/((m - 2)*x^2
)
```

Fricas [A]
time = 0.11, size = 77, normalized size = 0.81

$$\frac{4\,bxx^{m-3} + (i\,m - 2i)e^{\left(-(m-3)\log(2i\,b) - 2i\,a\right)}\Gamma(m-2, 2i\,bx) + (-i\,m + 2i)e^{\left(-(m-3)\log(-2i\,b) + 2i\,a\right)}\Gamma(m-2, -2i\,bx)}{8\,(bm - 2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 3) + (I*m - 2*I)*e^(-(m - 3)*log(2*I*b) - 2*I*a)*gamma(m
- 2, 2*I*b*x) + (-I*m + 2*I)*e^(-(m - 3)*log(-2*I*b) + 2*I*a)*gamma(m - 2,
-2*I*b*x))/(b*m - 2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3}\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 3)*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-3}\cos\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)*cos(a + b*x)^2,x)
```

```
[Out] int(x^(m - 3)*cos(a + b*x)^2, x)
```

3.118 $\int (c+dx)^3(a+a\cos(e+fx))\,dx$

Optimal. Leaf size=89

$$\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cos(e+fx)}{f^4}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}-\frac{6ad^2(c+dx)\sin(e+fx)}{f^3}+\frac{a(c+dx)^3\sin(e+fx)}{f}$$

```
[Out] 1/4*a*(d*x+c)^4/d-6*a*d^3*cos(f*x+e)/f^4+3*a*d*(d*x+c)^2*cos(f*x+e)/f^2-6*a
*d^2*(d*x+c)*sin(f*x+e)/f^3+a*(d*x+c)^3*sin(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2718}

$$-\frac{6ad^2(c+dx)\sin(e+fx)}{f^3}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}+\frac{a(c+dx)^3\sin(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cos(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cos[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Cos
[e + f*x])/f^2 - (6*a*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (a*(c + d*x)^3*Sin[
e + f*x])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3(a+a\cos(e+fx))\,dx &= \int \left(a(c+dx)^3+a(c+dx)^3\cos(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^4}{4d}+a\int (c+dx)^3\cos(e+fx)\,dx \\
&= \frac{a(c+dx)^4}{4d}+\frac{a(c+dx)^3\sin(e+fx)}{f}-\frac{(3ad)\int (c+dx)^2\sin(e+fx)\,d \cdots}{f} \\
&= \frac{a(c+dx)^4}{4d}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}+\frac{a(c+dx)^3\sin(e+fx)}{f}-\frac{(6 \cdots}{} \\
&= \frac{a(c+dx)^4}{4d}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}-\frac{6ad^2(c+dx)\sin(e+fx)}{f^3}+ \cdots \\
&= \frac{a(c+dx)^4}{4d}-\frac{6ad^3\cos(e+fx)}{f^4}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}-\frac{6ad^2(c \cdots}{}
\end{aligned}
$$

Mathematica [A]
time = 0.31, size = 122, normalized size = 1.37

$$
a\left(\frac{1}{4}x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+\frac{3d(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)}{f^4}+\frac{(c+dx)\,(c^2f^2+2cdf^2x+d^2(-6+f^2x^2))\sin(e+fx)}{f^3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Cos[e + f*x]),x]
```

```
[Out] a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (3*d*(c^2*f^2 + 2*c*
d*f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x])/f^4 + ((c + d*x)*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(-6 + f^2*x^2))*Sin[e + f*x])/f^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(87) = 174$.
time = 0.11, size = 476, normalized size = 5.35

method	result
risch	$\frac{a d^3x^4}{4}+ac d^2x^3+\frac{3ad c^2x^2}{2}+a c^3x+\frac{a c^4}{4d}+\frac{3ad(d^2x^2f^2+2cd f^2x+c^2f^2-2d^2)\cos(fx+e)}{f^4}+\frac{a(d^3f^2x^3+ \cdots}{}$
norman	$\frac{\frac{6a c^2d f^2-12a d^3}{f^4}+ac d^2x^3+ac d^2x^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{ac\left(c^2f^2+6d^2\right)x}{f^2}+\frac{ac\left(c^2f^2-6d^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a d^3x^4}{4}+\frac{a d^3x^4\left(\cdots\right.}{} \cdots}{\cdots}$
derivativedivides	$\frac{a c^3\sin(fx+e)-\frac{3a c^2de\sin(fx+e)}{f}+\frac{3a c^2d(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{3ac d^2e^2\sin(fx+e)}{f^2}-\frac{6ac d^2e(\cos(fx+e)+(fx+e)\sin \cdots}{f^2} \cdots}{\cdots}$
default	$\frac{a c^3\sin(fx+e)-\frac{3a c^2de\sin(fx+e)}{f}+\frac{3a c^2d(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{3ac d^2e^2\sin(fx+e)}{f^2}-\frac{6ac d^2e(\cos(fx+e)+(fx+e)\sin \cdots}{f^2} \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*cos(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^3*sin(f*x+e)-3*a/f*c^2*d*e*sin(f*x+e)+3*a/f*c^2*d*(cos(f*x+e)+(f*x
+e)*sin(f*x+e))+3*a/f^2*c*d^2*e^2*sin(f*x+e)-6*a/f^2*c*d^2*e*(cos(f*x+e)+(f
*x+e)*sin(f*x+e))+3*a/f^2*c*d^2*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x+e
)*cos(f*x+e))-a/f^3*d^3*e^3*sin(f*x+e)+3*a/f^3*d^3*e^2*(cos(f*x+e)+(f*x+e)*
sin(f*x+e))-3*a/f^3*d^3*e*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x+e)*cos(
f*x+e))+a/f^3*d^3*((f*x+e)^3*sin(f*x+e)+3*(f*x+e)^2*cos(f*x+e)-6*cos(f*x+e)
-6*(f*x+e)*sin(f*x+e))+a*c^3*(f*x+e)-3*a/f*c^2*d*e*(f*x+e)+3/2*a/f*c^2*d*(f
*x+e)^2+3*a/f^2*c*d^2*e^2*(f*x+e)-3*a/f^2*c*d^2*e*(f*x+e)^2+a/f^2*c*d^2*(f*
x+e)^3-a/f^3*d^3*e^3*(f*x+e)+3/2*a/f^3*d^3*e^2*(f*x+e)^2-a/f^3*d^3*e*(f*x+e
)^3+1/4*a/f^3*d^3*(f*x+e)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(91) = 182$.
time = 0.34, size = 492, normalized size = 5.53

$$\frac{4\,(fx+e)ac^3+\frac{(fx+e)^4ad^3}{f^3}+\frac{4\,(fx+e)^3acd^2}{f^2}+\frac{6\,(fx+e)^2ac^2d}{f}-\frac{4\,(fx+e)^3ad^3e}{f^3}-\frac{12\,(fx+e)^2acd^2e}{f^2}-\frac{12\,(fx+e)ac^2de}{f}+4\,ac^3\sin(fx+e)-\frac{12\,ac^2de\sin(fx+e)}{f}+\frac{12\,((fx+e)\sin(fx+e)+\cos(fx+e))ac^2d}{f}+\frac{6\,(fx+e)^2ad^3e^2}{f^3}+\frac{12\,(fx+e)acd^2e^2}{f^2}-\frac{24\,((fx+e)\sin(fx+e)+\cos(fx+e))acd^2e}{f^2}+\frac{12\,acd^2e^2\sin(fx+e)}{f^2}+\frac{12\left(2\,(fx+e)\cos(fx+e)+\left((fx+e)^2-2\right)\sin(fx+e)\right)acd^2}{f^2}-\frac{4\,(fx+e)ad^3e^3}{f^3}+\frac{12\,((fx+e)\sin(fx+e)+\cos(fx+e))ad^3e^2}{f^3}-\frac{12\left(2\,(fx+e)\cos(fx+e)+\left((fx+e)^2-2\right)\sin(fx+e)\right)ad^3e}{f^3}-\frac{4\,ad^3e^3\sin(fx+e)}{f^3}+\frac{4\left(3\left((fx+e)^2-2\right)\cos(fx+e)+\left((fx+e)^3-6\,fx-6\,e\right)\sin(fx+e)\right)ad^3}{f^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 + 4*(f*x + e)^3*a*c*d^2/f^2
+ 6*(f*x + e)^2*a*c^2*d/f - 4*(f*x + e)^3*a*d^3*e/f^3 - 12*(f*x + e)^2*a*c*
d^2*e/f^2 - 12*(f*x + e)*a*c^2*d*e/f + 4*a*c^3*sin(f*x + e) - 12*a*c^2*d*e*
sin(f*x + e)/f + 12*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*c^2*d/f + 6*(
f*x + e)^2*a*d^3*e^2/f^3 + 12*(f*x + e)*a*c*d^2*e^2/f^2 - 24*((f*x + e)*sin
(f*x + e) + cos(f*x + e))*a*c*d^2*e/f^2 + 12*a*c*d^2*e^2*sin(f*x + e)/f^2 +
 12*(2*(f*x + e)*cos(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x + e))*a*c*d^2/f^2
 - 4*(f*x + e)*a*d^3*e^3/f^3 + 12*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a
*d^3*e^2/f^3 - 12*(2*(f*x + e)*cos(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x + e
))*a*d^3*e/f^3 - 4*a*d^3*e^3*sin(f*x + e)/f^3 + 4*(3*((f*x + e)^2 - 2)*cos(
f*x + e) + ((f*x + e)^3 - 6*f*x - 6*e)*sin(f*x + e))*a*d^3/f^3)/f
```

Fricas [A]
time = 0.40, size = 170, normalized size = 1.91

$$\frac{ad^3f^4x^4+4\,acd^2f^4x^3+6\,ac^2df^4x^2+4\,ac^3f^4x+12\,(ad^3f^2x^2+2\,acd^2f^2x+ac^2df^2-2\,ad^3)\cos(fx+e)+4\,(ad^3f^3x^3+3\,acd^2f^3x^2+ac^3f^3-6\,acd^2f+3\,(ac^2df^3-2\,ad^3f)x)\sin(fx+e)}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
+ 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a*d^3)*cos(f*x + e)
 + 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 - 6*a*c*d^2*f + 3*(a*c^
2*d*f^3 - 2*a*d^3*f)*x)*sin(f*x + e))/f^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(88) = 176.
time = 0.25, size = 264, normalized size = 2.97

$$\begin{cases} ac^3x + \frac{ac^3\sin(e+fx)}{f} + \frac{3ac^2dx^2}{2} + \frac{3ac^2dx\sin(e+fx)}{f} + \frac{3ac^2d\cos(e+fx)}{f^2} + acd^2x^3 + \frac{3acd^2x^2\sin(e+fx)}{f} + \frac{6acd^2x\cos(e+fx)}{f^2} - \frac{6acd^2\sin(e+fx)}{f^3} + \frac{ad^3x^4}{4} + \frac{ad^3x^3\sin(e+fx)}{f} + \frac{3ad^3x^2\cos(e+fx)}{f^2} - \frac{6ad^3x\sin(e+fx)}{f^3} - \frac{6ad^3\cos(e+fx)}{f^4} & \text{for } f \neq 0 \\ (a\cos(e) + a)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + a*c**3*sin(e + f*x)/f + 3*a*c**2*d*x**2/2 + 3*a*c**2*
d*x*sin(e + f*x)/f + 3*a*c**2*d*cos(e + f*x)/f**2 + a*c*d**2*x**3 + 3*a*c*d
**2*x**2*sin(e + f*x)/f + 6*a*c*d**2*x*cos(e + f*x)/f**2 - 6*a*c*d**2*sin(e
 + f*x)/f**3 + a*d**3*x**4/4 + a*d**3*x**3*sin(e + f*x)/f + 3*a*d**3*x**2*c
os(e + f*x)/f**2 - 6*a*d**3*x*sin(e + f*x)/f**3 - 6*a*d**3*cos(e + f*x)/f**
4, Ne(f, 0)), ((a*cos(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**
3*x**4/4), True))
```

Giac [A]
time = 0.42, size = 156, normalized size = 1.75

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3\,(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 - 2\,ad^3)\cos(fx+e)}{f^4} + \frac{(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x + ac^3f^3 - 6\,ad^3fx - 6\,acd^2f)\sin(fx+e)}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3*(a*d^3*f^2*x^2
+ 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a*d^3)*cos(f*x + e)/f^4 + (a*d^3*f^3*x^
3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + a*c^3*f^3 - 6*a*d^3*f*x - 6*a*c*d
^2*f)*sin(f*x + e)/f^4
```

Mupad [B]
time = 0.26, size = 189, normalized size = 2.12

$$\frac{\sin(e+f\,x)\,\left(a\,c^3\,f^2-6\,a\,c\,d^2\right)}{f^3}-\frac{3\,\cos(e+f\,x)\,\left(2\,a\,d^3-a\,c^2\,d\,f^2\right)}{f^4}+\frac{a\,d^3\,x^4}{4}+a\,c^3\,x-\frac{3\,x\,\sin(e+f\,x)\,\left(2\,a\,d^3-a\,c^2\,d\,f^2\right)}{f^3}+\frac{3\,a\,c^2\,d\,x^2}{2}+a\,c\,d^2\,x^3+\frac{3\,a\,d^3\,x^2\,\cos(e+f\,x)}{f^2}+\frac{a\,d^3\,x^3\,\sin(e+f\,x)}{f}+\frac{6\,a\,c\,d^2\,x\,\cos(e+f\,x)}{f^2}+\frac{3\,a\,c\,d^2\,x^2\,\sin(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))*(c + d*x)^3,x)
```

```
[Out] (sin(e + f*x)*(a*c^3*f^2 - 6*a*c*d^2))/f^3 - (3*cos(e + f*x)*(2*a*d^3 - a*c
^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x - (3*x*sin(e + f*x)*(2*a*d^3 - a*c
^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (3*a*d^3*x^2*cos(e + f*x
))/f^2 + (a*d^3*x^3*sin(e + f*x))/f + (6*a*c*d^2*x*cos(e + f*x))/f^2 + (3*a
*c*d^2*x^2*sin(e + f*x))/f
```

3.119 $\int (c+dx)^2(a+a\cos(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{a(c+dx)^3}{3d}+\frac{2ad(c+dx)\cos(e+fx)}{f^2}-\frac{2ad^2\sin(e+fx)}{f^3}+\frac{a(c+dx)^2\sin(e+fx)}{f}$$

```
[Out] 1/3*a*(d*x+c)^3/d+2*a*d*(d*x+c)*cos(f*x+e)/f^2-2*a*d^2*sin(f*x+e)/f^3+a*(d*
x+c)^2*sin(f*x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2717}

$$\frac{2ad(c+dx)\cos(e+fx)}{f^2}+\frac{a(c+dx)^2\sin(e+fx)}{f}+\frac{a(c+dx)^3}{3d}-\frac{2ad^2\sin(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*a*d*(c + d*x)*Cos[e + f*x])/f^2 - (2*a*d^2*Sin[e
 + f*x])/f^3 + (a*(c + d*x)^2*Sin[e + f*x])/f
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2(a+a\cos(e+fx))\,dx &= \int \left(a(c+dx)^2 + a(c+dx)^2\cos(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^3}{3d} + a\int (c+dx)^2\cos(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d} + \frac{a(c+dx)^2\sin(e+fx)}{f} - \frac{(2ad)\int (c+dx)\sin(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^3}{3d} + \frac{2ad(c+dx)\cos(e+fx)}{f^2} + \frac{a(c+dx)^2\sin(e+fx)}{f} - \frac{(2a \\
&= \frac{a(c+dx)^3}{3d} + \frac{2ad(c+dx)\cos(e+fx)}{f^2} - \frac{2ad^2\sin(e+fx)}{f^3} + \frac{a(c+dx
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 80, normalized size = 1.19

$$
a\left(c^2x + cdx^2 + \frac{d^2x^3}{3} + \frac{2d(c+dx)\cos(e+fx)}{f^2} + \frac{(c^2f^2 + 2cdf^2x + d^2(-2+f^2x^2))\sin(e+fx)}{f^3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Cos[e + f*x]),x]
```

```
[Out] a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 + (2*d*(c + d*x)*Cos[e + f*x])/f^2 + ((c^2
*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(65) = 130$.
time = 0.08, size = 236, normalized size = 3.52

method	result
risch	$\frac{a\,d^2x^3}{3} + acd\,x^2 + a\,c^2x + \frac{a\,c^3}{3d} + \frac{2ad(dx+c)\cos(fx+e)}{f^2} + \frac{a\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\sin(fx+e)}{f^3}$
norman	$\frac{acd\,x^2+\frac{a\left(c^2f^2+2d^2\right)x}{f^2}+acd\,x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{a\left(c^2f^2-2d^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a\,d^2x^3}{3}-\frac{4acd\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a\,d^2x^3(\mathrm{t}}{}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$
derivativedivides	$\frac{a\,c^2\sin(fx+e)-\frac{2acde\sin(fx+e)}{f}+\frac{2acd(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{a\,d^2e^2\sin(fx+e)}{f^2}-\frac{2a\,d^2e(\cos(fx+e)+(fx+e)\sin(fx+e)}{f^2}}{}$
default	$\frac{a\,c^2\sin(fx+e)-\frac{2acde\sin(fx+e)}{f}+\frac{2acd(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{a\,d^2e^2\sin(fx+e)}{f^2}-\frac{2a\,d^2e(\cos(fx+e)+(fx+e)\sin(fx+e)}{f^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*cos(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^2*sin(f*x+e)-2*a/f*c*d*e*sin(f*x+e)+2*a/f*c*d*(cos(f*x+e)+(f*x+e)*
sin(f*x+e))+a/f^2*d^2*e^2*sin(f*x+e)-2*a/f^2*d^2*e*(cos(f*x+e)+(f*x+e)*sin(
f*x+e))+a/f^2*d^2*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x+e)*cos(f*x+e))+
a*c^2*(f*x+e)-2*a/f*c*d*e*(f*x+e)+a/f*c*d*(f*x+e)^2+a/f^2*d^2*e^2*(f*x+e)-a
/f^2*d^2*e*(f*x+e)^2+1/3*a/f^2*d^2*(f*x+e)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(68) = 136$.
time = 0.32, size = 256, normalized size = 3.82

$$\frac{3(fx+e)ac^2 + \frac{(fx+e)^3ad^2}{f^2} + \frac{3(fx+e)^2acd}{f} - \frac{3(fx+e)^2ad^2e}{f^2} - \frac{6(fx+e)acde}{f} + 3ac^2\sin(fx+e) - \frac{6acde\sin(fx+e)}{f} + \frac{6((fx+e)\sin(fx+e)+\cos(fx+e))acd}{f} + \frac{3(fx+e)ad^2e^2}{f^2} - \frac{6((fx+e)\sin(fx+e)+\cos(fx+e))ad^2e}{f^2} + \frac{3ad^2e^2\sin(fx+e)}{f^2} + \frac{3\left(2(fx+e)\cos(fx+e)+\left((fx+e)^2-2\right)\sin(fx+e)\right)ad^2}{f^2}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 3*
(f*x + e)^2*a*d^2*e/f^2 - 6*(f*x + e)*a*c*d*e/f + 3*a*c^2*sin(f*x + e) - 6*
a*c*d*e*sin(f*x + e)/f + 6*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*c*d/f
+ 3*(f*x + e)*a*d^2*e^2/f^2 - 6*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*d
^2*e/f^2 + 3*a*d^2*e^2*sin(f*x + e)/f^2 + 3*(2*(f*x + e)*cos(f*x + e) + ((f
*x + e)^2 - 2)*sin(f*x + e))*a*d^2/f^2)/f
```

Fricas [A]
time = 0.40, size = 104, normalized size = 1.55

$$\frac{ad^2f^3x^3 + 3\,acdf^3x^2 + 3\,ac^2f^3x + 6\left(ad^2fx + acdf\right)\cos(fx+e) + 3\left(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2\right)\sin(fx+e)}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x + 6*(a*d^2*f*x + a*c*d
*f)*cos(f*x + e) + 3*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*
sin(f*x + e))/f^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.
time = 0.14, size = 151, normalized size = 2.25

$$\begin{cases} ac^2x + \frac{ac^2\sin(e+fx)}{f} + acdx^2 + \frac{2acdx\sin(e+fx)}{f} + \frac{2acd\cos(e+fx)}{f^2} + \frac{ad^2x^3}{3} + \frac{ad^2x^2\sin(e+fx)}{f} + \frac{2ad^2x\cos(e+fx)}{f^2} - \frac{2ad^2\sin(e+fx)}{f^3} & \text{for } f \neq 0 \\ (a\cos(e)+a)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c**2*sin(e + f*x)/f + a*c*d*x**2 + 2*a*c*d*x*sin(e
+ f*x)/f + 2*a*c*d*cos(e + f*x)/f**2 + a*d**2*x**3/3 + a*d**2*x**2*sin(e +
```

```
f*x)/f + 2*a*d**2*x*cos(e + f*x)/f**2 - 2*a*d**2*sin(e + f*x)/f**3, Ne(f, 0
)), ((a*cos(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [A]
time = 0.45, size = 94, normalized size = 1.40

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + \frac{2\,(ad^2fx + acdf)\cos{(fx+e)}}{f^3} + \frac{(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2)\sin{(fx+e)}}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e)/
f^3 + (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*sin(f*x + e)/f^
3
```

Mupad [B]
time = 0.29, size = 112, normalized size = 1.67

$$\frac{a\,d^2\,x^3}{3} - \frac{\sin{(e+f\,x)}\,\left(2\,a\,d^2 - a\,c^2\,f^2\right)}{f^3} + a\,c^2\,x + a\,c\,d\,x^2 + \frac{2\,a\,d^2\,x\,\cos{(e+f\,x)}}{f^2} + \frac{a\,d^2\,x^2\,\sin{(e+f\,x)}}{f} + \frac{2\,a\,c\,d\,\cos{(e+f\,x)}}{f^2} + \frac{2\,a\,c\,d\,x\,\sin{(e+f\,x)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))*(c + d*x)^2,x)
```

```
[Out] (a*d^2*x^3)/3 - (sin(e + f*x)*(2*a*d^2 - a*c^2*f^2))/f^3 + a*c^2*x + a*c*d*
x^2 + (2*a*d^2*x*cos(e + f*x))/f^2 + (a*d^2*x^2*sin(e + f*x))/f + (2*a*c*d*
cos(e + f*x))/f^2 + (2*a*c*d*x*sin(e + f*x))/f
```

3.120 $\int (c+dx)(a+a\cos(e+fx))\,dx$

Optimal. Leaf size=44

$$\frac{a(c+dx)^2}{2d}+\frac{ad\cos(e+fx)}{f^2}+\frac{a(c+dx)\sin(e+fx)}{f}$$

```
[Out] 1/2*a*(d*x+c)^2/d+a*d*cos(f*x+e)/f^2+a*(d*x+c)*sin(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3398, 3377, 2718}

$$\frac{a(c+dx)\sin(e+fx)}{f}+\frac{a(c+dx)^2}{2d}+\frac{ad\cos(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) + (a*d*Cos[e + f*x])/f^2 + (a*(c + d*x)*Sin[e + f*x])
/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+a\cos(e+fx))\,dx &= \int (a(c+dx)+a(c+dx)\cos(e+fx))\,dx \\
&= \frac{a(c+dx)^2}{2d}+a\int (c+dx)\cos(e+fx)\,dx \\
&= \frac{a(c+dx)^2}{2d}+\frac{a(c+dx)\sin(e+fx)}{f}-\frac{(ad)\int \sin(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^2}{2d}+\frac{ad\cos(e+fx)}{f^2}+\frac{a(c+dx)\sin(e+fx)}{f}
\end{aligned}
$$

Mathematica [**A**]
time = 0.14, size = 52, normalized size = 1.18

$$
\frac{a(-2(e+fx)(de-2cf-dfx)+4d\cos(e+fx)+4f(c+dx)\sin(e+fx))}{4f^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(-2*(e + f*x)*(d*e - 2*c*f - d*f*x) + 4*d*Cos[e + f*x] + 4*f*(c + d*x)*S
in[e + f*x]))/(4*f^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.
time = 0.05, size = 89, normalized size = 2.02

method	result	siz
risch	$\frac{d a\,x^2}{2}+acx+\frac{ad\cos(fx+e)}{f^2}+\frac{a(dx+c)\sin(fx+e)}{f}$	41
derivativedivides	$\frac{ac\sin(fx+e)-\frac{ade\sin(fx+e)}{f}+\frac{ad(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+ac(fx+e)-\frac{ade(fx+e)}{f}+\frac{ad(fx+e)^2}{2f}}{f}$	89
default	$\frac{ac\sin(fx+e)-\frac{ade\sin(fx+e)}{f}+\frac{ad(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+ac(fx+e)-\frac{ade(fx+e)}{f}+\frac{ad(fx+e)^2}{2f}}{f}$	89
norman	$\frac{acx+acx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{d a\,x^2}{2}-\frac{2da\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{2ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{d a\,x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{2dax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*cos(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c*sin(f*x+e)-a/f*d*e*sin(f*x+e)+a/f*d*(cos(f*x+e)+(f*x+e)*sin(f*x+e)
)+a*c*(f*x+e)-a/f*d*e*(f*x+e)+1/2*a/f*d*(f*x+e)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(44) = 88$.
time = 0.30, size = 101, normalized size = 2.30

$$\frac{2\,(fx+e)ac+\frac{(fx+e)^2ad}{f}-\frac{2\,(fx+e)ade}{f}+2\,ac\sin\left(fx+e\right)-\frac{2\,ade\sin(fx+e)}{f}+\frac{2\,((fx+e)\sin(fx+e)+\cos(fx+e))ad}{f}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*(f*x + e)*a*d*e/f + 2*a*c*sin(
f*x + e) - 2*a*d*e*sin(f*x + e)/f + 2*((f*x + e)*sin(f*x + e) + cos(f*x + e
))*a*d/f)/f
```

Fricas [A]
time = 0.38, size = 53, normalized size = 1.20

$$\frac{adf^2x^2+2\,acf^2x+2\,ad\cos\left(fx+e\right)+2\,(adfx+acf)\sin\left(fx+e\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*a*d*cos(f*x + e) + 2*(a*d*f*x + a*c*f)*s
in(f*x + e))/f^2
```

Sympy [A]
time = 0.10, size = 68, normalized size = 1.55

$$\begin{cases} acx+\frac{ac\sin\left(e+fx\right)}{f}+\frac{adx^2}{2}+\frac{adx\sin\left(e+fx\right)}{f}+\frac{ad\cos\left(e+fx\right)}{f^2} & \text{for } f\neq 0\\ \left(a\cos\left(e\right)+a\right)\left(cx+\frac{dx^2}{2}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*c*sin(e + f*x)/f + a*d*x**2/2 + a*d*x*sin(e + f*x)/f +
 a*d*cos(e + f*x)/f**2, Ne(f, 0)), ((a*cos(e) + a)*(c*x + d*x**2/2), True))
```

Giac [A]
time = 0.42, size = 46, normalized size = 1.05

$$\frac{1}{2}\,adx^2+acx+\frac{ad\cos\left(fx+e\right)}{f^2}+\frac{(adfx+acf)\sin\left(fx+e\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + a*d*cos(f*x + e)/f^2 + (a*d*f*x + a*c*f)*sin(f*x + e)
/f^2
```

Mupad [B]
time = 0.09, size = 52, normalized size = 1.18

$$\frac{\frac{a\,f\,(2\,c\,\sin(e+f\,x)+2\,d\,x\,\sin(e+f\,x))}{2}+a\,d\,\cos\left(e+f\,x\right)}{f^2}+\frac{a\,\left(d\,x^2+2\,c\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))*(c + d*x),x)
```

```
[Out] ((a*f*(2*c*sin(e + f*x) + 2*d*x*sin(e + f*x)))/2 + a*d*cos(e + f*x))/f^2 +
(a*(2*c*x + d*x^2))/2
```

3.121 $\int \frac{a+a\cos(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=65

$$\frac{a\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}-\frac{a\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}$$

```
[Out] a*Ci(c*f/d+f*x)*cos(-e+c*f/d)/d+a*ln(d*x+c)/d+a*Si(c*f/d+f*x)*sin(-e+c*f/d)
/d
```

Rubi [A]
time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3384, 3380, 3383}

$$\frac{a\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d}-\frac{a\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d - (a
*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cos(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{a\cos(e+fx)}{c+dx}\right)dx\\ &= \frac{a\log(c+dx)}{d}+a\int \frac{\cos(e+fx)}{c+dx}\,dx\\ &= \frac{a\log(c+dx)}{d}+\left(a\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx-\left(a\sin\left(e-\frac{cf}{d}\right)\right)\int\\ &= \frac{a\cos\left(e-\frac{cf}{d}\right)\mathrm{Ci}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}-\frac{a\sin\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(\frac{cf}{d}+fx\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.85

$$\frac{a\left(\cos\left(e-\frac{cf}{d}\right)\mathrm{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)+\log(c+dx)-\sin\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])/(c + d*x),x]
```

```
[Out] (a*(Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] + Log[c + d*x] - Sin[e - (c*f
)/d]*SinIntegral[f*(c/d + x)]))/d
```

Maple [A]
time = 0.08, size = 102, normalized size = 1.57

method	result
derivativedivides	$\frac{fa\left(\frac{\mathrm{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{fa\ln(cf-de+d(fx+e))}{d}}{f}$
default	$\frac{fa\left(\frac{\mathrm{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{fa\ln(cf-de+d(fx+e))}{d}}{f}$
risch	$\frac{a\ln(dx+c)}{d}-\frac{a\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{2d}-\frac{a\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,-ifx-ie-\frac{icf-ide}{d}\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(f*a*(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*co
s((c*f-d*e)/d)/d)+f*a*ln(c*f-d*e+d*(f*x+e))/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 182, normalized size = 2.80

$$\frac{\frac{2\,af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}-\frac{\left(f\left(E_1\left(\frac{i(fx+e)d+i\,cf-i\,de}{d}\right)+E_1\left(-\frac{i(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f\left(i\,E_1\left(\frac{i(fx+e)d+i\,cf-i\,de}{d}\right)-i\,E_1\left(-\frac{i(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a}{d}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d - (f*(exp_integral_e(1, (I*(f*x
 + e)*d + I*c*f - I*d*e)/d) + exp_integral_e(1, -(I*(f*x + e)*d + I*c*f - I
*d*e)/d))*cos((c*f - d*e)/d) + f*(I*exp_integral_e(1, (I*(f*x + e)*d + I*c*
f - I*d*e)/d) - I*exp_integral_e(1, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*si
n((c*f - d*e)/d))*a/d)/f
```

Fricas [A]
time = 0.39, size = 95, normalized size = 1.46

$$-\frac{2\,a\sin\left(-\frac{cf-de}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)-\left(a\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+a\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\cos\left(-\frac{cf-de}{d}\right)-2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*sin(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) - (a*cos_integr
al((d*f*x + c*f)/d) + a*cos_integral(-(d*f*x + c*f)/d))*cos(-(c*f - d*e)/d)
 - 2*a*log(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\cos\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(cos(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 692, normalized size = 10.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*real_part(cos_
integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*real_part(cos_inte
gral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*imag_part(cos_integ
ral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*a*sin_integral((d*f*x + c*f)
/d)*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*imag_part(cos_integral(f*x + c*f/d))*
tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*imag_part(cos_integral(-f*x - c*f/d))*tan
(1/2*c*f/d)*tan(1/2*e)^2 - 4*a*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)
*tan(1/2*e)^2 + 2*a*log(abs(d*x + c))*tan(1/2*c*f/d)^2 - a*real_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*c*f/d)^2 - a*real_part(cos_integral(-f*x - c*f
/d))*tan(1/2*c*f/d)^2 + 4*a*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*
f/d)*tan(1/2*e) + 4*a*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*
tan(1/2*e) + 2*a*log(abs(d*x + c))*tan(1/2*e)^2 - a*real_part(cos_integral(
f*x + c*f/d))*tan(1/2*e)^2 - a*real_part(cos_integral(-f*x - c*f/d))*tan(1/
2*e)^2 + 2*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*a*imag
_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 4*a*sin_integral((d*f*x
+ c*f)/d)*tan(1/2*c*f/d) - 2*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*e) + 2*a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) - 4*a*sin_integr
al((d*f*x + c*f)/d)*tan(1/2*e) + 2*a*log(abs(d*x + c)) + a*real_part(cos_in
tegral(f*x + c*f/d)) + a*real_part(cos_integral(-f*x - c*f/d)))/(d*tan(1/2*
c*f/d)^2*tan(1/2*e)^2 + d*tan(1/2*c*f/d)^2 + d*tan(1/2*e)^2 + d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + a\,\cos\left(e + f\,x\right)}{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + a*cos(e + f*x))/(c + d*x), x)
```

3.122 $\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{a}{d(c+dx)}-\frac{a\cos(e+fx)}{d(c+dx)}-\frac{af\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a/d/(d*x+c)-a*cos(f*x+e)/d/(d*x+c)-a*f*cos(-e+c*f/d)*Si(c*f/d+f*x)/d^2+a*f
*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3380, 3383}

$$-\frac{af\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\cos(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) - (a*Cos[e + f*x])/(d*(c + d*x)) - (a*f*CosIntegral[(c*f
)/d + f*x]*Sin[e - (c*f)/d])/d^2 - (a*f*Cos[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{a\cos(e+fx)}{(c+dx)^2}\right)dx\\ &= -\frac{a}{d(c+dx)}+a\int \frac{\cos(e+fx)}{(c+dx)^2}\,dx\\ &= -\frac{a}{d(c+dx)}-\frac{a\cos(e+fx)}{d(c+dx)}-\frac{(af)\int \frac{\sin(e+fx)}{c+dx}\,dx}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{a\cos(e+fx)}{d(c+dx)}-\frac{\left(af\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}-\frac{\left(af\sin\left(e-\right.\right.}{}\\ &= -\frac{a}{d(c+dx)}-\frac{a\cos(e+fx)}{d(c+dx)}-\frac{af\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 78, normalized size = 0.88

$$-\frac{a\left(d(1+\cos(e+fx))+f(c+dx)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+f(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -((a*(d*(1 + Cos[e + f*x]) + f*(c + d*x)*CosIntegral[f*(c/d + x)]*Sin[e - (
c*f)/d] + f*(c + d*x)*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)]))/(d^2*(c +
 d*x)))
```

Maple [A]
time = 0.12, size = 143, normalized size = 1.61

method	result

derivativedivides	$\frac{f^2 a\left(-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-\frac{f^2 a}{(cf-de+d(\ldots}}{f}$
default	$\frac{f^2 a\left(-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-\frac{f^2 a}{(cf-de+d(\ldots}}{f}$
risch	$-\frac{a}{d(dx+c)}+\frac{iaf\, e^{\frac{i(cf-de)}{d}}\,\text{expIntegral}\left(1, ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^2}-\frac{ifa\, e^{-\frac{i(cf-de)}{d}}\,\text{expIntegral}\left(1, -ifx-ie-\frac{icf-ide}{d}\right)}{2d^2}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(f^2*a*(-cos(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c
*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)-f^2*a/(c*f-d*e+d*
(f*x+e))/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.42, size = 208, normalized size = 2.34

$$-\frac{\frac{2\,af^2}{(fx+e)d^2+cdf-d^2e}+\frac{\left(f^2\left(E_2\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+E_2\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)-f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+i\,E_2\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a}{(fx+e)d^2+cdf-d^2e}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*f^2/((f*x + e)*d^2 + c*d*f - d^2*e) + (f^2*(exp_integral_e(2, (I*
(f*x + e)*d + I*c*f - I*d*e)/d) + exp_integral_e(2, -(I*(f*x + e)*d + I*c*f
 - I*d*e)/d))*cos((c*f - d*e)/d) - f^2*(-I*exp_integral_e(2, (I*(f*x + e)*d
 + I*c*f - I*d*e)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d + I*c*f - I*d*e)
/d))*sin((c*f - d*e)/d))*a/((f*x + e)*d^2 + c*d*f - d^2*e))/f
```

Fricas [A]
time = 0.40, size = 137, normalized size = 1.54

$$-\frac{2\,ad\cos\left(fx+e\right)+2\left(adfx+acf\right)\cos\left(-\frac{cf-de}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)+2\,ad+\left(\left(adfx+acf\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+\left(adfx+acf\right)\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cos(f*x + e) + 2*(a*d*f*x + a*c*f)*cos(-(c*f - d*e)/d)*sin_inte
gral((d*f*x + c*f)/d) + 2*a*d + ((a*d*f*x + a*c*f)*cos_integral((d*f*x + c*
f)/d) + (a*d*f*x + a*c*f)*cos_integral(-(d*f*x + c*f)/d))*sin(-(c*f - d*e)/
d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\cos(e+fx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)**2,x)
```

```
[Out] a*(Integral(cos(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**
2 + 2*c*d*x + d**2*x**2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(93) = 186.
time = 0.43, size = 578, normalized size = 6.49

$$\frac{\left((dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\operatorname{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)\sin\left(\frac{cf-de}{d}\right)-cf^3\operatorname{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)\sin\left(\frac{cf-de}{d}\right)+df^2\operatorname{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)e\sin\left(\frac{cf-de}{d}\right)-(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\cos\left(\frac{cf-de}{d}\right)\operatorname{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)+cf^3\cos\left(\frac{cf-de}{d}\right)\operatorname{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)-df^2\cos\left(\frac{cf-de}{d}\right)e\operatorname{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)+df^2\cos\left(\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)}{d}\right)\right)ad^2}{\left((dx+c)d^4\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cd^4f+d^5e\right)f}-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos_integral(-((d*x + c)
*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*sin((c*f - d*e)/d) - c
*f^3*cos_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d
*e)/d)*sin((c*f - d*e)/d) + d*f^2*cos_integral(-((d*x + c)*(c*f/(d*x + c) -
 f - d*e/(d*x + c)) - c*f + d*e)/d)*e*sin((c*f - d*e)/d) - (d*x + c)*(c*f/(
d*x + c) - f - d*e/(d*x + c))*f^2*cos((c*f - d*e)/d)*sin_integral(-((d*x +
c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + c*f^3*cos((c*f - d
*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f +
 d*e)/d) - d*f^2*cos((c*f - d*e)/d)*e*sin_integral(-((d*x + c)*(c*f/(d*x +
c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + d*f^2*cos((d*x + c)*(c*f/(d*x + c
) - f - d*e/(d*x + c))/d))*a*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*d^4*f + d^5*e)*f) - a/((d*x + c)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\cos\left(e+f\,x\right)}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + a*cos(e + f*x))/(c + d*x)^2, x)
```

3.123 $\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$

Optimal. Leaf size=237

$$-\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{12a^2d^3\cos(e+fx)}{f^4}+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2}-\frac{3a^2d^3\cos^2(e+fx)}{8f^4}+3$$

```
[Out] -3/4*a^2*c*d^2*x/f^2-3/8*a^2*d^3*x^2/f^2+3/8*a^2*(d*x+c)^4/d-12*a^2*d^3*cos
(f*x+e)/f^4+6*a^2*d*(d*x+c)^2*cos(f*x+e)/f^2-3/8*a^2*d^3*cos(f*x+e)^2/f^4+3
/4*a^2*d*(d*x+c)^2*cos(f*x+e)^2/f^2-12*a^2*d^2*(d*x+c)*sin(f*x+e)/f^3+2*a^2
*(d*x+c)^3*sin(f*x+e)/f-3/4*a^2*d^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/f^3+1/2*a
^2*(d*x+c)^3*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A]
time = 0.17, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3398, 3377, 2718, 3392, 32, 3391}

$$-\frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3}-\frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2}+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2}+\frac{2a^2(c+dx)^3\sin(e+fx)}{f}+\frac{a^2(c+dx)^3\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2(c+dx)^4}{8d}-\frac{3a^2d^3\cos^2(e+fx)}{8f^4}-\frac{12a^2d^3\cos(e+fx)}{f^4}-\frac{3a^2d^3x^2}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (-3*a^2*c*d^2*x)/(4*f^2) - (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8
*d) - (12*a^2*d^3*Cos[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Cos[e + f*x])/f^
2 - (3*a^2*d^3*Cos[e + f*x]^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Cos[e + f*x]^
2)/(4*f^2) - (12*a^2*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (2*a^2*(c + d*x)^3*S
in[e + f*x])/f - (3*a^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) +
(a^2*(c + d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3(a+a\cos(e+fx))^2\,dx &= \int \left(a^2(c+dx)^3+2a^2(c+dx)^3\cos(e+fx)+a^2(c+dx)^3\cos^2(e+fx\right.\\ &= \frac{a^2(c+dx)^4}{4d}+a^2\int (c+dx)^3\cos^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^3\cos\\ &= \frac{a^2(c+dx)^4}{4d}+\frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)^3\sin(e+fx}{f}\\ &= \frac{3a^2(c+dx)^4}{8d}+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2}-\frac{3a^2d^3\cos^2(e+fx)}{8f^4}+\frac{3}{}\\ &= -\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2}-\\ &= -\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{12a^2d^3\cos(e+fx)}{f^4}+\frac{6a^2d(c}{}\end{aligned}$$

Mathematica [A]
time = 0.88, size = 217, normalized size = 0.92

$$\frac{a^2\left(96d\left(c^2f^2+2cdf^2x+d^2\left(-2+f^2x^2\right)\right)\cos(e+fx)+3d\left(2c^2f^2+4cdf^2x+d^2\left(-1+2f^2x^2\right)\right)\cos(2(e+fx))+2f\left(3f^3x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+16(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(-6+f^2x^2\right)\right)\sin(e+fx)+(c+dx)\left(2c^2f^2+4cdf^2x+d^2\left(-3+2f^2x^2\right)\right)\sin(2(e+fx))\right)\right)}{16f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*(96*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x] + 3*d*
(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 2*f*(3*
f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*(c + d*x)*(c^2*f^2 +
 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Sin[e + f*x] + (c + d*x)*(2*c^2*f^2 + 4*
c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)])))/(16*f^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(223) = 446$.
time = 0.20, size = 1129, normalized size = 4.76

method	result
risch	$\frac{3a^2d^3x^4}{8}+\frac{3a^2c\,d^2x^3}{2}+\frac{9a^2d\,c^2x^2}{4}+\frac{3a^2c^3x}{2}+\frac{3a^2c^4}{8d}+\frac{6a^2d\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\cos(fx+e)}{f^4}+\frac{2a^2(\cdots}{}$
norman	$\frac{\frac{12a^2c^2d\,f^2-24a^2d^3}{f^4}+\frac{3a^2d^3x^4}{8}+\frac{\left(18a^2c^2d\,f^2-45a^2d^3\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f^4}+\frac{3a^2c\,d^2x^3}{2}+\frac{3a^2d^3x^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{3a^2d^3x^4\left(\tan^4\left(\cdots\right.\right.}{8}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*cos(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c^3*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+a^2/f^3*d^3*((f*x+e)
^3*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3/4*(f*x+e)^2*cos(f*x+e)^2-3/2
*(f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3/8*(f*x+e)^2+3/8*sin(f*
x+e)^2-3/8*(f*x+e)^4)+1/4*a^2/f^3*d^3*(f*x+e)^4+2*a^2/f^3*d^3*((f*x+e)^3*si
n(f*x+e)+3*(f*x+e)^2*cos(f*x+e)-6*cos(f*x+e)-6*(f*x+e)*sin(f*x+e))+a^2*c^3*
(f*x+e)+2*a^2*c^3*sin(f*x+e)+3*a^2/f^2*c*d^2*e^2*(1/2*cos(f*x+e)*sin(f*x+e)
+1/2*f*x+1/2*e)-3*a^2/f^2*c*d^2*e*(f*x+e)^2-6*a^2/f^2*c*d^2*e*((f*x+e)*(1/2
*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)-12*a^
2/f^2*c*d^2*e*(cos(f*x+e)+(f*x+e)*sin(f*x+e))+3*a^2/f^2*c*d^2*e^2*(f*x+e)-3
*a^2/f*c^2*d*e*(f*x+e)-6*a^2/f*c^2*d*e*sin(f*x+e)+6*a^2/f^2*c*d^2*e^2*sin(f
*x+e)-3*a^2/f*c^2*d*e*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a^2/f^3*d
^3*e^3*sin(f*x+e)+a^2/f^2*c*d^2*(f*x+e)^3-a^2/f^3*d^3*e^3*(1/2*cos(f*x+e)*s
in(f*x+e)+1/2*f*x+1/2*e)+3/2*a^2/f^3*d^3*e^2*(f*x+e)^2-a^2/f^3*d^3*e*(f*x+e
)^3+3/2*a^2/f*c^2*d*(f*x+e)^2-a^2/f^3*d^3*e^3*(f*x+e)+3*a^2/f^2*c*d^2*((f*x
+e)^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+1/2*(f*x+e)*cos(f*x+e)^2-1/
4*cos(f*x+e)*sin(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+e)^3)+6*a^2/f*c^2*d*(cos(f*x
+e)+(f*x+e)*sin(f*x+e))+6*a^2/f^2*c*d^2*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+
2*(f*x+e)*cos(f*x+e))+3*a^2/f^3*d^3*e^2*((f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)
+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)-3*a^2/f^3*d^3*e*((f*x+e)^2*
(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+1/2*(f*x+e)*cos(f*x+e)^2-1/4*cos(
```

```
f*x+e)*sin(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+e)^3)+6*a^2/f^3*d^3*e^2*(cos(f*x+e
)+(f*x+e)*sin(f*x+e))-6*a^2/f^3*d^3*e*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*
(f*x+e)*cos(f*x+e))+3*a^2/f*c^2*d*((f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f
*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1021 vs. $2(233) = 466$.
time = 0.35, size = 1021, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^3 + 16*(f*x + e)*a^2*c^3 + 4
*(f*x + e)^4*a^2*d^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 + 24*(f*x + e)^2*a^
2*c^2*d/f - 16*(f*x + e)^3*a^2*d^3*e/f^3 - 48*(f*x + e)^2*a^2*c*d^2*e/f^2 -
 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^2*d*e/f - 48*(f*x + e)*a^2*c^2*d
*e/f + 32*a^2*c^3*sin(f*x + e) - 96*a^2*c^2*d*e*sin(f*x + e)/f + 6*(2*(f*x
+ e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*c^2*d/f + 96*
((f*x + e)*sin(f*x + e) + cos(f*x + e))*a^2*c^2*d/f + 24*(f*x + e)^2*a^2*d^
3*e^2/f^3 + 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c*d^2*e^2/f^2 + 48*(f*x
 + e)*a^2*c*d^2*e^2/f^2 - 12*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e)
+ cos(2*f*x + 2*e))*a^2*c*d^2*e/f^2 - 192*((f*x + e)*sin(f*x + e) + cos(f*x
 + e))*a^2*c*d^2*e/f^2 + 96*a^2*c*d^2*e^2*sin(f*x + e)/f^2 + 2*(4*(f*x + e)
^3 + 6*(f*x + e)*cos(2*f*x + 2*e) + 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))
*a^2*c*d^2/f^2 + 96*(2*(f*x + e)*cos(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x +
 e))*a^2*c*d^2/f^2 - 4*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*d^3*e^3/f^3 - 1
6*(f*x + e)*a^2*d^3*e^3/f^3 + 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f*x + 2*
e) + cos(2*f*x + 2*e))*a^2*d^3*e^2/f^3 + 96*((f*x + e)*sin(f*x + e) + cos(f
*x + e))*a^2*d^3*e^2/f^3 - 2*(4*(f*x + e)^3 + 6*(f*x + e)*cos(2*f*x + 2*e)
+ 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^3*e/f^3 - 96*(2*(f*x + e)*c
os(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x + e))*a^2*d^3*e/f^3 - 32*a^2*d^3*e^
3*sin(f*x + e)/f^3 + (2*(f*x + e)^4 + 3*(2*(f*x + e)^2 - 1)*cos(2*f*x + 2*e
) + 2*(2*(f*x + e)^3 - 3*f*x - 3*e)*sin(2*f*x + 2*e))*a^2*d^3/f^3 + 32*(3*(
(f*x + e)^2 - 2)*cos(f*x + e) + ((f*x + e)^3 - 6*f*x - 6*e)*sin(f*x + e))*a
^2*d^3/f^3)/f
```

Fricas [A]
time = 0.39, size = 373, normalized size = 1.57

$$\frac{3\,a^{2}d^{3}f^{4}x^{4}+12\,a^{2}cd^{2}f^{4}x^{3}+3\,(6\,a^{2}c^{2}df^{4}-a^{2}d^{3}f^{2})x^{2}+3\,(2\,a^{2}d^{3}f^{2}x^{2}+4\,a^{2}cd^{2}f^{2}x+2\,a^{2}c^{2}df^{2}-a^{2}d^{3})\cos(fx+e)^{2}+6\,(2\,a^{2}c^{3}f^{4}-a^{2}cd^{2}f^{2})x+48\,(a^{2}d^{3}f^{2}x^{2}+2\,a^{2}cd^{2}f^{2}x+a^{2}c^{2}df^{2}-2\,a^{2}d^{3})\cos(fx+e)+2\,(8\,a^{2}d^{3}f^{3}x^{3}+24\,a^{2}cd^{2}f^{3}x^{2}+8\,a^{2}c^{3}f^{3}-48\,a^{2}cd^{2}f+24\,(a^{2}c^{2}df^{3}-2\,a^{2}d^{3}f)x+(2\,a^{2}d^{3}f^{3}x^{3}+6\,a^{2}cd^{2}f^{3}x^{2}+2\,a^{2}c^{3}f^{3}-3\,a^{2}cd^{2}f+3\,(2\,a^{2}c^{2}df^{3}-a^{2}d^{3}f)x)\cos(fx+e))\sin(fx+e)}{8\,f^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^2*d^3*f^4*x^4 + 12*a^2*c*d^2*f^4*x^3 + 3*(6*a^2*c^2*d*f^4 - a^2*d^
3*f^2)*x^2 + 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a
^2*d^3)*cos(f*x + e)^2 + 6*(2*a^2*c^3*f^4 - a^2*c*d^2*f^2)*x + 48*(a^2*d^3*
f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2 - 2*a^2*d^3)*cos(f*x + e) + 2*(
8*a^2*d^3*f^3*x^3 + 24*a^2*c*d^2*f^3*x^2 + 8*a^2*c^3*f^3 - 48*a^2*c*d^2*f +
 24*(a^2*c^2*d*f^3 - 2*a^2*d^3*f)*x + (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*
x^2 + 2*a^2*c^3*f^3 - 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 - a^2*d^3*f)*x)*co
s(f*x + e))*sin(f*x + e))/f^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(243) = 486$.
time = 0.44, size = 779, normalized size = 3.29

$$\begin{cases} \frac{a^2c^3x\sin^2(e+fx)}{2} + \frac{a^2c^3x\cos^2(e+fx)}{2} + a^2c^3x + \frac{a^2c^3\sin(e+fx)\cos(e+fx)}{2f} + \frac{2a^2c^3\sin(e+fx)}{f} + \frac{3a^2c^2dx^2\sin^2(e+fx)}{4} + \frac{3a^2c^2dx^2\cos^2(e+fx)}{4} + \frac{3a^2c^2dx^2}{2} + \frac{3a^2c^2dx\sin(e+fx)\cos(e+fx)}{2f} + \frac{6a^2c^2dx\sin(e+fx)}{f} - \frac{3a^2c^2d\sin^2(e+fx)}{4f^2} + \frac{6a^2c^2d\cos(e+fx)}{f^2} + \frac{a^2cd^2x^3\sin^2(e+fx)}{2} + \frac{a^2cd^2x^3\cos^2(e+fx)}{2} + a^2cd^2x^3 + \frac{3a^2cd^2x^2\sin(e+fx)\cos(e+fx)}{2f} + \frac{6a^2cd^2x^2\sin(e+fx)}{f} - \frac{3a^2cd^2x\sin^2(e+fx)}{4f^2} + \frac{3a^2cd^2x\cos^2(e+fx)}{4f^2} + \frac{12a^2cd^2x\cos(e+fx)}{f^2} - \frac{3a^2cd^2\sin(e+fx)\cos(e+fx)}{4f^3} - \frac{12a^2cd^2\sin(e+fx)}{f^3} + \frac{a^2d^3x^4\sin^2(e+fx)}{8} + \frac{a^2d^3x^4\cos^2(e+fx)}{8} + \frac{a^2d^3x^4}{4} + \frac{a^2d^3x^3\sin(e+fx)\cos(e+fx)}{2f} + \frac{2a^2d^3x^3\sin(e+fx)}{f} - \frac{3a^2d^3x^2\sin^2(e+fx)}{8f^2} + \frac{3a^2d^3x^2\cos^2(e+fx)}{8f^2} + \frac{6a^2d^3x^2\cos(e+fx)}{f^2} - \frac{3a^2d^3x\sin(e+fx)\cos(e+fx)}{4f^3} - \frac{12a^2d^3x\sin(e+fx)}{f^3} + \frac{3a^2d^3\sin^2(e+fx)}{8f^4} - \frac{12a^2d^3\cos(e+fx)}{f^4} & \text{for}\: f \neq 0 \\ (a\cos(e)+a)^2\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
a**2*c**3*x + a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c**3*sin(e
 + f*x)/f + 3*a**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*a**2*c**2*d*x**2*cos(e
 + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 + 3*a**2*c**2*d*x*sin(e + f*x)*cos(e +
f*x)/(2*f) + 6*a**2*c**2*d*x*sin(e + f*x)/f - 3*a**2*c**2*d*sin(e + f*x)**2
/(4*f**2) + 6*a**2*c**2*d*cos(e + f*x)/f**2 + a**2*c*d**2*x**3*sin(e + f*x)
**2/2 + a**2*c*d**2*x**3*cos(e + f*x)**2/2 + a**2*c*d**2*x**3 + 3*a**2*c*d*
*2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 6*a**2*c*d**2*x**2*sin(e + f*x)/f
 - 3*a**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) + 3*a**2*c*d**2*x*cos(e + f*x)*
*2/(4*f**2) + 12*a**2*c*d**2*x*cos(e + f*x)/f**2 - 3*a**2*c*d**2*sin(e + f*
x)*cos(e + f*x)/(4*f**3) - 12*a**2*c*d**2*sin(e + f*x)/f**3 + a**2*d**3*x**
4*sin(e + f*x)**2/8 + a**2*d**3*x**4*cos(e + f*x)**2/8 + a**2*d**3*x**4/4 +
 a**2*d**3*x**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*d**3*x**3*sin(e +
f*x)/f - 3*a**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) + 3*a**2*d**3*x**2*cos(e
 + f*x)**2/(8*f**2) + 6*a**2*d**3*x**2*cos(e + f*x)/f**2 - 3*a**2*d**3*x*si
n(e + f*x)*cos(e + f*x)/(4*f**3) - 12*a**2*d**3*x*sin(e + f*x)/f**3 + 3*a**
2*d**3*sin(e + f*x)**2/(8*f**4) - 12*a**2*d**3*cos(e + f*x)/f**4, Ne(f, 0))
, ((a*cos(e) + a)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)
, True))
```

Giac [**A**]
time = 0.47, size = 339, normalized size = 1.43

$$\frac{3}{8}a^2d^3x^4 + \frac{3}{2}a^2cd^2x^3 + \frac{9}{4}a^2c^2dx^2 + \frac{3}{2}a^2c^3x + \frac{3\,(2\,a^2d^3f^2x^2 + 4\,a^2cd^2f^2x + 2\,a^2c^2df^2 - a^2d^3)\cos(2\,fx + 2\,e)}{16\,f^4} + \frac{6\,(a^2d^3f^2x^2 + 2\,a^2cd^2f^2x + a^2c^2df^2 - 2\,a^2d^3)\cos(fx + e)}{f^4} + \frac{(2\,a^2d^3f^3x^3 + 6\,a^2cd^2f^3x^2 + 6\,a^2c^2df^3x + 2\,a^2c^3f^3 - 3\,a^2d^3fx - 3\,a^2cd^2f)\sin(2\,fx + 2\,e)}{8\,f^4} + \frac{2\,(a^2d^3f^3x^3 + 3\,a^2cd^2f^3x^2 + 3\,a^2c^2df^3x + a^2c^3f^3 - 6\,a^2d^3fx - 6\,a^2cd^2f)\sin(fx + e)}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x + 3
/16*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a^2*d^3)*cos
(2*f*x + 2*e)/f^4 + 6*(a^2*d^3*f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2
- 2*a^2*d^3)*cos(f*x + e)/f^4 + 1/8*(2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^
2 + 6*a^2*c^2*d*f^3*x + 2*a^2*c^3*f^3 - 3*a^2*d^3*f*x - 3*a^2*c*d^2*f)*sin(
2*f*x + 2*e)/f^4 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f
^3*x + a^2*c^3*f^3 - 6*a^2*d^3*f*x - 6*a^2*c*d^2*f)*sin(f*x + e)/f^4
```

Mupad [B]
time = 0.91, size = 452, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))^2*(c + d*x)^3,x)
```

```
[Out] (16*a^2*c^3*f^3*sin(e + f*x) - (3*a^2*d^3*cos(2*e + 2*f*x))/2 - 96*a^2*d^3*
cos(e + f*x) + 12*a^2*c^3*f^4*x + 2*a^2*c^3*f^3*sin(2*e + 2*f*x) + 3*a^2*d^
3*f^4*x^4 - 96*a^2*c*d^2*f*sin(e + f*x) - 96*a^2*d^3*f*x*sin(e + f*x) + 3*a
^2*d^3*f^2*x^2*cos(2*e + 2*f*x) + 2*a^2*d^3*f^3*x^3*sin(2*e + 2*f*x) + 48*a
^2*c^2*d*f^2*cos(e + f*x) - 3*a^2*c*d^2*f*sin(2*e + 2*f*x) - 3*a^2*d^3*f*x*
sin(2*e + 2*f*x) + 3*a^2*c^2*d*f^2*cos(2*e + 2*f*x) + 18*a^2*c^2*d*f^4*x^2
+ 12*a^2*c*d^2*f^4*x^3 + 48*a^2*d^3*f^2*x^2*cos(e + f*x) + 16*a^2*d^3*f^3*x
^3*sin(e + f*x) + 6*a^2*c*d^2*f^2*x*cos(2*e + 2*f*x) + 6*a^2*c^2*d*f^3*x*si
n(2*e + 2*f*x) + 48*a^2*c*d^2*f^3*x^2*sin(e + f*x) + 6*a^2*c*d^2*f^3*x^2*si
n(2*e + 2*f*x) + 96*a^2*c*d^2*f^2*x*cos(e + f*x) + 48*a^2*c^2*d*f^3*x*sin(e
 + f*x))/(8*f^4)
```

3.124 $\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}-\frac{4a^2d^2\sin(e+fx)}{f^3}+2a^2(c+dx$$

```
[Out] -1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d+4*a^2*d*(d*x+c)*cos(f*x+e)/f^2+1/2*a
^2*d*(d*x+c)*cos(f*x+e)^2/f^2-4*a^2*d^2*sin(f*x+e)/f^3+2*a^2*(d*x+c)^2*sin(
f*x+e)/f-1/4*a^2*d^2*cos(f*x+e)*sin(f*x+e)/f^3+1/2*a^2*(d*x+c)^2*cos(f*x+e)
*sin(f*x+e)/f
```

Rubi [A]
time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3377, 2717, 3392, 32, 2715, 8}

$$\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{2a^2(c+dx)^2\sin(e+fx)}{f}+\frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}-\frac{4a^2d^2\sin(e+fx)}{f^3}-\frac{a^2d^2\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{a^2d^2x}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] -1/4*(a^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d*(c + d*x)*Cos[e +
 f*x])/f^2 + (a^2*d*(c + d*x)*Cos[e + f*x]^2)/(2*f^2) - (4*a^2*d^2*Sin[e +
f*x])/f^3 + (2*a^2*(c + d*x)^2*Sin[e + f*x])/f - (a^2*d^2*Cos[e + f*x]*Sin[
e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+a\cos(e+fx))^2\,dx &= \int \left(a^2(c+dx)^2+2a^2(c+dx)^2\cos(e+fx)+a^2(c+dx)^2\cos^2(e+fx\right. \\
&= \frac{a^2(c+dx)^3}{3d}+a^2\int (c+dx)^2\cos^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^2\cos \\
&= \frac{a^2(c+dx)^3}{3d}+\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}+\frac{2a^2(c+dx)^2\sin(e+fx)}{f} \\
&= \frac{a^2(c+dx)^3}{2d}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2} \\
&= -\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{a^2d(c+dx)\cos}{2f^2}
\end{aligned}$$

Mathematica [A]
time = 0.41, size = 193, normalized size = 1.15

$$\frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3+32df(c+dx)\cos(e+fx)+2df(c+dx)\cos(2(e+fx))-32d^2\sin(e+fx)+16c^2f^2\sin(e+fx)+32cdf^2x\sin(e+fx)+16d^2f^2x^2\sin(e+fx)-d^2\sin(2(e+fx))+2c^2f^2\sin(2(e+fx))+4cdf^2x\sin(2(e+fx))+2d^2f^2x^2\sin(2(e+fx)))}{8f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 + 32*d*f*(c + d*x)*Cos[
e + f*x] + 2*d*f*(c + d*x)*Cos[2*(e + f*x)] - 32*d^2*Sin[e + f*x] + 16*c^2*
f^2*Sin[e + f*x] + 32*c*d*f^2*x*Sin[e + f*x] + 16*d^2*f^2*x^2*Sin[e + f*x]
- d^2*Sin[2*(e + f*x)] + 2*c^2*f^2*Sin[2*(e + f*x)] + 4*c*d*f^2*x*Sin[2*(e
+ f*x)] + 2*d^2*f^2*x^2*Sin[2*(e + f*x)]))/(8*f^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(158) = 316$.
time = 0.13, size = 564, normalized size = 3.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*cos(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a^2/f*c*d*e*(1/2*c
os(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2*a^2/f*c*d*((f*x+e)*(1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)+a^2/f^2*d^2*e^2*(1/
2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a^2/f^2*d^2*e*((f*x+e)*(1/2*cos(f*
x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)+a^2/f^2*d^2*
((f*x+e)^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+1/2*(f*x+e)*cos(f*x+e)
^2-1/4*cos(f*x+e)*sin(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+e)^3)+2*a^2*c^2*sin(f*x
+e)-4*a^2/f*c*d*e*sin(f*x+e)+4*a^2/f*c*d*(cos(f*x+e)+(f*x+e)*sin(f*x+e))+2*
a^2/f^2*d^2*e^2*sin(f*x+e)-4*a^2/f^2*d^2*e*(cos(f*x+e)+(f*x+e)*sin(f*x+e))+
2*a^2/f^2*d^2*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x+e)*cos(f*x+e))+a^2*
c^2*(f*x+e)-2*a^2/f*c*d*e*(f*x+e)+a^2/f*c*d*(f*x+e)^2+a^2/f^2*d^2*e^2*(f*x+
e)-a^2/f^2*d^2*e*(f*x+e)^2+1/3*a^2/f^2*d^2*(f*x+e)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(166) = 332$.
time = 0.32, size = 535, normalized size = 3.18

$$\frac{6\,(2\,fx+2\,e+\sin(2\,fx+2\,e))a^2c^2+24\,(fx+e)a^2c^2+\frac{8\,(fx+e)^3a^2d^2}{f^2}+\frac{24\,(fx+e)^2a^2cd}{f}-\frac{24\,(fx+e)^2a^2d^2e}{f^2}-\frac{12\,(2\,fx+2\,e+\sin(2\,fx+2\,e))a^2cde}{f}-\frac{48\,(fx+e)a^2cde}{f}+48\,a^2c^2\sin(fx+e)-\frac{96\,a^2cde\sin(fx+e)}{f}+\frac{6\left(2\,(fx+e)^2+2\,(fx+e)\sin(2\,fx+2\,e)+\cos(2\,fx+2\,e)\right)a^2cd}{f}+\frac{96\,((fx+e)\sin(fx+e)+\cos(fx+e))a^2cd}{f}+\frac{6\,(2\,fx+2\,e+\sin(2\,fx+2\,e))a^2d^2e^2}{f^2}+\frac{24\,(fx+e)a^2d^2e^2}{f^2}-\frac{6\left(2\,(fx+e)^2+2\,(fx+e)\sin(2\,fx+2\,e)+\cos(2\,fx+2\,e)\right)a^2d^2e}{f^2}-\frac{96\,((fx+e)\sin(fx+e)+\cos(fx+e))a^2d^2e}{f^2}+\frac{48\,a^2d^2e^2\sin(fx+e)}{f^2}+\frac{\left(4\,(fx+e)^3+6\,(fx+e)\cos(2\,fx+2\,e)+3\left(2\,(fx+e)^2-1\right)\sin(2\,fx+2\,e)\right)a^2d^2}{f^2}+\frac{48\left(2\,(fx+e)\cos(fx+e)+\left((fx+e)^2-2\right)\sin(fx+e)\right)a^2d^2}{f^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^2 + 24*(f*x + e)*a^2*c^2 + 8
*(f*x + e)^3*a^2*d^2/f^2 + 24*(f*x + e)^2*a^2*c*d/f - 24*(f*x + e)^2*a^2*d^
2*e/f^2 - 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c*d*e/f - 48*(f*x + e)*a^
2*c*d*e/f + 48*a^2*c^2*sin(f*x + e) - 96*a^2*c*d*e*sin(f*x + e)/f + 6*(2*(f
*x + e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*c*d/f + 96
*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a^2*c*d/f + 6*(2*f*x + 2*e + sin(2
*f*x + 2*e))*a^2*d^2*e^2/f^2 + 24*(f*x + e)*a^2*d^2*e^2/f^2 - 6*(2*(f*x + e
)^2 + 2*(f*x + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d^2*e/f^2 - 96*(
(f*x + e)*sin(f*x + e) + cos(f*x + e))*a^2*d^2*e/f^2 + 48*a^2*d^2*e^2*sin(f
*x + e)/f^2 + (4*(f*x + e)^3 + 6*(f*x + e)*cos(2*f*x + 2*e) + 3*(2*(f*x + e
)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^2/f^2 + 48*(2*(f*x + e)*cos(f*x + e) + ((f
*x + e)^2 - 2)*sin(f*x + e))*a^2*d^2/f^2)/f
```

Fricas [A]
time = 0.40, size = 216, normalized size = 1.29

$$\frac{2\,a^{2}d^{2}f^{3}x^{3}+6\,a^{2}cdf^{3}x^{2}+2\,(a^{2}d^{2}fx+a^{2}cdf)\cos\left(fx+e\right)^{2}+(6\,a^{2}c^{2}f^{3}-a^{2}d^{2}f)x+16\,(a^{2}d^{2}fx+a^{2}cdf)\cos\left(fx+e\right)+(8\,a^{2}d^{2}f^{2}x^{2}+16\,a^{2}cdf^{2}x+8\,a^{2}c^{2}f^{2}-16\,a^{2}d^{2}+(2\,a^{2}d^{2}f^{2}x^{2}+4\,a^{2}cdf^{2}x+2\,a^{2}c^{2}f^{2}-a^{2}d^{2})\cos\left(fx+e\right))\sin\left(fx+e\right)}{4\,f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*co
s(f*x + e)^2 + (6*a^2*c^2*f^3 - a^2*d^2*f)*x + 16*(a^2*d^2*f*x + a^2*c*d*f)
*cos(f*x + e) + (8*a^2*d^2*f^2*x^2 + 16*a^2*c*d*f^2*x + 8*a^2*c^2*f^2 - 16*
a^2*d^2 + (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*c
os(f*x + e))*sin(f*x + e))/f^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(163) = 326$.
time = 0.30, size = 456, normalized size = 2.71

$$\begin{cases} \frac{a^{2}c^{2}x\sin^{2}(e+fx)}{2}+\frac{a^{2}c^{2}x\cos^{2}(e+fx)}{2}+a^{2}c^{2}x+\frac{a^{2}c^{2}\sin(e+fx)\cos(e+fx)}{2f}+\frac{2a^{2}c^{2}\sin(e+fx)}{f}+\frac{a^{2}cdx^{2}\sin^{2}(e+fx)}{2}+\frac{a^{2}cdx^{2}\cos^{2}(e+fx)}{2}+a^{2}cdx^{2}+\frac{a^{2}cdx\sin(e+fx)\cos(e+fx)}{f}+\frac{4a^{2}cdx\sin(e+fx)}{f}-\frac{a^{2}cd\sin^{2}(e+fx)}{2f^{2}}+\frac{4a^{2}cd\cos(e+fx)}{f^{2}}+\frac{a^{2}d^{2}x^{3}\sin^{2}(e+fx)}{6}+\frac{a^{2}d^{2}x^{3}\cos^{2}(e+fx)}{6}+\frac{a^{2}d^{2}x^{3}}{3}+\frac{a^{2}d^{2}x^{2}\sin(e+fx)\cos(e+fx)}{2f}+\frac{2a^{2}d^{2}x^{2}\sin(e+fx)}{f}-\frac{a^{2}d^{2}x\sin^{2}(e+fx)}{4f^{2}}+\frac{a^{2}d^{2}x\cos^{2}(e+fx)}{4f^{2}}+\frac{4a^{2}d^{2}x\cos(e+fx)}{f^{2}}-\frac{a^{2}d^{2}\sin(e+fx)\cos(e+fx)}{4f^{3}}-\frac{4a^{2}d^{2}\sin(e+fx)}{f^{3}} & \text{for}\: f\neq 0 \\ (a\cos(e)+a)^{2}\left(c^{2}x+cdx^{2}+\frac{d^{2}x^{3}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
a**2*c**2*x + a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c**2*sin(e
 + f*x)/f + a**2*c*d*x**2*sin(e + f*x)**2/2 + a**2*c*d*x**2*cos(e + f*x)**2
/2 + a**2*c*d*x**2 + a**2*c*d*x*sin(e + f*x)*cos(e + f*x)/f + 4*a**2*c*d*x*
sin(e + f*x)/f - a**2*c*d*sin(e + f*x)**2/(2*f**2) + 4*a**2*c*d*cos(e + f*x
)/f**2 + a**2*d**2*x**3*sin(e + f*x)**2/6 + a**2*d**2*x**3*cos(e + f*x)**2/
6 + a**2*d**2*x**3/3 + a**2*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a
**2*d**2*x**2*sin(e + f*x)/f - a**2*d**2*x*sin(e + f*x)**2/(4*f**2) + a**2*
d**2*x*cos(e + f*x)**2/(4*f**2) + 4*a**2*d**2*x*cos(e + f*x)/f**2 - a**2*d*
*2*sin(e + f*x)*cos(e + f*x)/(4*f**3) - 4*a**2*d**2*sin(e + f*x)/f**3, Ne(f
, 0)), ((a*cos(e) + a)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [A]
time = 0.39, size = 207, normalized size = 1.23

$$\frac{1}{2}\,a^{2}d^{2}x^{3}+\frac{3}{2}\,a^{2}cdx^{2}+\frac{3}{2}\,a^{2}c^{2}x+\frac{(a^{2}d^{2}fx+a^{2}cdf)\cos\left(2\,fx+2\,e\right)}{4\,f^{3}}+\frac{4\,(a^{2}d^{2}fx+a^{2}cdf)\cos\left(fx+e\right)}{f^{3}}+\frac{(2\,a^{2}d^{2}f^{2}x^{2}+4\,a^{2}cdf^{2}x+2\,a^{2}c^{2}f^{2}-a^{2}d^{2})\sin\left(2\,fx+2\,e\right)}{8\,f^{3}}+\frac{2\,(a^{2}d^{2}f^{2}x^{2}+2\,a^{2}cdf^{2}x+a^{2}c^{2}f^{2}-2\,a^{2}d^{2})\sin\left(fx+e\right)}{f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x + 1/4*(a^2*d^2*f*x + a^2*
c*d*f)*cos(2*f*x + 2*e)/f^3 + 4*(a^2*d^2*f*x + a^2*c*d*f)*cos(f*x + e)/f^3
```

```
+ 1/8*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*sin(2
*f*x + 2*e)/f^3 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2 - 2*a^
2*d^2)*sin(f*x + e)/f^3
```

Mupad [B]
time = 0.59, size = 255, normalized size = 1.52

$$\frac{8\,a^2\,c^2\,f^2\,\sin\left(e+f\,x\right)-\frac{a^2\,d^2\,\sin\left(2\,e+2\,f\,x\right)}{2}-16\,a^2\,d^2\,\sin\left(e+f\,x\right)+6\,a^2\,c^2\,f^3\,x+a^2\,c^2\,f^2\,\sin\left(2\,e+2\,f\,x\right)+2\,a^2\,d^2\,f^3\,x^3+a^2\,c\,d\,f\,\cos\left(2\,e+2\,f\,x\right)+16\,a^2\,d^2\,f\,x\,\cos\left(e+f\,x\right)+a^2\,d^2\,f^2\,x^2\,\sin\left(2\,e+2\,f\,x\right)+6\,a^2\,c\,d\,f^3\,x^2+a^2\,d^2\,f\,x\,\cos\left(2\,e+2\,f\,x\right)+16\,a^2\,c\,d\,f\,\cos\left(e+f\,x\right)+8\,a^2\,d^2\,f^2\,x^2\,\sin\left(e+f\,x\right)+16\,a^2\,c\,d\,f^2\,x\,\sin\left(e+f\,x\right)+2\,a^2\,c\,d\,f^2\,x\,\sin\left(2\,e+2\,f\,x\right)}{4\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] (8*a^2*c^2*f^2*sin(e + f*x) - (a^2*d^2*sin(2*e + 2*f*x))/2 - 16*a^2*d^2*sin
(e + f*x) + 6*a^2*c^2*f^3*x + a^2*c^2*f^2*sin(2*e + 2*f*x) + 2*a^2*d^2*f^3*
x^3 + a^2*c*d*f*cos(2*e + 2*f*x) + 16*a^2*d^2*f*x*cos(e + f*x) + a^2*d^2*f^
2*x^2*sin(2*e + 2*f*x) + 6*a^2*c*d*f^3*x^2 + a^2*d^2*f*x*cos(2*e + 2*f*x) +
 16*a^2*c*d*f*cos(e + f*x) + 8*a^2*d^2*f^2*x^2*sin(e + f*x) + 16*a^2*c*d*f^
2*x*sin(e + f*x) + 2*a^2*c*d*f^2*x*sin(2*e + 2*f*x))/(4*f^3)
```

3.125 $\int (c+dx)(a+a\cos(e+fx))^2\,dx$

Optimal. Leaf size=118

$$\frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2a^2d\cos(e+fx)}{f^2}+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)\sin(e+fx)}{f}+\frac{a^2(c+dx)\cos$$

```
[Out] 1/2*a^2*c*x+1/4*a^2*d*x^2+1/2*a^2*(d*x+c)^2/d+2*a^2*d*cos(f*x+e)/f^2+1/4*a^
2*d*cos(f*x+e)^2/f^2+2*a^2*(d*x+c)*sin(f*x+e)/f+1/2*a^2*(d*x+c)*cos(f*x+e)*
sin(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3377, 2718, 3391}

$$\frac{2a^2(c+dx)\sin(e+fx)}{f}+\frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2d\cos(e+fx)}{f^2}+\frac{1}{4}a^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a^2*d*Cos[e + f*
x])/f^2 + (a^2*d*Cos[e + f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sin[e + f*x])/f
 + (a^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+a\cos(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2a^2(c+dx)\cos(e+fx)+a^2(c+dx)\cos^2(e+fx)\right)\,dx\\ &= \frac{a^2(c+dx)^2}{2d}+a^2\int (c+dx)\cos^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)\cos(e+\\ &= \frac{a^2(c+dx)^2}{2d}+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)\sin(e+fx)}{f}+\frac{a^2(c+dx\\ &= \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2a^2d\cos(e+fx)}{f^2}+\frac{a^2d\cos^2(e+fx)}{4f^2}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 80, normalized size = 0.68

$$\frac{a^2(-6(e+fx)(-2cf+d(e-fx))+16d\cos(e+fx)+d\cos(2(e+fx))+16f(c+dx)\sin(e+fx)+2f(c+dx)\sin(2(e+fx)))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*(-6*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*d*Cos[e + f*x] + d*Cos[2*(e
+ f*x)] + 16*f*(c + d*x)*Sin[e + f*x] + 2*f*(c + d*x)*Sin[2*(e + f*x)]))/(8
*f^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(108) = 216.
time = 0.09, size = 218, normalized size = 1.85

method	result
risch	$\frac{3a^2dx^2}{4}+\frac{3a^2cx}{2}+\frac{2a^2d\cos(fx+e)}{f^2}+\frac{2a^2(dx+c)\sin(fx+e)}{f}+\frac{a^2d\cos(2fx+2e)}{8f^2}+\frac{a^2(dx+c)\sin(2fx+2e)}{4f}$
norman	$\frac{\frac{4a^2d}{f^2}+\frac{3a^2d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{3a^2cx}{2}+\frac{3a^2dx^2}{4}+\frac{5a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{3a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+3a^2cx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{3a^2cx(\tan}{}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$
derivativedivides	$\frac{a^2c\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2de\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2d\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{(fx+e}{4}\right)}{f}}{}$

default	$\frac{a^2 c\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2 de\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2 d\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-(fx \cdots\right)}{f}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*cos(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a^2/f*d*e*(1/2*cos(f*x
+e)*sin(f*x+e)+1/2*f*x+1/2*e)+a^2/f*d*((f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1
/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)+2*a^2*c*sin(f*x+e)-2*a^2/f*d*
e*sin(f*x+e)+2*a^2/f*d*(cos(f*x+e)+(f*x+e)*sin(f*x+e))+a^2*c*(f*x+e)-a^2/f*
d*e*(f*x+e)+1/2*a^2/f*d*(f*x+e)^2)
```

Maxima [A]
time = 0.28, size = 216, normalized size = 1.83

$$\frac{2\,(2\,fx+2\,e+\sin{(2\,fx+2\,e)})a^2c+8\,(fx+e)a^2c+\frac{4\,(fx+e)^2a^2d}{f}-\frac{2\,(2\,fx+2\,e+\sin(2\,fx+2\,e))a^2de}{f}-\frac{8\,(fx+e)a^2de}{f}+16\,a^2c\sin{(fx+e)}-\frac{16\,a^2de\sin(fx+e)}{f}+\frac{\left(2\,(fx+e)^2+2\,(fx+e)\sin(2\,fx+2\,e)+\cos(2\,fx+2\,e)\right)a^2d}{f}+\frac{16\,((fx+e)\sin(fx+e)+\cos(fx+e))a^2d}{f}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c + 8*(f*x + e)*a^2*c + 4*(f*x
+ e)^2*a^2*d/f - 2*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*d*e/f - 8*(f*x + e)
*a^2*d*e/f + 16*a^2*c*sin(f*x + e) - 16*a^2*d*e*sin(f*x + e)/f + (2*(f*x +
e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d/f + 16*((f*x
+ e)*sin(f*x + e) + cos(f*x + e))*a^2*d/f)/f
```

Fricas [A]
time = 0.39, size = 102, normalized size = 0.86

$$\frac{3\,a^2df^2x^2+6\,a^2cf^2x+a^2d\cos{(fx+e)}^2+8\,a^2d\cos{(fx+e)}+2\,(4\,a^2dfx+4\,a^2cf+(a^2dfx+a^2cf)\cos{(fx+e)})\sin{(fx+e)}}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*d*f^2*x^2 + 6*a^2*c*f^2*x + a^2*d*cos(f*x + e)^2 + 8*a^2*d*cos(f
*x + e) + 2*(4*a^2*d*f*x + 4*a^2*c*f + (a^2*d*f*x + a^2*c*f)*cos(f*x + e))*
sin(f*x + e))/f^2
```

Sympy [A]
time = 0.15, size = 219, normalized size = 1.86

$$\begin{cases} \frac{a^2cx\sin^2{(e+fx)}}{2}+\frac{a^2cx\cos^2{(e+fx)}}{2}+a^2cx+\frac{a^2c\sin{(e+fx)}\cos{(e+fx)}}{2f}+\frac{2a^2c\sin{(e+fx)}}{f}+\frac{a^2dx^2\sin^2{(e+fx)}}{4}+\frac{a^2dx^2\cos^2{(e+fx)}}{4}+\frac{a^2dx^2}{2}+\frac{a^2dx\sin{(e+fx)}\cos{(e+fx)}}{2f}+\frac{2a^2dx\sin{(e+fx)}}{f}-\frac{a^2d\sin^2{(e+fx)}}{4f^2}+\frac{2a^2d\cos{(e+fx)}}{f^2} & \text{for}\; f \neq 0 \\ (a\cos{(e)}+a)^2\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*c
*x + a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c*sin(e + f*x)/f + a**
2*d*x**2*sin(e + f*x)**2/4 + a**2*d*x**2*cos(e + f*x)**2/4 + a**2*d*x**2/2
+ a**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*d*x*sin(e + f*x)/f - a*
*2*d*sin(e + f*x)**2/(4*f**2) + 2*a**2*d*cos(e + f*x)/f**2, Ne(f, 0)), ((a*
cos(e) + a)**2*(c*x + d*x**2/2), True))
```

Giac [A]
time = 0.45, size = 107, normalized size = 0.91

$$\frac{3}{4}\,a^2dx^2+\frac{3}{2}\,a^2cx+\frac{a^2d\cos\left(2\,fx+2\,e\right)}{8\,f^2}+\frac{2\,a^2d\cos\left(fx+e\right)}{f^2}+\frac{\left(a^2dfx+a^2cf\right)\sin\left(2\,fx+2\,e\right)}{4\,f^2}+\frac{2\,\left(a^2dfx+a^2cf\right)\sin\left(fx+e\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x + 1/8*a^2*d*cos(2*f*x + 2*e)/f^2 + 2*a^2*d*cos(
f*x + e)/f^2 + 1/4*(a^2*d*f*x + a^2*c*f)*sin(2*f*x + 2*e)/f^2 + 2*(a^2*d*f*
x + a^2*c*f)*sin(f*x + e)/f^2
```

Mupad [B]
time = 0.20, size = 117, normalized size = 0.99

$$\frac{3\,a^2\,d\,f^2\,x^2-16\,a^2\,d\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\,d\sin\left(e+f\,x\right)^2+8\,a^2\,c\,f\,\sin\left(e+f\,x\right)+a^2\,c\,f\,\sin\left(2\,e+2\,f\,x\right)+6\,a^2\,c\,f^2\,x+a^2\,d\,f\,x\,\sin\left(2\,e+2\,f\,x\right)+8\,a^2\,d\,f\,x\,\sin\left(e+f\,x\right)}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))^2*(c + d*x),x)
```

```
[Out] (3*a^2*d*f^2*x^2 - 16*a^2*d*sin(e/2 + (f*x)/2)^2 - a^2*d*sin(e + f*x)^2 + 8
*a^2*c*f*sin(e + f*x) + a^2*c*f*sin(2*e + 2*f*x) + 6*a^2*c*f^2*x + a^2*d*f*
x*sin(2*e + 2*f*x) + 8*a^2*d*f*x*sin(e + f*x))/(4*f^2)
```

3.126 $\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=145

$$\frac{2a^2\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}-\frac{2a^2\sin}{}$$

[Out] 1/2*a^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/d+2*a^2*Ci(c*f/d+f*x)*cos(-e+c*f/d)/d+3/2*a^2*ln(d*x+c)/d+1/2*a^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/d+2*a^2*Si(c*f/d+f*x)*sin(-e+c*f/d)/d

Rubi [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3393, 3384, 3380, 3383}

$$\frac{2a^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d}+\frac{a^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d}-\frac{2a^2\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}-\frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[e + f*x])^2/(c + d*x),x]

[Out] (2*a^2*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*Log[c + d*x])/(2*d) - (2*a^2*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d - (a^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)}{c+dx}\,dx\\
&= \left(4a^2\right)\int \left(\frac{3}{8(c+dx)}+\frac{\cos(e+fx)}{2(c+dx)}+\frac{\cos(2e+2fx)}{8(c+dx)}\right)dx\\
&= \frac{3a^2\log(c+dx)}{2d}+\frac{1}{2}a^2\int \frac{\cos(2e+2fx)}{c+dx}\,dx+\left(2a^2\right)\int \frac{\cos(e+fx)}{c+dx}\,dx\\
&= \frac{3a^2\log(c+dx)}{2d}+\frac{1}{2}\left(a^2\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx+\left(2a^2\cos\left(\right.\right.\\
&= \frac{2a^2\cos\left(e-\frac{cf}{d}\right)\operatorname{Ci}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 114, normalized size = 0.79

$$\frac{a^2\left(4\cos\left(e-\frac{cf}{d}\right)\operatorname{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)+\cos\left(2e-\frac{2cf}{d}\right)\operatorname{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+3\log(c+dx)-4\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)-\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])^2/(c + d*x),x]
```

```
[Out] (a^2*(4*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] + Cos[2*e - (2*c*f)/d]*Co
sIntegral[(2*f*(c + d*x))/d] + 3*Log[c + d*x] - 4*Sin[e - (c*f)/d]*SinInteg
ral[f*(c/d + x)] - Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))/(2
*d)
```

Maple [A]
time = 0.10, size = 197, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{f\,a^2\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+\frac{3f\,a^2\ln(cf-de+c}{2d}}{f}$
default	$\frac{\frac{f\,a^2\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+\frac{3f\,a^2\ln(cf-de+c}{2d}}{f}$
risch	$-\frac{a^2\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{d}-\frac{a^2\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,-ifx-ie-\frac{icf-ide}{d}\right)}{d}+\frac{3a^2\ln(dx+}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*f*a^2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f
*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d)+3/2*f*a^2*ln(c*f-d*e+d*(f*x+e))
/d+2*f*a^2*(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*
cos((c*f-d*e)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 361, normalized size = 2.49

$$\frac{\frac{4a^2f\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}-\frac{\left(f\left(E_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+E_1\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\cos\left(\frac{2(cf-de)}{d}\right)-f\left(iE_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-iE_1\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\sin\left(\frac{2(cf-de)}{d}\right)-2f\log((fx+e)d+cf-de)\right)a^2}{d}-\frac{4\left(f\left(E_1\left(\frac{i(fx+e)d+icf-ide}{d}\right)+E_1\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f\left(iE_1\left(\frac{i(fx+e)d+icf-ide}{d}\right)-iE_1\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a^2}{d}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d - (f*(exp_integral_e(1, 2*(-I
*(f*x + e)*d - I*c*f + I*d*e)/d) + exp_integral_e(1, -2*(-I*(f*x + e)*d - I
*c*f + I*d*e)/d))*cos(2*(c*f - d*e)/d) - f*(I*exp_integral_e(1, 2*(-I*(f*x
+ e)*d - I*c*f + I*d*e)/d) - I*exp_integral_e(1, -2*(-I*(f*x + e)*d - I*c*f
 + I*d*e)/d))*sin(2*(c*f - d*e)/d) - 2*f*log((f*x + e)*d + c*f - d*e))*a^2/
d - 4*(f*(exp_integral_e(1, (I*(f*x + e)*d + I*c*f - I*d*e)/d) + exp_integr
al_e(1, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*cos((c*f - d*e)/d) + f*(I*exp_
integral_e(1, (I*(f*x + e)*d + I*c*f - I*d*e)/d) - I*exp_integral_e(1, -(I*
(f*x + e)*d + I*c*f - I*d*e)/d))*sin((c*f - d*e)/d))*a^2/d)/f
```

Fricas [A]
time = 0.39, size = 191, normalized size = 1.32

$$-\frac{2a^2\sin\left(-\frac{2(cf-de)}{d}\right)\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+8a^2\sin\left(-\frac{cf-de}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)-6a^2\log(dx+c)-4\left(a^2\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+a^2\mathrm{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\cos\left(-\frac{cf-de}{d}\right)-\left(a^2\mathrm{Ci}\left(\frac{2(dfx+cf)}{d}\right)+a^2\mathrm{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\right)\cos\left(-\frac{2(cf-de)}{d}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*sin(-2*(c*f - d*e)/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*a^2*s
in(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) - 6*a^2*log(d*x + c) - 4*(
a^2*cos_integral((d*f*x + c*f)/d) + a^2*cos_integral(-(d*f*x + c*f)/d))*cos
(-(c*f - d*e)/d) - (a^2*cos_integral(2*(d*f*x + c*f)/d) + a^2*cos_integral(
-2*(d*f*x + c*f)/d))*cos(-2*(c*f - d*e)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\cos\left(e+fx\right)}{c+dx}\,dx+\int \frac{\cos^2\left(e+fx\right)}{c+dx}\,dx+\int \frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**2/(d*x+c),x)
```

```
[Out] a**2*(Integral(2*cos(e + f*x)/(c + d*x), x) + Integral(cos(e + f*x)**2/(c +
 d*x), x) + Integral(1/(c + d*x), x))
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 6933, normalized size = 47.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan
(e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c
*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*
tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2
+ a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 2*a^2*imag_part(cos_inte
gral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) +
 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/
2*e)^2*tan(e) + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan
(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a^2*imag_part(cos_integral(-f*x - c*f
/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 + 16*a^2*sin_integra
l((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a^
2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*
e)^2*tan(e)^2 + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*ta
n(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 - 16*a^2*sin_integral((d*f*x + c*f)/d)*t
an(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 - 2*a^2*imag_part(cos_inte
gral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 +
2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2
```

```
*tan(1/2*e)^2*tan(e)^2 - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*t
an(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/
d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*real_part(cos_integr
al(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*real_pa
rt(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
 a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^
2*tan(1/2*e)^2 + 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*
tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 4*a^2*real_part(cos_integral(-2*f*x
- 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 6*a^2*log(abs
(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + a^2*real_part(cos_integ
ral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 4*a^2*real_p
art(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 4*a
^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(
e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c
*f/d)^2*tan(e)^2 + 16*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2
*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 16*a^2*real_part(cos_integral(-f*x -
c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 6*a^2*log(abs(d*x
 + c))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + a^2*real_part(cos_integral(2*f*
x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 4*a^2*real_part(cos_inte
gral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 4*a^2*real_part(cos
_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + a^2*real_part
(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 6*a^2
*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*real_part(c
os_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^
2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)
^2 + 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e
)^2*tan(e)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c
*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 8*a^2*imag_part(cos_integral(-f*x - c
*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 16*a^2*sin_integral((d*f*
x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 8*a^2*imag_part(cos_
integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 8*a^2*ima
g_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2
 - 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2
*e)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c
*f/d)^2*tan(1/2*e)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(
c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d
)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a^2*imag_part(cos_integral(2
*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) - 2*a^2*imag_part(cos
_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) + 4*a^2*s
in_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) + 2*a^2
*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)
- 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2
*tan(e) + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2*t
```

```
an(e) - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan
(1/2*e)^2*tan(e) + 2*a^2*imag_part(cos_integral...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\cos(e + f\,x))^2}{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))^2/(c + d*x),x)
[Out] int((a + a*cos(e + f*x))^2/(c + d*x), x)
```

3.127 $\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}-\frac{a^2f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2a^2f\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-2$$

```
[Out] -4*a^2*cos(1/2*f*x+1/2*e)^4/d/(d*x+c)-2*a^2*f*cos(-e+c*f/d)*Si(c*f/d+f*x)/d
^2-a^2*f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/d^2+a^2*f*Ci(2*c*f/d+2*f*x)*si
n(-2*e+2*c*f/d)/d^2+2*a^2*f*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^2
```

Rubi [A]
time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3394, 3384, 3380, 3383}

$$-\frac{a^2f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2a^2f\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2a^2f\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a^2f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-4*a^2*Cos[e/2 + (f*x)/2]^4)/(d*(c + d*x)) - (a^2*f*CosIntegral[(2*c*f)/d
+ 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (2*a^2*f*CosIntegral[(c*f)/d + f*x]*Si
n[e - (c*f)/d])/d^2 - (2*a^2*f*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])
/d^2 - (a^2*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)}{(c+dx)^2}\,dx\\
&= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(8a^2f\right)\int\left(-\frac{\sin(e+fx)}{4(c+dx)}-\frac{\sin(2e+2fx)}{8(c+dx)}\right)dx}{d}\\
&= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}-\frac{\left(a^2f\right)\int\frac{\sin(2e+2fx)}{c+dx}\,dx}{d}-\frac{\left(2a^2f\right)\int\frac{\sin(e+fx)}{c+dx}\,dx}{d}\\
&= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}-\frac{\left(a^2f\cos\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{d}-\frac{\left(2a^2f\cos\left(e-\right.\right.}{}\\
&= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}-\frac{a^2f\,\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2a^2f\,\mathrm{Ci}\left(\frac{cf}{d}+fx\right)\sin}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 206, normalized size = 1.30

$$-\frac{a^2\left(3d+4d\cos(e+fx)+d\cos(2(e+fx))+2f(c+dx)\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)+4f(c+dx)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4cf\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+4dfx\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+2cf\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)+2dfx\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] -1/2*(a^2*(3*d + 4*d*Cos[e + f*x] + d*Cos[2*(e + f*x)] + 2*f*(c + d*x)*CosI
ntegral[(2*f*(c + d*x))/d]*Sin[2*e - (2*c*f)/d] + 4*f*(c + d*x)*CosIntegral
[f*(c/d + x)]*Sin[e - (c*f)/d] + 4*c*f*Cos[e - (c*f)/d]*SinIntegral[f*(c/d
+ x)] + 4*d*f*x*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 2*c*f*Cos[2*e -
 (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cos[2*e - (2*c*f)/d]*S
inIntegral[(2*f*(c + d*x))/d]))/(d^2*(c + d*x))
```

Maple [A]
time = 0.15, size = 276, normalized size = 1.74

method	result
derivativedivides	$\frac{\frac{f^2 a^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-}{}\right.}{d}\right.}{d}\right.}{4}}{}$
default	$\frac{\frac{f^2 a^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-}{}\right.}{d}\right.}{d}\right.}{4}}{}$
risch	$\frac{ia^2 f\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{d^2}-\frac{if\,a^2\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,-ifx-ie-\frac{icf-ide}{d}\right)}{d^2}-\frac{3a^2}{2d(dx+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*f^2*a^2*(-2*cos(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e
+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c
*f-d*e)/d)/d)/d)-3/2*f^2*a^2/(c*f-d*e+d*(f*x+e))/d+2*f^2*a^2*(-cos(f*x+e)/(
c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*
f-d*e)/d)*sin((c*f-d*e)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.41, size = 396, normalized size = 2.49

$$-\frac{\frac{4a^2f^2}{(fx+e)d^2+cdf-d^2e}+\frac{\left(f^2\left(E_2\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+E_2\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\cos\left(\frac{2(cf-de)}{d}\right)-f^2\left(iE_2\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-iE_2\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\sin\left(\frac{2(cf-de)}{d}\right)+2f^2\right)a^2}{(fx+e)d^2+cdf-d^2e}+\frac{4\left(f^2\left(E_2\left(\frac{i(fx+e)d+icf-ide}{d}\right)+E_2\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)-f^2\left(-iE_2\left(\frac{i(fx+e)d+icf-ide}{d}\right)+iE_2\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a^2}{(fx+e)d^2+cdf-d^2e}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^2*f^2/((f*x + e)*d^2 + c*d*f - d^2*e) + (f^2*(exp_integral_e(2, 2
*(-I*(f*x + e)*d - I*c*f + I*d*e)/d) + exp_integral_e(2, -2*(-I*(f*x + e)*d
 - I*c*f + I*d*e)/d))*cos(2*(c*f - d*e)/d) - f^2*(I*exp_integral_e(2, 2*(-I
*(f*x + e)*d - I*c*f + I*d*e)/d) - I*exp_integral_e(2, -2*(-I*(f*x + e)*d -
 I*c*f + I*d*e)/d))*sin(2*(c*f - d*e)/d) + 2*f^2)*a^2/((f*x + e)*d^2 + c*d*
f - d^2*e) + 4*(f^2*(exp_integral_e(2, (I*(f*x + e)*d + I*c*f - I*d*e)/d) +
 exp_integral_e(2, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*cos((c*f - d*e)/d)
- f^2*(-I*exp_integral_e(2, (I*(f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_inte
gral_e(2, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*sin((c*f - d*e)/d))*a^2/((f*
x + e)*d^2 + c*d*f - d^2*e))/f
```

Fricas [A]
time = 0.42, size = 289, normalized size = 1.82

$$-\frac{2\,a^2 d\cos\left(fx+e\right)^2+4\,a^2 d\cos\left(fx+e\right)+2\,a^2 d+2\,\left(a^2 dfx+a^2 cf\right)\cos\left(-\frac{2\,(cf-de)}{d}\right)\operatorname{Si}\left(\frac{2\,(dfx+cf)}{d}\right)+4\,\left(a^2 dfx+a^2 cf\right)\cos\left(-\frac{cf-de}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)+2\,\left(\left(a^2 dfx+a^2 cf\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+\left(a^2 dfx+a^2 cf\right)\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)+\left(\left(a^2 dfx+a^2 cf\right)\operatorname{Ci}\left(\frac{2\,(dfx+cf)}{d}\right)+\left(a^2 dfx+a^2 cf\right)\operatorname{Ci}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\sin\left(-\frac{2\,(cf-de)}{d}\right)}{2\,\left(d^3 x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*cos(f*x + e)^2 + 4*a^2*d*cos(f*x + e) + 2*a^2*d + 2*(a^2*d*f*
x + a^2*c*f)*cos(-2*(c*f - d*e)/d)*sin_integral(2*(d*f*x + c*f)/d) + 4*(a^2
*d*f*x + a^2*c*f)*cos(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) + 2*((a
^2*d*f*x + a^2*c*f)*cos_integral((d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*c
os_integral(-(d*f*x + c*f)/d))*sin(-(c*f - d*e)/d) + ((a^2*d*f*x + a^2*c*f)
*cos_integral(2*(d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*cos_integral(-2*(d
*f*x + c*f)/d))*sin(-2*(c*f - d*e)/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{2\cos\left(e+fx\right)}{c^{2}+2cdx+d^{2}x^{2}}\,dx+\int\frac{\cos^{2}\left(e+fx\right)}{c^{2}+2cdx+d^{2}x^{2}}\,dx+\int\frac{1}{c^{2}+2cdx+d^{2}x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*cos(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(c
os(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*d*
x + d**2*x**2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(161) = 322$.
time = 0.61, size = 1133, normalized size = 7.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos_integral(-
2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*f
 - d*e)/d) - 2*a^2*c*f^3*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*
e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*f - d*e)/d) + 2*a^2*d*f^2*cos_integra
l(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e*sin(2
*(c*f - d*e)/d) + 4*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*c
os_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)
```

```
*sin((c*f - d*e)/d) - 4*a^2*c*f^3*cos_integral(-((d*x + c)*(c*f/(d*x + c) -
 f - d*e/(d*x + c)) - c*f + d*e)/d)*sin((c*f - d*e)/d) + 4*a^2*d*f^2*cos_in
tegral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e*si
n((c*f - d*e)/d) - 4*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*
cos((c*f - d*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
 c)) - c*f + d*e)/d) + 4*a^2*c*f^3*cos((c*f - d*e)/d)*sin_integral(-((d*x +
 c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 4*a^2*d*f^2*cos((
c*f - d*e)/d)*e*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)
) - c*f + d*e)/d) - 2*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2
*cos(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*f + d*e)/d) + 2*a^2*c*f^3*cos(2*(c*f - d*e)/d)*sin_integral(-
2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 2*a^2*d*
f^2*cos(2*(c*f - d*e)/d)*e*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f -
d*e/(d*x + c)) - c*f + d*e)/d) + a^2*d*f^2*cos(2*(d*x + c)*(c*f/(d*x + c) -
 f - d*e/(d*x + c))/d) + 4*a^2*d*f^2*cos((d*x + c)*(c*f/(d*x + c) - f - d*e
/(d*x + c))/d) + 3*a^2*d*f^2)*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/
(d*x + c)) - c*d^4*f + d^5*e)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\cos(e + f\,x))^2}{(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))^2/(c + d*x)^2,x)
[Out] int((a + a*cos(e + f*x))^2/(c + d*x)^2, x)
```

3.128 $\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$

Optimal. Leaf size=134

$$-\frac{i(c+dx)^3}{af}+\frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2}-\frac{12id^2(c+dx)\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,-e^{i(e+fx)}\right)}{af^4}$$

```
[Out] -I*(d*x+c)^3/a/f+6*d*(d*x+c)^2*ln(1+exp(I*(f*x+e)))/a/f^2-12*I*d^2*(d*x+c)*
polylog(2,-exp(I*(f*x+e)))/a/f^3+12*d^3*polylog(3,-exp(I*(f*x+e)))/a/f^4+(d
*x+c)^3*tan(1/2*f*x+1/2*e)/a/f
```

Rubi [A]
time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3399, 4269, 3800, 2221, 2611, 2320, 6724}

$$-\frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3}+\frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^3}{af}+\frac{12d^3\text{Li}_3\left(-e^{i(e+fx)}\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) + (6*d*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])/(a*f^
2) - ((12*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(e + f*x))])/(a*f^3) + (12*d^3*
PolyLog[3, -E^(I*(e + f*x))])/(a*f^4) + ((c + d*x)^3*Tan[e/2 + (f*x)/2])/(a
*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = \frac{\int (c+dx)^3 \csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{2a}$$

$$= \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(3d)\int (c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(6id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(12d^2)\int}{}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3} + \frac{(c}{}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3} + \frac{(c}{}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3} + \frac{12}{}$$

Mathematica [A]
time = 0.22, size = 151, normalized size = 1.13

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-\frac{i\cos\left(\frac{1}{2}(e+fx)\right)\left(f^2(c+dx)^2\left(f(c+dx)+6id\log\left(1+e^{i(e+fx)}\right)\right)+12d^2f(c+dx)\text{PolyLog}\left(2,-e^{i(e+fx)}\right)+12id^3\text{PolyLog}\left(3,-e^{i(e+fx)}\right)\right)}{f^3}+(c+dx)^3\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(((-I)*Cos[(e + f*x)/2]*(f^2*(c + d*x)^2*(f*(c + d*x) +
 (6*I)*d*Log[1 + E^(I*(e + f*x))]) + 12*d^2*f*(c + d*x)*PolyLog[2, -E^(I*(e
 + f*x))] + (12*I)*d^3*PolyLog[3, -E^(I*(e + f*x))]))/f^3 + (c + d*x)^3*Sin
[(e + f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(122) = 244$.
time = 0.14, size = 364, normalized size = 2.72

method	result
risch	$\frac{2i(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}{fa\left(e^{i(fx+e)}+1\right)} + \frac{6d\,c^2\ln\left(e^{i(fx+e)}+1\right)}{a\,f^2} - \frac{6d\,c^2\ln\left(e^{i(fx+e)}\right)}{a\,f^2} - \frac{6d^3e^2\ln\left(e^{i(fx+e)}\right)}{a\,f^4} - \frac{6id^2c\,e^2}{a\,f^3} + \frac{4id^3e^3}{a\,f^4} + \frac{6d^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*cos(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))+1)+6/a/f^2*d*c^
2*ln(exp(I*(f*x+e))+1)-6/a/f^2*d*c^2*ln(exp(I*(f*x+e)))-6/a/f^4*d^3*e^2*ln(
exp(I*(f*x+e)))-6*I/a/f^3*d^2*c*e^2-12*I/a/f^3*d^2*c*polylog(2,-exp(I*(f*x+
e)))+6/a/f^2*d^3*ln(exp(I*(f*x+e))+1)*x^2+4*I/a/f^4*d^3*e^3+12*d^3*polylog(
3,-exp(I*(f*x+e)))/a/f^4+12/a/f^3*d^2*c*e*ln(exp(I*(f*x+e)))-2*I/a/f*d^3*x^
3-12*I/a/f^2*d^2*c*e*x-12*I/a/f^3*d^3*polylog(2,-exp(I*(f*x+e)))*x-6*I/a/f*
d^2*c*x^2+6*I/a/f^3*d^3*e^2*x+12/a/f^2*d^2*c*ln(exp(I*(f*x+e))+1)*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(123) = 246$.
time = 0.40, size = 1006, normalized size = 7.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f*x + e
)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e))*c*d^
2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 + 2*a*f^2*cos(f*x + e) + a
*f^2) - 3*((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f
*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e)
)*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*cos(f*x + e) + a*f
) - c^3*sin(f*x + e)/(a*(cos(f*x + e) + 1)) + 3*c^2*d*e*sin(f*x + e)/(a*f*(
cos(f*x + e) + 1)) - 3*c*d^2*e^2*sin(f*x + e)/(a*f^2*(cos(f*x + e) + 1)) +
(2*d^3*e^3 - 6*((f*x + e)^2*d^3 + d^3*e^2 + 2*(c*d^2*f - d^3*e)*(f*x + e) +
 ((f*x + e)^2*d^3 + d^3*e^2 + 2*(c*d^2*f - d^3*e)*(f*x + e))*cos(f*x + e) -
 (-I*(f*x + e)^2*d^3 - I*d^3*e^2 + 2*(-I*c*d^2*f + I*d^3*e)*(f*x + e))*sin(
f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*((f*x + e)^3*d^3 + 3*
(f*x + e)*d^3*e^2 + 3*(c*d^2*f - d^3*e)*(f*x + e)^2)*cos(f*x + e) + 12*((f*
x + e)*d^3 + c*d^2*f - d^3*e + ((f*x + e)*d^3 + c*d^2*f - d^3*e)*cos(f*x +
e) + (I*(f*x + e)*d^3 + I*c*d^2*f - I*d^3*e)*sin(f*x + e))*dilog(-e^(I*f*x
+ I*e)) + 3*(I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(I*c*d^2*f - I*d^3*e)*(f*x +
 e) + (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(I*c*d^2*f - I*d^3*e)*(f*x + e))*c
os(f*x + e) - ((f*x + e)^2*d^3 + d^3*e^2 + 2*(c*d^2*f - d^3*e)*(f*x + e))*s
in(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 12
*(I*d^3*cos(f*x + e) - d^3*sin(f*x + e) + I*d^3)*polylog(3, -e^(I*f*x + I*e
)) + 2*(I*(f*x + e)^3*d^3 + 3*I*(f*x + e)*d^3*e^2 + 3*(I*c*d^2*f - I*d^3*e)
*(f*x + e)^2)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) - I
*a*f^3))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(123) = 246$.
time = 0.39, size = 442, normalized size = 3.30

$$-\frac{6\left(-i\,d^{3}fx-i\,cd^{2}f+\left(-i\,d^{3}fx-i\,cd^{2}f\right)\cos\left(fx+e\right)\right)\mathrm{Li}_2\left(-\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)+6\left(i\,d^{3}fx+i\,cd^{2}f+\left(i\,d^{3}fx+i\,cd^{2}f\right)\cos\left(fx+e\right)\right)\mathrm{Li}_2\left(-\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)-3\left(d^{3}f^{2}x^{2}+2\,cd^{2}f^{2}x+c^{2}df^{2}+\left(d^{3}f^{2}x^{2}+2\,cd^{2}f^{2}x+c^{2}df^{2}\right)\cos\left(fx+e\right)\right)\log\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)+1\right)-3\left(d^{3}f^{2}x^{2}+2\,cd^{2}f^{2}x+c^{2}df^{2}+\left(d^{3}f^{2}x^{2}+2\,cd^{2}f^{2}x+c^{2}df^{2}\right)\cos\left(fx+e\right)\right)\log\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)+1\right)-6\left(d^{3}\cos\left(fx+e\right)+d^{3}\right)\mathrm{polylog}\left(3,-\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)-6\left(d^{3}\cos\left(fx+e\right)+d^{3}\right)\mathrm{polylog}\left(3,-\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)-\left(d^{3}f^{3}x^{3}+3\,cd^{2}f^{3}x^{2}+3\,c^{2}df^{3}x+c^{3}f^{3}\right)\sin\left(fx+e\right)}{af^{4}\cos\left(fx+e\right)+af^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(6*(-I*d^3*f*x - I*c*d^2*f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e))*dilog(
-cos(f*x + e) + I*sin(f*x + e)) + 6*(I*d^3*f*x + I*c*d^2*f + (I*d^3*f*x + I
*c*d^2*f)*cos(f*x + e))*dilog(-cos(f*x + e) - I*sin(f*x + e)) - 3*(d^3*f^2*
x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)
*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*x + e) + 1) - 3*(d^3*f^2*x^2 + 2*
c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(f*x
 + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) - 6*(d^3*cos(f*x + e) + d^3)*
polylog(3, -cos(f*x + e) + I*sin(f*x + e)) - 6*(d^3*cos(f*x + e) + d^3)*pol
ylog(3, -cos(f*x + e) - I*sin(f*x + e)) - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 +
3*c^2*d*f^3*x + c^3*f^3)*sin(f*x + e))/(a*f^4*cos(f*x + e) + a*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^{3}}{\cos{\left(e+fx \right)}+1}\, dx + \int \frac{d^{3} x^{3}}{\cos{\left(e+fx \right)}+1}\, dx + \int \frac{3 c d^{2} x^{2}}{\cos{\left(e+fx \right)}+1}\, dx + \int \frac{3 c^{2} d x}{\cos{\left(e+fx \right)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*cos(f*x+e)),x)
```

```
[Out] (Integral(c**3/(cos(e + f*x) + 1), x) + Integral(d**3*x**3/(cos(e + f*x) +
1), x) + Integral(3*c*d**2*x**2/(cos(e + f*x) + 1), x) + Integral(3*c**2*d*
x/(cos(e + f*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cos(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^3}{a+a\,\cos\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cos(e + f*x)),x)
[Out] int((c + d*x)^3/(a + a*cos(e + f*x)), x)
```

3.129 $\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx$

Optimal. Leaf size=101

$$-\frac{i(c+dx)^2}{af}+\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2}-\frac{4id^2\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] -I*(d*x+c)^2/a/f+4*d*(d*x+c)*ln(1+exp(I*(f*x+e)))/a/f^2-4*I*d^2*polylog(2,-
exp(I*(f*x+e)))/a/f^3+(d*x+c)^2*tan(1/2*f*x+1/2*e)/a/f
```

Rubi [A]
time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3399, 4269, 3800, 2221, 2317, 2438}

$$\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^2}{af}-\frac{4id^2\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) + (4*d*(c + d*x)*Log[1 + E^(I*(e + f*x))])/(a*f^2)
 - ((4*I)*d^2*PolyLog[2, -E^(I*(e + f*x))])/(a*f^3) + ((c + d*x)^2*Tan[e/2
+ (f*x)/2])/(a*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \frac{\int (c+dx)^2 \csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{2a}$$

$$= \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(2d)\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(4id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(4d^2)\int}{}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(4id^2)\,\mathrm{S}}{}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{4id^2\mathrm{Li}_2\left(-e^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^2\,\mathrm{ta}}{a}$$

Mathematica [A]
time = 0.23, size = 125, normalized size = 1.24

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-4id^2\cos\left(\frac{1}{2}(e+fx)\right)\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)+f(c+dx)\left(\cos\left(\frac{1}{2}(e+fx)\right)\left(-if(c+dx)+4d\log\left(1+e^{i(e+fx)}\right)\right)+f(c+dx)\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af^3(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*((-4*I)*d^2*Cos[(e + f*x)/2]*PolyLog[2, -E^(I*(e + f*x)
)] + f*(c + d*x)*(Cos[(e + f*x)/2]*((-I)*f*(c + d*x) + 4*d*Log[1 + E^(I*(e
+ f*x))]) + f*(c + d*x)*Sin[(e + f*x)/2])))/(a*f^3*(1 + Cos[e + f*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(91) = 182$.
time = 0.09, size = 197, normalized size = 1.95

method	result
risch	$\frac{2i(d^2x^2+2cdx+c^2)}{fa(e^{i(fx+e)}+1)} + \frac{4dc\ln(e^{i(fx+e)}+1)}{a\,f^2} - \frac{4dc\ln(e^{i(fx+e)})}{a\,f^2} - \frac{2id^2x^2}{af} - \frac{4id^2ex}{a\,f^2} - \frac{2id^2e^2}{a\,f^3} + \frac{4d^2\ln(e^{i(fx+e)}+1)x}{a\,f^2} - \frac{4id^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*cos(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))+1)+4/a/f^2*d*c*ln(exp(I*(f*x+
e))+1)-4/a/f^2*d*c*ln(exp(I*(f*x+e)))-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2*I
/a/f^3*d^2*e^2+4/a/f^2*d^2*ln(exp(I*(f*x+e))+1)*x-4*I*d^2*polylog(2,-exp(I*
(f*x+e)))/a/f^3+4/a/f^3*d^2*e*ln(exp(I*(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(91) = 182$.
time = 0.41, size = 300, normalized size = 2.97

$$\frac{2\left(c^2f^2+2\left(d^2fx+cdf+\left(d^2fx+cdf\right)\cos\left(fx+e\right)-\left(-i\,d^2fx-i\,cdf\right)\sin\left(fx+e\right)\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)-\left(d^2f^2x^2+2\,cdf^2x\right)\cos\left(fx+e\right)-2\left(d^2\cos\left(fx+e\right)+i\,d^2\sin\left(fx+e\right)+d^2\right)\mathrm{Li}_2\left(-e^{(i\,fx+i\,e)}\right)-\left(i\,d^2fx+i\,cdf+\left(i\,d^2fx+i\,cdf\right)\cos\left(fx+e\right)-\left(d^2fx+cdf\right)\sin\left(fx+e\right)\right)\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\,\cos\left(fx+e\right)+1\right)-\left(i\,d^2f^2x^2+2i\,cdf^2x\right)\sin\left(fx+e\right)\right)}{-i\,af^3\cos\left(fx+e\right)+af^3\sin\left(fx+e\right)-i\,af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(c^2*f^2 + 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e) - (-I*d^2*
f*x - I*c*d*f)*sin(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - (d^2
*f^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) - 2*(d^2*cos(f*x + e) + I*d^2*sin(f*x
+ e) + d^2)*dilog(-e^(I*f*x + I*e)) - (I*d^2*f*x + I*c*d*f + (I*d^2*f*x + I
*c*d*f)*cos(f*x + e) - (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 +
 sin(f*x + e)^2 + 2*cos(f*x + e) + 1) - (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin
(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) - I*a*f^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(91) = 182$.
time = 0.39, size = 240, normalized size = 2.38

$$-\frac{2\left(-i\,d^2\cos\left(fx+e\right)-i\,d^2\right)\mathrm{Li}_2\left(-\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)+2\left(i\,d^2\cos\left(fx+e\right)+i\,d^2\right)\mathrm{Li}_2\left(-\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)-2\left(d^2fx+cdf+\left(d^2fx+cdf\right)\cos\left(fx+e\right)\right)\log\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)+1\right)-2\left(d^2fx+cdf+\left(d^2fx+cdf\right)\cos\left(fx+e\right)\right)\log\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)+1\right)-\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)\sin\left(fx+e\right)}{af^3\cos\left(fx+e\right)+af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(2*(-I*d^2*cos(f*x + e) - I*d^2)*dilog(-cos(f*x + e) + I*sin(f*x + e)) + 2
*(I*d^2*cos(f*x + e) + I*d^2)*dilog(-cos(f*x + e) - I*sin(f*x + e)) - 2*(d^
2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*
x + e) + 1) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(
f*x + e) - I*sin(f*x + e) + 1) - (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*sin(
f*x + e))/(a*f^3*cos(f*x + e) + a*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\cos(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cos(e+fx)+1}\,dx + \int \frac{2cdx}{\cos(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*cos(f*x+e)),x)
```

```
[Out] (Integral(c**2/(cos(e + f*x) + 1), x) + Integral(d**2*x**2/(cos(e + f*x) +
1), x) + Integral(2*c*d*x/(cos(e + f*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cos(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{a+a\,\cos(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cos(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a + a*cos(e + f*x)), x)
```

3.130 $\int \frac{c+dx}{a+a\cos(e+fx)}\,dx$

Optimal. Leaf size=49

$$\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] 2*d*ln(cos(1/2*f*x+1/2*e))/a/f^2+(d*x+c)*tan(1/2*f*x+1/2*e)/a/f
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3399, 4269, 3556}

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[e/2 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tan[e/2 + (f*x)/2])/(a*f
)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \frac{\int (c+dx)\csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{2a}$$
$$= \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{d\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}$$
$$= \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} + \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

Mathematica [A]
time = 0.06, size = 70, normalized size = 1.43

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(2d\cos\left(\frac{1}{2}(e+fx)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+f(c+dx)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af^2(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Cos[e + f*x]),x]
[Out] (2*Cos[(e + f*x)/2]*(2*d*Cos[(e + f*x)/2]*Log[Cos[(e + f*x)/2]] + f*(c + d*
x)*Sin[(e + f*x)/2]))/(a*f^2*(1 + Cos[e + f*x]))
```

Maple [A]
time = 0.08, size = 60, normalized size = 1.22

method	result	size
norman	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a f^2}$	60
risch	$-\frac{2idx}{af}-\frac{2ide}{a f^2}+\frac{2i(dx+c)}{fa\left(e^{i(fx+e)}+1\right)}+\frac{2d\ln\left(e^{i(fx+e)}+1\right)}{a f^2}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*cos(f*x+e)),x,method=_RETURNVERBOSE)
[Out] c/a/f*tan(1/2*f*x+1/2*e)+d/a/f*x*tan(1/2*f*x+1/2*e)-d/a/f^2*ln(1+tan(1/2*f*
x+1/2*e)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(43) = 86.
time = 0.33, size = 176, normalized size = 3.59

$$\frac{\frac{\left(\left(\cos(fx+e)^2+\sin(fx+e)^2+2\cos(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2+2\cos(fx+e)+1\right)+2(fx+e)\sin(fx+e)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2+2af\cos(fx+e)+af}+\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}-\frac{de\sin(fx+e)}{af(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] (((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f*x + e)^2
 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e))*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*cos(f*x + e) + a*f) + c*sin(f*x
 + e)/(a*(cos(f*x + e) + 1)) - d*e*sin(f*x + e)/(a*f*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 0.38, size = 62, normalized size = 1.27

$$\frac{(d\cos(fx+e)+d)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+(dfx+cf)\sin(fx+e)}{af^2\cos(fx+e)+af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((d*cos(f*x + e) + d)*log(1/2*cos(f*x + e) + 1/2) + (d*f*x + c*f)*sin(f*x +
 e))/(a*f^2*cos(f*x + e) + a*f^2)
```

Sympy [A]
time = 0.31, size = 70, normalized size = 1.43

$$\begin{cases} \frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{dx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2} & \text{for } f\neq 0 \\ \frac{cx+\frac{dx^2}{2}}{a\cos(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise(((c*tan(e/2 + f*x/2)/(a*f) + d*x*tan(e/2 + f*x/2)/(a*f) - d*log(ta
n(e/2 + f*x/2)**2 + 1)/(a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cos(e) + a
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(43) = 86.
time = 0.47, size = 234, normalized size = 4.78

$$-\frac{dfx\tan\left(\frac{1}{2}fx\right)+dfx\tan\left(\frac{1}{2}e\right)-d\log\left(\frac{4\left(\tan\left(\frac{1}{2}fx\right)^4\tan\left(\frac{1}{2}e\right)^2-2\tan\left(\frac{1}{2}fx\right)^3\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx\right)^2-2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+1\right)}{\tan\left(\frac{1}{2}e\right)^2+1}\right)\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+cf\tan\left(\frac{1}{2}fx\right)+cf\tan\left(\frac{1}{2}e\right)+d\log\left(\frac{4\left(\tan\left(\frac{1}{2}fx\right)^4\tan\left(\frac{1}{2}e\right)^2-2\tan\left(\frac{1}{2}fx\right)^3\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx\right)^2-2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+1\right)}{\tan\left(\frac{1}{2}e\right)^2+1}\right)}{af^2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)-af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] -(d*f*x*tan(1/2*f*x) + d*f*x*tan(1/2*e) - d*log(4*(tan(1/2*f*x)^4*tan(1/2*e
)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f
*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)*tan
(1/2*e) + c*f*tan(1/2*f*x) + c*f*tan(1/2*e) + d*log(4*(tan(1/2*f*x)^4*tan(1
```

```
/2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1
/2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1)))/(a*f^2*tan(
1/2*f*x)*tan(1/2*e) - a*f^2)
```

Mupad [B]
time = 0.66, size = 65, normalized size = 1.33

$$\frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\right)}{a\,f^2}+\frac{\left(c+d\,x\right)\,2\mathrm{i}}{a\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}-\frac{d\,x\,2\mathrm{i}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cos(e + f*x)),x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) + 1))/(a*f^2) + ((c + d*x)*2i)/(a*f*(exp(e*1
i + f*x*1i) + 1)) - (d*x*2i)/(a*f)
```

3.131 $\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+a\cos(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Cos[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a + a*Cos[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

Mathematica [A]
time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+a\cos(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f*x
 + a*c*d*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e))*integrat
e(sin(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*a*c*
d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin
(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)), x) + s
in(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (a*d*f*x
 + a*c*f)*sin(f*x + e)^2 + 2*(a*d*f*x + a*c*f)*cos(f*x + e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (a*d*x + a*c)*cos(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\cos{(e+fx)}+c+dx\cos{(e+fx)}+dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

```
[Out] Integral(1/(c*cos(e + f*x) + c + d*x*cos(e + f*x) + d*x), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*cos(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+a\,\cos\left(e+f\,x\right)\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cos(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cos(e + f*x))*(c + d*x)), x)
```

3.132 $\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+a\cos(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+a*cos(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Cos[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cos[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

Mathematica [A]
time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+a\cos(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*cos(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*cos(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*f*
x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*s
in(f*x + e)^2 + 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*cos(f*x + e))*i
ntegrate(sin(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^
3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x + e
)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f*x + e
)^2 + 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x +
 e)), x) + sin(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^
2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*
c^2*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e
))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*cos(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\cos(e+fx)+c^2+2cdx\cos(e+fx)+2cdx+d^2x^2\cos(e+fx)+d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cos(f*x+e)),x)
```

```
[Out] Integral(1/(c**2*cos(e + f*x) + c**2 + 2*c*d*x*cos(e + f*x) + 2*c*d*x + d**
2*x**2*cos(e + f*x) + d**2*x**2), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*x + c)^2*(a*cos(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+a\,\cos\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cos(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a + a*cos(e + f*x))*(c + d*x)^2), x)
```

3.133 $\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2} \, dx$

Optimal. Leaf size=271

$$-\frac{i(c+dx)^3}{3a^2f}+\frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2}+\frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4}-\frac{4id^2(c+dx)\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{a^2f^3}+\frac{4d}{}$$

```
[Out] -1/3*I*(d*x+c)^3/a^2/f+2*d*(d*x+c)^2*ln(1+exp(I*(f*x+e)))/a^2/f^2+4*d^3*ln(
cos(1/2*f*x+1/2*e))/a^2/f^4-4*I*d^2*(d*x+c)*polylog(2,-exp(I*(f*x+e)))/a^2/
f^3+4*d^3*polylog(3,-exp(I*(f*x+e)))/a^2/f^4-1/2*d*(d*x+c)^2*sec(1/2*f*x+1/
2*e)^2/a^2/f^2+2*d^2*(d*x+c)*tan(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^3*tan(1
/2*f*x+1/2*e)/a^2/f+1/6*(d*x+c)^3*sec(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2*e)/a
^2/f
```

Rubi [A]

time = 0.24, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3399, 4271, 4269, 3556, 3800, 2221, 2611, 2320, 6724}

$$-\frac{4id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{a^2f^3}+\frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}+\frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2}-\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}-\frac{i(c+dx)^3}{3a^2f}+\frac{4d^3\text{Li}_3\left(-e^{i(e+fx)}\right)}{a^2f^4}+\frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] ((-1/3*I)*(c + d*x)^3)/(a^2*f) + (2*d*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])
/(a^2*f^2) + (4*d^3*Log[Cos[e/2 + (f*x)/2]])/(a^2*f^4) - ((4*I)*d^2*(c + d*
x)*PolyLog[2, -E^(I*(e + f*x))])/(a^2*f^3) + (4*d^3*PolyLog[3, -E^(I*(e + f
*x))])/(a^2*f^4) - (d*(c + d*x)^2*Sec[e/2 + (f*x)/2]^2)/(2*a^2*f^2) + (2*d^
2*(c + d*x)*Tan[e/2 + (f*x)/2])/(a^2*f^3) + ((c + d*x)^3*Tan[e/2 + (f*x)/2]
)/(3*a^2*f) + ((c + d*x)^3*Sec[e/2 + (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*
f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \frac{\int (c+dx)^3 \csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{4a^2}$$

$$= -\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)}{}$$

$$= -\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{2d^2(c+dx}{}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{d(c+}{}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4id^2(c}{}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4id^2(c}{}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4id^2(c}{}$$

Mathematica [A]

time = 0.69, size = 250, normalized size = 0.92

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-3df^2(c+dx)^2\cos\left(\frac{1}{2}(e+fx)\right)+f^3(c+dx)^3\sin\left(\frac{1}{2}(e+fx)\right)+12d^2\cos^3\left(\frac{1}{2}(e+fx)\right)\left(2d\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+f(c+dx)\tan\left(\frac{1}{2}(e+fx)\right)\right)-2\cos^3\left(\frac{1}{2}(e+fx)\right)\left(if^3(c+dx)^3-6d\left(f^2(c+dx)^2\log\left(1+e^{i(e+fx)}\right)-2idf(c+dx)\text{PolyLog}\left(2,-e^{i(e+fx)}\right)+2d^2\text{PolyLog}\left(3,-e^{i(e+fx)}\right)\right)-f^3(c+dx)^3\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{3a^2f^4(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-3*d*f^2*(c + d*x)^2*Cos[(e + f*x)/2] + f^3*(c + d*x)^
3*Sin[(e + f*x)/2] + 12*d^2*Cos[(e + f*x)/2]^3*(2*d*Log[Cos[(e + f*x)/2]] +
 f*(c + d*x)*Tan[(e + f*x)/2]) - 2*Cos[(e + f*x)/2]^3*(I*f^3*(c + d*x)^3 -
6*d*(f^2*(c + d*x)^2*Log[1 + E^(I*(e + f*x))] - (2*I)*d*f*(c + d*x)*PolyLog
```

```
[2, -E^(I*(e + f*x))] + 2*d^2*PolyLog[3, -E^(I*(e + f*x))]) - f^3*(c + d*x)
^3*Tan[(e + f*x)/2])))/(3*a^2*f^4*(1 + Cos[e + f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(231) = 462$.
time = 0.49, size = 678, normalized size = 2.50

method	result
risch	$\frac{4id^3e^3}{3a^2f^4} - \frac{2id^3x^3}{3a^2f} + \frac{2i\left(3ic^2df\,e^{2i(fx+e)}+3d^3f^2x^3e^{i(fx+e)}+3ic^2df\,e^{i(fx+e)}+3id^3f\,x^2e^{i(fx+e)}+9c\,d^2f^2x^2e^{i(fx+e)}+d^3f^2x^3+6ic\,d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*cos(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I/a^2/f^4*d^3*e^3-2/3*I/a^2/f*d^3*x^3+2/3*I*(3*I*c^2*d*f*exp(2*I*(f*x+e
))+3*d^3*f^2*x^3*exp(I*(f*x+e))+3*I*c^2*d*f*exp(I*(f*x+e))+3*I*d^3*f*x^2*ex
p(I*(f*x+e))+9*c*d^2*f^2*x^2*exp(I*(f*x+e))+d^3*f^2*x^3+6*I*c*d^2*f*x*exp(I
*(f*x+e))+3*I*d^3*f*x^2*exp(2*I*(f*x+e))+9*c^2*d*f^2*x*exp(I*(f*x+e))+3*c*d
^2*f^2*x^2+6*I*c*d^2*f*x*exp(2*I*(f*x+e))+3*c^3*f^2*exp(I*(f*x+e))+3*c^2*d*
f^2*x+6*d^3*x*exp(2*I*(f*x+e))+c^3*f^2+6*c*d^2*exp(2*I*(f*x+e))+12*d^3*x*ex
p(I*(f*x+e))+12*c*d^2*exp(I*(f*x+e))+6*d^3*x+6*c*d^2)/f^3/a^2/(exp(I*(f*x+e
))+1)^3+2*I/a^2/f^3*d^3*e^2*x+2/a^2/f^2*d^3*ln(exp(I*(f*x+e))+1)*x^2-2*I/a^
2/f*d^2*c*x^2-4*I/a^2/f^3*d^3*polylog(2,-exp(I*(f*x+e)))*x-4*I/a^2/f^3*d^2*
c*polylog(2,-exp(I*(f*x+e)))+2/a^2/f^2*d*c^2*ln(exp(I*(f*x+e))+1)-2/a^2/f^2
*d*c^2*ln(exp(I*(f*x+e)))-2/a^2/f^4*d^3*e^2*ln(exp(I*(f*x+e)))-4*I/a^2/f^2*
d^2*c*e*x+4/a^2/f^2*d^2*ln(exp(I*(f*x+e))+1)*c*x+4*d^3*polylog(3,-exp(I*(f*
x+e)))/a^2/f^4+4/a^2/f^4*d^3*ln(exp(I*(f*x+e))+1)-4/a^2/f^4*d^3*ln(exp(I*(f
*x+e)))+4/a^2/f^3*d^2*c*e*ln(exp(I*(f*x+e)))-2*I/a^2/f^3*d^2*c*e^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3441 vs. $2(237) = 474$.
time = 0.92, size = 3441, normalized size = 12.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*e) + cos(f*x + e))*cos
(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f*x
 + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (2*(3*cos(2*f*x + 2*e)
+ 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x + 3*e)^2 + 6*(3*cos(f*x
+ e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9*cos(f*x + e)^2 + 6*(s
in(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + sin(3*f*x + 3*e)^2 + 9*s
in(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e) + 9*sin(f*x + e)^2 + 6
```

```
*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1
) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x + 2*e) - sin(f*x + e)
)*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e) + e - 2*sin(f*x + e)
)*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x + e)^2 + 2*cos(f*x +
e))*c*d^2*e/(a^2*f^2*cos(3*f*x + 3*e)^2 + 9*a^2*f^2*cos(2*f*x + 2*e)^2 + 9*
a^2*f^2*cos(f*x + e)^2 + a^2*f^2*sin(3*f*x + 3*e)^2 + 9*a^2*f^2*sin(2*f*x +
 2*e)^2 + 18*a^2*f^2*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f^2*sin(f*x + e)
^2 + 6*a^2*f^2*cos(f*x + e) + a^2*f^2 + 2*(3*a^2*f^2*cos(2*f*x + 2*e) + 3*a
^2*f^2*cos(f*x + e) + a^2*f^2)*cos(3*f*x + 3*e) + 6*(3*a^2*f^2*cos(f*x + e)
 + a^2*f^2)*cos(2*f*x + 2*e) + 6*(a^2*f^2*sin(2*f*x + 2*e) + a^2*f^2*sin(f*
x + e))*sin(3*f*x + 3*e)) - 6*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*
e) + cos(f*x + e))*cos(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f
*x + e) + 1)*cos(2*f*x + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (
2*(3*cos(2*f*x + 2*e) + 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x +
3*e)^2 + 6*(3*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9
*cos(f*x + e)^2 + 6*(sin(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + si
n(3*f*x + 3*e)^2 + 9*sin(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e)
+ 9*sin(f*x + e)^2 + 6*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^
2 + 2*cos(f*x + e) + 1) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x
 + 2*e) - sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e
) + e - 2*sin(f*x + e))*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x
 + e)^2 + 2*cos(f*x + e))*c^2*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f
*x + 2*e)^2 + 9*a^2*f*cos(f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 9*a^2*f*s
in(2*f*x + 2*e)^2 + 18*a^2*f*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f*sin(f*
x + e)^2 + 6*a^2*f*cos(f*x + e) + a^2*f + 2*(3*a^2*f*cos(2*f*x + 2*e) + 3*a
^2*f*cos(f*x + e) + a^2*f)*cos(3*f*x + 3*e) + 6*(3*a^2*f*cos(f*x + e) + a^2
*f)*cos(2*f*x + 2*e) + 6*(a^2*f*sin(2*f*x + 2*e) + a^2*f*sin(f*x + e))*sin(
3*f*x + 3*e)) + c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2 - 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f
*x + e)^3/(cos(f*x + e) + 1)^3)*e/(a^2*f) + 3*c*d^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*e^2/(a^2*f^2) + 6*(12*c*
d^2*f - 2*d^3*(e^3 + 6*e) + 6*((f*x + e)^2*d^3 + d^3*(e^2 + 2) + 2*(c*d^2*f
 - d^3*e)*(f*x + e) + ((f*x + e)^2*d^3 + d^3*(e^2 + 2) + 2*(c*d^2*f - d^3*e
)*(f*x + e))*cos(3*f*x + 3*e) + 3*((f*x + e)^2*d^3 + d^3*(e^2 + 2) + 2*(c*d
^2*f - d^3*e)*(f*x + e))*cos(2*f*x + 2*e) + 3*((f*x + e)^2*d^3 + d^3*(e^2 +
 2) + 2*(c*d^2*f - d^3*e)*(f*x + e))*cos(f*x + e) - (-I*(f*x + e)^2*d^3 + d
^3*(-I*e^2 - 2*I) + 2*(-I*c*d^2*f + I*d^3*e)*(f*x + e))*sin(3*f*x + 3*e) -
3*(-I*(f*x + e)^2*d^3 + d^3*(-I*e^2 - 2*I) + 2*(-I*c*d^2*f + I*d^3*e)*(f*x
+ e))*sin(2*f*x + 2*e) - 3*(-I*(f*x + e)^2*d^3 + d^3*(-I*e^2 - 2*I) + 2*(-I
*c*d^2*f + I*d^3*e)*(f*x + e))*sin(f*x + e))*arctan2(sin(f*x + e), cos(f*x
+ e) + 1) - 2*((f*x + e)^3*d^3 + 3*(f*x + e)*d^3*(e^2 + 2) + 3*(c*d^2*f - d
^3*e)*(f*x + e)^2)*cos(3*f*x + 3*e) - 6*((f*x + e)^3*d^3 - 2*c*d^2*f + d^3*
(-I*e^2 + 2*e) + (3*c*d^2*f - d^3*(3*e + I))*(f*x + e)^2 + (-2*I*c*d^2*f +
d^3*(3*e^2 + 2*I*e + 4))*(f*x + e))*cos(2*f*x + 2*e) - 6*(-I*(f*x + e)^2*d^
3 - 4*c*d^2*f + d^3*(e^3 - I*e^2 + 4*e) + 2*(-I*c*d^2*f + d^3*(I*e + 1))*(f
```

```
*x + e))*cos(f*x + e) - 12*((f*x + e)*d^3 + c*d^2*f - d^3*e + ((f*x + e)*d^
3 + c*d^2*f - d^3*e)*cos(3*f*x + 3*e) + 3*((f*x + e)*d^3 + c*d^2*f - d^3*e)
*cos(2*f*x + 2*e) + 3*((f*x + e)*d^3 + c*d^2*f - d^3*e)*cos(f*x + e) + (I*(
f*x + e)*d^3 + I*c*d^2*f - I*d^3*e)*sin(3*f*x + 3*e) + 3*(I*(f*x + e)*d^3 +
 I*c*d^2*f - I*d^3*e)*sin(2*f*x + 2*e) + 3*(I*(f*x + e)*d^3 + I*c*d^2*f - I
*d^3*e)*sin(f*x + e))*dilog(-e^(I*f*x + I*e)) - 3*(I*(f*x + e)^2*d^3 + d^3*
(I*e^2 + 2*I) + 2*(I*c*d^2*f - I*d^3*e)*(f*x + e) + (I*(f*x + e)^2*d^3 + d^
3*(I*e^2 + 2*I) + 2*(I*c*d^2*f - I*d^3*e)*(f*x + e))*cos(3*f*x + 3*e) + 3*(
I*(f*x + e)^2*d^3 + d^3*(I*e^2 + 2*I) + 2*(I*c*d^2*f - I*d^3*e)*(f*x + e))*
cos(2*f*x + 2*e) + 3*(I*(f*x + e)^2*d^3 + d^3*(I*e^2 + 2*I) + 2*(I*c*d^2*f
- I*d^3*e)*(f*x + e))*cos(f*x + e) - ((f*x + e)^2*d^3 + d^3*(e^2 + 2) + 2*(
c*d^2*f - d^3*e)*(f*x + e))*sin(3*f*x + 3*e) - 3*((f*x + e)^2*d^3 + d^3*(e^
2 + 2) + 2*(c*d^2*f - d^3*e)*(f*x + e))*sin(2*f*x + 2*e) - 3*((f*x + e)^2*d
^3 + d^3*(e^2 + 2) + 2*(c*d^2*f - d^3*e)*(f*x + e))*sin(f*x + e))*log(cos(f
*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(237) = 474$.
time = 0.41, size = 800, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d^3*f^2*x^2 + 6*c*d^2*f^2*x + 3*c^2*d*f^2 + 3*(d^3*f^2*x^2 + 2*c*d^
2*f^2*x + c^2*d*f^2)*cos(f*x + e) + 6*(-I*d^3*f*x - I*c*d^2*f + (-I*d^3*f*x
 - I*c*d^2*f)*cos(f*x + e)^2 + 2*(-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e))*dil
og(-cos(f*x + e) + I*sin(f*x + e)) + 6*(I*d^3*f*x + I*c*d^2*f + (I*d^3*f*x
+ I*c*d^2*f)*cos(f*x + e)^2 + 2*(I*d^3*f*x + I*c*d^2*f)*cos(f*x + e))*dilog
(-cos(f*x + e) - I*sin(f*x + e)) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f
^2 + 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x + e)
^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x + e))*log(
cos(f*x + e) + I*sin(f*x + e) + 1) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d
*f^2 + 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x +
e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x + e))*lo
g(cos(f*x + e) - I*sin(f*x + e) + 1) - 6*(d^3*cos(f*x + e)^2 + 2*d^3*cos(f*
x + e) + d^3)*polylog(3, -cos(f*x + e) + I*sin(f*x + e)) - 6*(d^3*cos(f*x +
 e)^2 + 2*d^3*cos(f*x + e) + d^3)*polylog(3, -cos(f*x + e) - I*sin(f*x + e)
) - (2*d^3*f^3*x^3 + 6*c*d^2*f^3*x^2 + 2*c^3*f^3 + 6*c*d^2*f + 6*(c^2*d*f^3
 + d^3*f)*x + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3 + 6*c*d^2*f + 3*(c^2
*d*f^3 + 2*d^3*f)*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f^4*cos(f*x + e)^2 +
2*a^2*f^4*cos(f*x + e) + a^2*f^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*cos(f*x+e))**2,x)
```

```
[Out] (Integral(c**3/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(d**3*x
**3/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(co
s(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(3*c**2*d*x/(cos(e + f*x)
**2 + 2*cos(e + f*x) + 1), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cos(f*x + e) + a)^2, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cos(e + f*x))^2,x)
```

```
[Out] \text{Hanged}
```

3.134 $\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$

Optimal. Leaf size=212

$$-\frac{i(c+dx)^2}{3a^2f}+\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2}-\frac{4id^2\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{3a^2f^3}-\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}+\frac{2d^2\tan\left(\right.}{3a^2}$$

```
[Out] -1/3*I*(d*x+c)^2/a^2/f+4/3*d*(d*x+c)*ln(1+exp(I*(f*x+e)))/a^2/f^2-4/3*I*d^2
*polylog(2,-exp(I*(f*x+e)))/a^2/f^3-1/3*d*(d*x+c)*sec(1/2*f*x+1/2*e)^2/a^2/
f^2+2/3*d^2*tan(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^2*tan(1/2*f*x+1/2*e)/a^2
/f+1/6*(d*x+c)^2*sec(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2*e)/a^2/f
```

Rubi [A]
time = 0.17, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3399, 4271, 3852, 8, 4269, 3800, 2221, 2317, 2438}

$$\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2}-\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}-\frac{i(c+dx)^2}{3a^2f}-\frac{4id^2\text{Li}_2\left(-e^{i(e+fx)}\right)}{3a^2f^3}+\frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] ((-1/3*I)*(c + d*x)^2)/(a^2*f) + (4*d*(c + d*x)*Log[1 + E^(I*(e + f*x))])/(
3*a^2*f^2) - (((4*I)/3)*d^2*PolyLog[2, -E^(I*(e + f*x))])/(a^2*f^3) - (d*(c
 + d*x)*Sec[e/2 + (f*x)/2]^2)/(3*a^2*f^2) + (2*d^2*Tan[e/2 + (f*x)/2])/(3*a
^2*f^3) + ((c + d*x)^2*Tan[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)^2*Sec[e/2
 + (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \frac{\int (c+dx)^2\csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{4a^2}$$

$$= -\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)}{}$$

$$= -\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}\right.}{3a^2f}$$

$$= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2}{}$$

$$= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2}{}$$

$$= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{4id^2\operatorname{Li}_2\left(-e^{i(e+fx)}\right)}{3a^2f^3} - \frac{d(c+dx}{}$$

Mathematica [A]
time = 0.72, size = 212, normalized size = 1.00

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-2df(c+dx)\cos\left(\frac{1}{2}(e+fx)\right)-2if(c+dx)\cos^3\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)+4id\log\left(1+e^{i(e+fx)}\right)\right)-8id^2\cos^3\left(\frac{1}{2}(e+fx)\right)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)+\left(2(c^2f^2+2cdf^2x+d^2(1+f^2x^2))+(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cos(e+fx)\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{3a^2f^3(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-2*d*f*(c + d*x)*Cos[(e + f*x)/2] - (2*I)*f*(c + d*x)*
Cos[(e + f*x)/2]^3*(f*(c + d*x) + (4*I)*d*Log[1 + E^(I*(e + f*x))]) - (8*I)
*d^2*Cos[(e + f*x)/2]^3*PolyLog[2, -E^(I*(e + f*x))] + (2*(c^2*f^2 + 2*c*d*
f^2*x + d^2*(1 + f^2*x^2)) + (c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Co
s[e + f*x])*Sin[(e + f*x)/2]))/(3*a^2*f^3*(1 + Cos[e + f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(172) = 344.
time = 0.40, size = 358, normalized size = 1.69

method	result
risch	$\frac{2i\left(2id^2fx\,e^{2i(fx+e)}+3d^2f^2x^2e^{i(fx+e)}+2icdf\,e^{2i(fx+e)}+2id^2fx\,e^{i(fx+e)}+6cd\,f^2x\,e^{i(fx+e)}+d^2x^2f^2+2icdf\,e^{i(fx+e)}+3c^2f^2e^{i(fx+e}\right.}{3f^3a^2\left(e^{i(fx+e)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*cos(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*(2*I*d^2*f*x*exp(2*I*(f*x+e))+3*d^2*f^2*x^2*exp(I*(f*x+e))+2*I*c*d*f*
exp(2*I*(f*x+e))+2*I*d^2*f*x*exp(I*(f*x+e))+6*c*d*f^2*x*exp(I*(f*x+e))+d^2*
x^2*f^2+2*I*c*d*f*exp(I*(f*x+e))+3*c^2*f^2*exp(I*(f*x+e))+2*c*d*f^2*x+c^2*f
^2+2*d^2*exp(2*I*(f*x+e))+4*d^2*exp(I*(f*x+e))+2*d^2)/f^3/a^2/(exp(I*(f*x+e
))+1)^3+4/3/a^2*d/f^2*c*ln(exp(I*(f*x+e))+1)-4/3/a^2*d/f^2*c*ln(exp(I*(f*x+
e)))-2/3*I/a^2*d^2/f*x^2-4/3*I/a^2*d^2/f^2*e*x-2/3*I/a^2*d^2/f^3*e^2+4/3/a^
2*d^2/f^2*ln(exp(I*(f*x+e))+1)*x-4/3*I*d^2*polylog(2,-exp(I*(f*x+e)))/a^2/f
^3+4/3/a^2*d^2/f^3*e*ln(exp(I*(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(176) = 352$.
time = 0.59, size = 812, normalized size = 3.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*(c^2*f^2 + 2*d^2 + 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(3*f*x + 3*e
) + 3*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e) + 3*(d^2*f*x + c*d*f)*cos(f*x + e)
 - (-I*d^2*f*x - I*c*d*f)*sin(3*f*x + 3*e) - 3*(-I*d^2*f*x - I*c*d*f)*sin(2
*f*x + 2*e) - 3*(-I*d^2*f*x - I*c*d*f)*sin(f*x + e))*arctan2(sin(f*x + e),
cos(f*x + e) + 1) - (d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(3*f*x + 3*e) - (3*d^2*f
^2*x^2 - 2*I*c*d*f - 2*d^2 + 2*(3*c*d*f^2 - I*d^2*f)*x)*cos(2*f*x + 2*e) +
(3*c^2*f^2 + 2*I*d^2*f*x + 2*I*c*d*f + 4*d^2)*cos(f*x + e) - 2*(d^2*cos(3*f
*x + 3*e) + 3*d^2*cos(2*f*x + 2*e) + 3*d^2*cos(f*x + e) + I*d^2*sin(3*f*x +
 3*e) + 3*I*d^2*sin(2*f*x + 2*e) + 3*I*d^2*sin(f*x + e) + d^2)*dilog(-e^(I*
f*x + I*e)) - (I*d^2*f*x + I*c*d*f + (I*d^2*f*x + I*c*d*f)*cos(3*f*x + 3*e)
 + 3*(I*d^2*f*x + I*c*d*f)*cos(2*f*x + 2*e) + 3*(I*d^2*f*x + I*c*d*f)*cos(f
*x + e) - (d^2*f*x + c*d*f)*sin(3*f*x + 3*e) - 3*(d^2*f*x + c*d*f)*sin(2*f*
x + 2*e) - 3*(d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x +
 e)^2 + 2*cos(f*x + e) + 1) - (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(3*f*x + 3
*e) - (3*I*d^2*f^2*x^2 + 2*c*d*f - 2*I*d^2 + 2*(3*I*c*d*f^2 + d^2*f)*x)*sin
(2*f*x + 2*e) - (-3*I*c^2*f^2 + 2*d^2*f*x + 2*c*d*f - 4*I*d^2)*sin(f*x + e)
)/(-3*I*a^2*f^3*cos(3*f*x + 3*e) - 9*I*a^2*f^3*cos(2*f*x + 2*e) - 9*I*a^2*f
^3*cos(f*x + e) + 3*a^2*f^3*sin(3*f*x + 3*e) + 9*a^2*f^3*sin(2*f*x + 2*e) +
 9*a^2*f^3*sin(f*x + e) - 3*I*a^2*f^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(176) = 352$.

time = 0.41, size = 411, normalized size = 1.94

$$-\frac{2\,d^2fx+2\,cdf+2\,(d^2fx+cdf)\cos(fx+e)+2\left(-i\,d^2\cos(fx+e)^2-2i\,d^2\cos(fx+e)-i\,d^2\right)\mathrm{Li}_2(-\cos(fx+e)+i\,\sin(fx+e))+2\left(i\,d^2\cos(fx+e)^2+2i\,d^2\cos(fx+e)+i\,d^2\right)\mathrm{Li}_2(-\cos(fx+e)-i\,\sin(fx+e))-2\left(d^2fx+cdf+(d^2fx+cdf)\cos(fx+e)^2+2\,(d^2fx+cdf)\cos(fx+e)\right)\log(\cos(fx+e)+i\,\sin(fx+e)+1)-2\left(d^2fx+cdf+(d^2fx+cdf)\cos(fx+e)^2+2\,(d^2fx+cdf)\cos(fx+e)\right)\log(\cos(fx+e)-i\,\sin(fx+e)+1)-(2\,d^2f^2x^2+4\,cdf^2x+2\,c^2f^2+2\,d^2+(d^2f^2x^2+2\,cdf^2x+c^2f^2+2\,d^2)\cos(fx+e))\sin(fx+e)}{3\left(a^2f^3\cos(fx+e)^2+2\,a^2f^3\cos(fx+e)+a^2f^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*d^2*f*x + 2*c*d*f + 2*(d^2*f*x + c*d*f)*cos(f*x + e) + 2*(-I*d^2*co
s(f*x + e)^2 - 2*I*d^2*cos(f*x + e) - I*d^2)*dilog(-cos(f*x + e) + I*sin(f*
x + e)) + 2*(I*d^2*cos(f*x + e)^2 + 2*I*d^2*cos(f*x + e) + I*d^2)*dilog(-co
s(f*x + e) - I*sin(f*x + e)) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f
*x + e)^2 + 2*(d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*x
+ e) + 1) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e)^2 + 2*(d^2*
f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) - (2*d^2*
f^2*x^2 + 4*c*d*f^2*x + 2*c^2*f^2 + 2*d^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^
2*f^2 + 2*d^2)*cos(f*x + e))*sin(f*x + e))/(a^2*f^3*cos(f*x + e)^2 + 2*a^2*
f^3*cos(f*x + e) + a^2*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx+\int \frac{d^2x^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx+\int \frac{2cdx}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*cos(f*x+e))**2,x)
```

```
[Out] (Integral(c**2/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(d**2*x
**2/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(2*c*d*x/(cos(e +
f*x)**2 + 2*cos(e + f*x) + 1), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cos(f*x + e) + a)^2, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cos(e + f*x))^2,x)
```

```
[Out] \text{Hanged}
```

3.135 $\int \frac{c+dx}{(a+a\cos(e+fx))^2} dx$

Optimal. Leaf size=123

$$\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}-\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

```
[Out] 2/3*d*ln(cos(1/2*f*x+1/2*e))/a^2/f^2-1/6*d*sec(1/2*f*x+1/2*e)^2/a^2/f^2+1/3
*(d*x+c)*tan(1/2*f*x+1/2*e)/a^2/f+1/6*(d*x+c)*sec(1/2*f*x+1/2*e)^2*tan(1/2*
f*x+1/2*e)/a^2/f
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3399, 4270, 4269, 3556}

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}-\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*d*Log[Cos[e/2 + (f*x)/2]])/(3*a^2*f^2) - (d*Sec[e/2 + (f*x)/2]^2)/(6*a^2
*f^2) + ((c + d*x)*Tan[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sec[e/2 + (f*
x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \frac{\int (c+dx)\csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{4a^2}$$
$$= -\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{\int (c+dx)\sec^2\left(\frac{e}{2}+\right.}{6a^2}$$
$$= -\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\right.}{6a^2f}$$
$$= \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}-\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\mathrm{se}}{}$$

Mathematica [A]

time = 0.29, size = 113, normalized size = 0.92

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(2d\cos\left(\frac{3}{2}(e+fx)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+2d\cos\left(\frac{1}{2}(e+fx)\right)\left(-1+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)+f(c+dx)\left(3\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)\right)\right)}{3a^2f^2(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Cos[e + f*x])^2,x]
[Out] (Cos[(e + f*x)/2]*(2*d*Cos[(3*(e + f*x))/2]*Log[Cos[(e + f*x)/2]] + 2*d*Cos
[(e + f*x)/2]*(-1 + 3*Log[Cos[(e + f*x)/2]]) + f*(c + d*x)*(3*Sin[(e + f*x)
/2] + Sin[(3*(e + f*x))/2])))/(3*a^2*f^2*(1 + Cos[e + f*x])^2)
```

Maple [A]

time = 0.22, size = 109, normalized size = 0.89

method	result	size
default	$\frac{\frac{c\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}\right)}{f}-\frac{d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f^2}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}+\frac{dx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f^2}}{a^2}$	109
norman	$\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}-\frac{d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6a\,f^2}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{dx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}}{a}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^2f^2}$	128
risch	$-\frac{2idx}{3a^2f}-\frac{2ide}{3a^2f^2}-\frac{2\left(-3idfx\,\mathrm{e}^{i(fx+e)}-3icf\,\mathrm{e}^{i(fx+e)}-idfx-icf+d\,\mathrm{e}^{2i(fx+e)}+d\,\mathrm{e}^{i(fx+e)}\right)}{3f^2a^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3}+\frac{2d\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{3a^2f^2}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*cos(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^2*(1/2*c/f*(1/6*tan(1/2*f*x+1/2*e)^3+1/2*tan(1/2*f*x+1/2*e))-1/12*d/f^2
*tan(1/2*f*x+1/2*e)^2+1/4*d/f*x*tan(1/2*f*x+1/2*e)+1/12*d/f*x*tan(1/2*f*x+1
/2*e)^3-1/6*d/f^2*ln(1+tan(1/2*f*x+1/2*e)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(100) = 200$.
time = 0.31, size = 834, normalized size = 6.78

$$-\frac{\frac{2\left(2\left(3\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(2fx+2e\right)+\cos\left(fx+e\right)\right)\cos\left(3fx+3e\right)+2\left(9\left(fx+e\right)\sin\left(fx+e\right)+6\cos\left(fx+e\right)+1\right)\cos\left(2fx+2e\right)+6\cos\left(2fx+2e\right)^2+6\cos\left(fx+e\right)^2-\left(2\left(3\cos\left(2fx+2e\right)+3\cos\left(fx+e\right)+1\right)\cos\left(3fx+3e\right)+\cos\left(3fx+3e\right)^2+6\left(3\cos\left(fx+e\right)+1\right)\cos\left(2fx+2e\right)+9\cos\left(2fx+2e\right)^2+9\cos\left(fx+e\right)^2+6\left(\sin\left(2fx+2e\right)+\sin\left(fx+e\right)\right)\sin\left(3fx+3e\right)+\sin\left(3fx+3e\right)^2+9\sin\left(2fx+2e\right)^2+18\sin\left(2fx+2e\right)\sin\left(fx+e\right)+9\sin\left(fx+e\right)^2+6\cos\left(fx+e\right)+1\right)\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\cos\left(fx+e\right)+1\right)-2\left(fx+3\left(fx+e\right)\cos\left(fx+e\right)+e-\sin\left(2fx+2e\right)-\sin\left(fx+e\right)\right)\sin\left(3fx+3e\right)-6\left(fx+3\left(fx+e\right)\cos\left(fx+e\right)+e-2\sin\left(fx+e\right)\right)\sin\left(2fx+2e\right)+6\sin\left(2fx+2e\right)^2+6\sin\left(fx+e\right)^2+2\cos\left(fx+e\right)\right)d}{a^2f\cos\left(3fx+3e\right)^2+9a^2f\cos\left(2fx+2e\right)^2+9a^2f\cos\left(fx+e\right)^2+a^2f\sin\left(3fx+3e\right)^2+9a^2f\sin\left(2fx+2e\right)^2+18a^2f\sin\left(2fx+2e\right)\sin\left(fx+e\right)+9a^2f\sin\left(fx+e\right)^2+6a^2f\cos\left(fx+e\right)+a^2f+2\left(3a^2f\cos\left(2fx+2e\right)+3a^2f\cos\left(fx+e\right)+a^2f\right)\cos\left(3fx+3e\right)+6\left(3a^2f\cos\left(fx+e\right)+a^2f\right)\cos\left(2fx+2e\right)+6\left(a^2f\sin\left(2fx+2e\right)+a^2f\sin\left(fx+e\right)\right)\sin\left(3fx+3e\right)}-\frac{c\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)}{a^2}+\frac{d\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)e}{a^2f}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*e) + cos(f*x + e))*cos
(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f*x
 + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (2*(3*cos(2*f*x + 2*e)
+ 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x + 3*e)^2 + 6*(3*cos(f*x
+ e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9*cos(f*x + e)^2 + 6*(s
in(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + sin(3*f*x + 3*e)^2 + 9*s
in(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e) + 9*sin(f*x + e)^2 + 6
*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1
) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x + 2*e) - sin(f*x + e)
)*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e) + e - 2*sin(f*x + e)
)*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x + e)^2 + 2*cos(f*x +
e))*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2 + 9*a^2*f*cos(
f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 9*a^2*f*sin(2*f*x + 2*e)^2 + 18*a^2
*f*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f*sin(f*x + e)^2 + 6*a^2*f*cos(f*x
 + e) + a^2*f + 2*(3*a^2*f*cos(2*f*x + 2*e) + 3*a^2*f*cos(f*x + e) + a^2*f)
*cos(3*f*x + 3*e) + 6*(3*a^2*f*cos(f*x + e) + a^2*f)*cos(2*f*x + 2*e) + 6*(
a^2*f*sin(2*f*x + 2*e) + a^2*f*sin(f*x + e))*sin(3*f*x + 3*e)) - c*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 + d*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*e/
(a^2*f))/f
```

Fricas [A]
time = 0.38, size = 126, normalized size = 1.02

$$-\frac{d\cos\left(fx+e\right)-\left(d\cos\left(fx+e\right)^2+2\,d\cos\left(fx+e\right)+d\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-\left(2\,dfx+2\,cf+\left(dfx+cf\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)+d}{3\left(a^2f^2\cos\left(fx+e\right)^2+2\,a^2f^2\cos\left(fx+e\right)+a^2f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(d*cos(f*x + e) - (d*cos(f*x + e)^2 + 2*d*cos(f*x + e) + d)*log(1/2*co
s(f*x + e) + 1/2) - (2*d*f*x + 2*c*f + (d*f*x + c*f)*cos(f*x + e))*sin(f*x
+ e) + d)/(a^2*f^2*cos(f*x + e)^2 + 2*a^2*f^2*cos(f*x + e) + a^2*f^2)
```

Sympy [A]
time = 0.48, size = 146, normalized size = 1.19

$$\begin{cases} \frac{c\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f}+\frac{dx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{dx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f}-\frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{3a^2f^2}-\frac{d\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} & \text{for } f\neq 0 \\ \frac{cx+\frac{dx^2}{2}}{(a\cos(e)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((c*tan(e/2 + f*x/2)**3/(6*a**2*f) + c*tan(e/2 + f*x/2)/(2*a**2*f)
 + d*x*tan(e/2 + f*x/2)**3/(6*a**2*f) + d*x*tan(e/2 + f*x/2)/(2*a**2*f) - d
*log(tan(e/2 + f*x/2)**2 + 1)/(3*a**2*f**2) - d*tan(e/2 + f*x/2)**2/(6*a**2
*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cos(e) + a)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. 2(100) = 200.
time = 0.61, size = 757, normalized size = 6.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*d*f*x*tan(1/2*f*x)^3*tan(1/2*e)^2 + 3*d*f*x*tan(1/2*f*x)^2*tan(1/2*
e)^3 - 2*d*log(4*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e)
 + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e)
 + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)^3*tan(1/2*e)^3 + 3*c*f*tan(1/2*f*x)^
3*tan(1/2*e)^2 + 3*c*f*tan(1/2*f*x)^2*tan(1/2*e)^3 + d*tan(1/2*f*x)^3*tan(1
/2*e)^3 + d*f*x*tan(1/2*f*x)^3 - 3*d*f*x*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*f*
x*tan(1/2*f*x)*tan(1/2*e)^2 + 6*d*log(4*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*ta
n(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 - 2*
tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)^2*tan(1/2*e)^
2 + d*f*x*tan(1/2*e)^3 + c*f*tan(1/2*f*x)^3 - 3*c*f*tan(1/2*f*x)^2*tan(1/2*
e) + d*tan(1/2*f*x)^3*tan(1/2*e) - 3*c*f*tan(1/2*f*x)*tan(1/2*e)^2 - d*tan(
1/2*f*x)^2*tan(1/2*e)^2 + c*f*tan(1/2*e)^3 + d*tan(1/2*f*x)*tan(1/2*e)^3 +
3*d*f*x*tan(1/2*f*x) + 3*d*f*x*tan(1/2*e) - 6*d*log(4*(tan(1/2*f*x)^4*tan(1
/2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1
/2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)
*tan(1/2*e) + 3*c*f*tan(1/2*f*x) - d*tan(1/2*f*x)^2 + 3*c*f*tan(1/2*e) + d*
```

```
tan(1/2*f*x)*tan(1/2*e) - d*tan(1/2*e)^2 + 2*d*log(4*(tan(1/2*f*x)^4*tan(1/
2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/
2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1)) - d)/(a^2*f^2
*tan(1/2*f*x)^3*tan(1/2*e)^3 - 3*a^2*f^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 3*a^
2*f^2*tan(1/2*f*x)*tan(1/2*e) - a^2*f^2)
```

Mupad [B]
time = 4.29, size = 175, normalized size = 1.42

$$\frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\right)}{3\,a^2\,f^2}+\frac{\left(c\,f+d\,f\,x-d\,1\mathrm{i}\right)\,2\mathrm{i}}{3\,a^2\,f^2\,\left(2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}-\frac{d\,x\,2\mathrm{i}}{3\,a^2\,f}-\frac{2\,d}{3\,a^2\,f^2\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(c+d\,x\right)\,4\mathrm{i}}{3\,a^2\,f\,\left(3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+3\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cos(e + f*x))^2,x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) + 1))/(3*a^2*f^2) + ((c*f - d*1i + d*f*x)*2i
)/(3*a^2*f^2*(2*exp(e*1i + f*x*1i) + exp(e*2i + f*x*2i) + 1)) - (d*x*2i)/(3
*a^2*f) - (2*d)/(3*a^2*f^2*(exp(e*1i + f*x*1i) + 1)) + (exp(e*1i + f*x*1i)*
(c + d*x)*4i)/(3*a^2*f*(3*exp(e*1i + f*x*1i) + 3*exp(e*2i + f*x*2i) + exp(e
*3i + f*x*3i) + 1))
```

3.136 $\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+a\cos(e+fx))^2},x\right)$$

[Out] Unintegrable(1/(d*x+c)/(a+a*cos(f*x+e))^2,x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c + d*x)*(a + a*Cos[e + f*x])^2),x]

[Out] Defer[Int][1/((c + d*x)*(a + a*Cos[e + f*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

Mathematica [A]
time = 7.65, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])^2),x]

[Out] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])^2), x]

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+a\cos\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*cos(f*x+e))^2,x)
[Out] int(1/(d*x+c)/(a+a*cos(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
[Out] 1/3*(6*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + 6*(d^2*f*x + c*d*f)*cos(f*x +
 e)^2 + 6*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 4*d^2*sin(f*x + e) + 6*(d^
2*f*x + c*d*f)*sin(f*x + e)^2 - 2*(2*d^2*sin(2*f*x + 2*e) - (d^2*f*x + c*d*
f)*cos(2*f*x + 2*e) - (d^2*f*x + c*d*f)*cos(f*x + e) + (3*d^2*f^2*x^2 + 6*c
*d*f^2*x + 3*c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) + 2*(d^2*f*x +
 c*d*f + 6*(d^2*f*x + c*d*f)*cos(f*x + e) - 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x
+ 3*c^2*f^2 + 2*d^2)*sin(f*x + e))*cos(2*f*x + 2*e) + 2*(d^2*f*x + c*d*f)*c
os(f*x + e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x
+ a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x
+ a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^
2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^
3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 +
 (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*
sin(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d
*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^
2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)*sin(f*x + e)
+ 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^
3)*sin(f*x + e)^2 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*
f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*
f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) +
 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3
 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f
^3)*cos(f*x + e))*cos(2*f*x + 2*e) + 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x
^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e) + 6*((a^2*d^3*f^3*x^3 +
3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e) + (
a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*si
n(f*x + e))*sin(3*f*x + 3*e))*integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x +
c^2*d*f^2 + 6*d^3)*sin(f*x + e)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*
a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 +
4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^2*d^4*f^3
*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^
```

```
2*c^4*f^3)*cos(f*x + e)), x) + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d
^2*cos(2*f*x + 2*e) + 2*d^2 + (3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 4*
d^2)*cos(f*x + e) + (d^2*f*x + c*d*f)*sin(2*f*x + 2*e) + (d^2*f*x + c*d*f)*
sin(f*x + e))*sin(3*f*x + 3*e) + 2*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2
 + 4*d^2 + 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 2*d^2)*cos(f*x + e)
 + 6*(d^2*f*x + c*d*f)*sin(f*x + e))*sin(2*f*x + 2*e))/(a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 +
9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)
*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*
x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*
e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*
c^3*f^3)*sin(2*f*x + 2*e)*sin(f*x + e) + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f
^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e)^2 + 2*(a^2*d^3*f^3*x
^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3
*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2
*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^
3*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^
3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*
f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e))*cos(2*f*x + 2*e) +
 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3
)*cos(f*x + e) + 6*((a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^
3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e) + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^
2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*sin(3*f*x + 3*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (a^2*d*x + a^2*c)*cos(f*x + e)^2 + 2*(a^2*d*x
 + a^2*c)*cos(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\cos^2(e+fx)+2c\cos(e+fx)+c+dx\cos^2(e+fx)+2dx\cos(e+fx)+dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Integral(1/(c*cos(e + f*x)**2 + 2*c*cos(e + f*x) + c + d*x*cos(e + f*x)**2
+ 2*d*x*cos(e + f*x) + d*x), x)/a**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*cos(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+a\,\cos\left(e+f\,x\right)\right)}^2\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cos(e + f*x))^2*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cos(e + f*x))^2*(c + d*x)), x)
```

3.137 $\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+a\cos(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cos[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

Mathematica [A]
time = 8.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)^{2}\left(a+a\cos\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(12*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + 12*(d^2*f*x + c*d*f)*cos(f*x
 + e)^2 + 12*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 12*d^2*sin(f*x + e) + 1
2*(d^2*f*x + c*d*f)*sin(f*x + e)^2 - 2*(6*d^2*sin(2*f*x + 2*e) - 2*(d^2*f*x
 + c*d*f)*cos(2*f*x + 2*e) - 2*(d^2*f*x + c*d*f)*cos(f*x + e) + 3*(d^2*f^2*
x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) + 2*(2*
d^2*f*x + 2*c*d*f + 12*(d^2*f*x + c*d*f)*cos(f*x + e) - 9*(d^2*f^2*x^2 + 2*
c*d*f^2*x + c^2*f^2 + 2*d^2)*sin(f*x + e))*cos(2*f*x + 2*e) + 4*(d^2*f*x +
c*d*f)*cos(f*x + e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*
d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*
d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(
3*f*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f
^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f
^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x +
a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 9
*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3
*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*
d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(
2*f*x + 2*e)*sin(f*x + e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^
2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f
^3*x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d
^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^4
*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x
+ a^2*c^4*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*
c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3
*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3
*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e))*cos(2*f*x + 2*e) + 6*(a^2*d^4*f^3*x^4
 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^
4*f^3)*cos(f*x + e) + 6*((a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2
*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e) + (a^2*d^4
*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x
+ a^2*c^4*f^3)*sin(f*x + e))*sin(3*f*x + 3*e))*integrate(4/3*(d^3*f^2*x^2 +
 2*c*d^2*f^2*x + c^2*d*f^2 + 12*d^3)*sin(f*x + e)/(a^2*d^5*f^3*x^5 + 5*a^2*
```

```
c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4
*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^
2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*c
os(f*x + e)^2 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3
*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*sin(f*x +
e)^2 + 2*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 +
10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*cos(f*x + e)), x)
 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 6*d^2*cos(2*f*x + 2*e) + 6*d^2
+ 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*cos(f*x + e) + 2*(d^2*f*x
 + c*d*f)*sin(2*f*x + 2*e) + 2*(d^2*f*x + c*d*f)*sin(f*x + e))*sin(3*f*x +
3*e) + 6*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2 + 3*(d^2*f^2*x^2 + 2*
c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e) + 4*(d^2*f*x + c*d*f)*sin(f*x + e
))*sin(2*f*x + 2*e))/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3
*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(3*f
*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*
x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*
x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2
*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 9*(a
^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*
f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3
*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f
*x + 2*e)*sin(f*x + e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^2*d
^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*
x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*
f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^4*f^
3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a
^2*c^4*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d
^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3*(a
^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*
f^3*x + a^2*c^4*f^3)*cos(f*x + e))*cos(2*f*x + ...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (a^2*d^2*x^2 + 2*a^2*c*d*
x + a^2*c^2)*cos(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*cos(f
*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\cos^2(e+fx)+2c^2\cos(e+fx)+c^2+2cdx\cos^2(e+fx)+4cdx\cos(e+fx)+2cdx+d^2x^2\cos^2(e+fx)+2d^2x^2\cos(e+fx)+d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Integral(1/(c**2*cos(e + f*x)**2 + 2*c**2*cos(e + f*x) + c**2 + 2*c*d*x*cos
(e + f*x)**2 + 4*c*d*x*cos(e + f*x) + 2*c*d*x + d**2*x**2*cos(e + f*x)**2 +
 2*d**2*x**2*cos(e + f*x) + d**2*x**2), x)/a**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cos(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+a\,\cos\left(e+f\,x\right)\right)}^2\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cos(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*cos(e + f*x))^2*(c + d*x)^2), x)
```

3.138 $\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx$

Optimal. Leaf size=133

$$-\frac{i(c+dx)^3}{af}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{12id^2(c+dx)\mathrm{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3}+\frac{1}{}$$

```
[Out] -I*(d*x+c)^3/a/f-(d*x+c)^3*cot(1/2*f*x+1/2*e)/a/f+6*d*(d*x+c)^2*ln(1-exp(I*
(f*x+e)))/a/f^2-12*I*d^2*(d*x+c)*polylog(2,exp(I*(f*x+e)))/a/f^3+12*d^3*pol
ylog(3,exp(I*(f*x+e)))/a/f^4
```

Rubi [A]

time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3399, 4269, 3798, 2221, 2611, 2320, 6724}

$$-\frac{12id^2(c+dx)\mathrm{Li}_2\left(e^{i(e+fx)}\right)}{af^3}+\frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^3}{af}+\frac{12d^3\mathrm{Li}_3\left(e^{i(e+fx)}\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a - a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) - ((c + d*x)^3*Cot[e/2 + (f*x)/2])/(a*f) + (6*d*(c
 + d*x)^2*Log[1 - E^(I*(e + f*x))])/(a*f^2) - ((12*I)*d^2*(c + d*x)*PolyLog
[2, E^(I*(e + f*x))])/(a*f^3) + (12*d^3*PolyLog[3, E^(I*(e + f*x))])/(a*f^4
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \frac{\int (c+dx)^3\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{2a}$$

$$= -\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(3d)\int (c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(6id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{(12d^2)}{}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c}{}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c}{}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c}{}$$

Mathematica [A]
time = 0.77, size = 198, normalized size = 1.49

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\left(f^3(c+dx)^3\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+2d\left(3f^2(c+dx)^2\log(1-\cos(e+fx)-i\sin(e+fx))-6idf(c+dx)\mathrm{PolyLog}(2,\cos(e+fx)+i\sin(e+fx))+6d^2\mathrm{PolyLog}(3,\cos(e+fx)+i\sin(e+fx))+\frac{f^3x(3c^2+3cdx+d^2x^2)(-i\cos(e)+\sin(e))}{-1+\cos(e)+i\sin(e)}\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f^4(a-a\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a - a*Cos[e + f*x]),x]
```

```
[Out] (2*Sin[(e + f*x)/2]*(f^3*(c + d*x)^3*Csc[e/2]*Sin[(f*x)/2] + 2*d*(3*f^2*(c
+ d*x)^2*Log[1 - Cos[e + f*x] - I*Sin[e + f*x]] - (6*I)*d*f*(c + d*x)*PolyL
og[2, Cos[e + f*x] + I*Sin[e + f*x]] + 6*d^2*PolyLog[3, Cos[e + f*x] + I*Si
n[e + f*x]] + (f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2)*((-I)*Cos[e] + Sin[e]))/(-
1 + Cos[e] + I*Sin[e]))*Sin[(e + f*x)/2]))/(f^4*(a - a*Cos[e + f*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(121) = 242.
time = 0.14, size = 468, normalized size = 3.52

method	result
risch	$-\frac{2i(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}{fa(\mathrm{e}^{i(fx+e)}-1)}-\frac{6d\,c^2\ln(\mathrm{e}^{i(fx+e)})}{a\,f^2}+\frac{6d\,c^2\ln(\mathrm{e}^{i(fx+e)}-1)}{a\,f^2}+\frac{6d^3e^2\ln(\mathrm{e}^{i(fx+e)}-1)}{a\,f^4}-\frac{6d^3e^2\ln(\mathrm{e}^{i(fx+e)})}{a\,f^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3/(a-a*cos(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))-1)-6/a/f^2*d*c^2*ln(exp(I*(f*x+e)))+6*d/a/f^2*c^2*ln(exp(I*(f*x+e))-1)+6*d^3/a/f^4*e^2*ln(exp(I*(f*x+e))-1)-6/a/f^4*d^3*e^2*ln(exp(I*(f*x+e)))-12*I*d^2/a/f^3*c*polylog(2,exp(I*(f*x+e)))-12*I*d^3/a/f^3*polylog(2,exp(I*(f*x+e)))*x+6*I/a/f^3*d^3*e^2*x+4*I/a/f^4*d^3*e^3+6*d^3/a/f^2*ln(1-exp(I*(f*x+e)))*x^2-6*d^3/a/f^4*ln(1-exp(I*(f*x+e)))*e^2-12*I/a/f^2*d^2*c*e*x+12*d^3*polylog(3,exp(I*(f*x+e)))/a/f^4-12*d^2/a/f^3*c*e*ln(exp(I*(f*x+e))-1)+12/a/f^3*d^2*c*e*ln(exp(I*(f*x+e)))-6*I/a/f^3*d^2*c*e^2-6*I/a/f*d^2*c*x^2-2*I/a/f*d^3*x^3+12*d^2/a/f^2*c*ln(1-exp(I*(f*x+e)))*x+12*d^2/a/f^3*c*ln(1-exp(I*(f*x+e)))*e

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(122) = 244$.
time = 0.39, size = 1042, normalized size = 7.83

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="maxima")

[Out] -(6*((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*c*d^2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 - 2*a*f^2*cos(f*x + e) + a*f^2) - 3*((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*cos(f*x + e) + a*f) + c^3*(cos(f*x + e) + 1)/(a*sin(f*x + e)) - 3*c^2*d*(cos(f*x + e) + 1)*e/(a*f*sin(f*x + e)) + 3*c*d^2*(cos(f*x + e) + 1)*e^2/(a*f^2*sin(f*x + e)) - (2*d^3*e^3 + 6*(d^3*cos(f*x + e)*e^2 + I*d^3*e^2*sin(f*x + e) - d^3*e^2)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 6*((f*x + e)^2*d^3 + 2*(c*d^2*f - d^3*e)*(f*x + e) - ((f*x + e)^2*d^3 + 2*(c*d^2*f - d^3*e)*(f*x + e))*cos(f*x + e) - (I*(f*x + e)^2*d^3 + 2*(I*c*d^2*f - I*d^3*e)*(f*x + e))*sin(f*x + e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - 2*((f*x + e)^3*d^3 + 3*(f*x + e)*d^3*e^2 + 3*(c*d^2*f - d^3*e)*(f*x + e)^2)*cos(f*x + e) + 12*((f*x + e)*d^3 + c*d^2*f - d^3*e - ((f*x + e)*d^3 + c*d^2*f - d^3*e)*cos(f*x + e) - (I*(f*x + e)*d^3 + I*c*d^2*f - I*d^3*e)*sin(f*x + e))*dilog(e^(I*f*x + I*e)) - 3*(-I*(f*x + e)^2*d^3 - I*d^3*e^2 + 2*(-I*c*d^2*f + I*d^3*e)*(f*x + e) + (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(I*c*d^2*f - I*d^3*e)*(f*x + e))*cos(f*x + e) - ((f*x + e)^2*d^3 + d^3*e^2 + 2*(c*d^2*f - d^3*e)*(f*x + e))*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 12*(I*d^3*cos(f*x + e) - d^3*sin(f*x + e) - I*d^3)*polylog(3, e^(I*f*x + I*e)) - 2

```
*(I*(f*x + e)^3*d^3 + 3*I*(f*x + e)*d^3*e^2 + 3*(I*c*d^2*f - I*d^3*e)*(f*x
+ e)^2)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + I*a*f^3
))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(122) = 244$.
time = 0.41, size = 493, normalized size = 3.71

$$-\frac{d^3f^3x^3+3cd^2f^3x^2+3c^2df^3x+c^3f^3-6d^3\mathrm{polylog}(3,\cos(fx+e)+i\sin(fx+e))\sin(fx+e)-6d^3\mathrm{polylog}(3,\cos(fx+e)-i\sin(fx+e))\sin(fx+e)+6(id^3fx+icd^2f)\mathrm{Li}_2(\cos(fx+e)+i\sin(fx+e))\sin(fx+e)+6(-id^3fx-icd^2f)\mathrm{Li}_2(\cos(fx+e)-i\sin(fx+e))\sin(fx+e)-3(c^2df^2-2cd^2fe+d^3e^2)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}i\sin(fx+e)+\frac{1}{2}\right)\sin(fx+e)-3(c^2df^2-2cd^2fe+d^3e^2)\log\left(-\frac{1}{2}\cos(fx+e)-\frac{1}{2}i\sin(fx+e)+\frac{1}{2}\right)\sin(fx+e)-3(d^3f^2x^2+2cd^2f^2x+2cd^2fe-d^3e^2)\log(-\cos(fx+e)+i\sin(fx+e)+1)\sin(fx+e)-3(d^3f^2x^2+2cd^2f^2x+2cd^2fe-d^3e^2)\log(-\cos(fx+e)-i\sin(fx+e)+1)\sin(fx+e)+(d^3f^3x^3+3cd^2f^3x^2+3c^2df^3x+c^3f^3)\cos(fx+e)}{af^4\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 - 6*d^3*polylog(3
, cos(f*x + e) + I*sin(f*x + e))*sin(f*x + e) - 6*d^3*polylog(3, cos(f*x +
e) - I*sin(f*x + e))*sin(f*x + e) + 6*(I*d^3*f*x + I*c*d^2*f)*dilog(cos(f*x
 + e) + I*sin(f*x + e))*sin(f*x + e) + 6*(-I*d^3*f*x - I*c*d^2*f)*dilog(cos
(f*x + e) - I*sin(f*x + e))*sin(f*x + e) - 3*(c^2*d*f^2 - 2*c*d^2*f*e + d^3
*e^2)*log(-1/2*cos(f*x + e) + 1/2*I*sin(f*x + e) + 1/2)*sin(f*x + e) - 3*(c
^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2)*log(-1/2*cos(f*x + e) - 1/2*I*sin(f*x + e
) + 1/2)*sin(f*x + e) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*
e^2)*log(-cos(f*x + e) + I*sin(f*x + e) + 1)*sin(f*x + e) - 3*(d^3*f^2*x^2
+ 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*log(-cos(f*x + e) - I*sin(f*x + e)
 + 1)*sin(f*x + e) + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f
^3)*cos(f*x + e))/(a*f^4*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{c^3}{\cos(e+fx)-1}\,dx+\int \frac{d^3x^3}{\cos(e+fx)-1}\,dx+\int \frac{3cd^2x^2}{\cos(e+fx)-1}\,dx+\int \frac{3c^2dx}{\cos(e+fx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a-a*cos(f*x+e)),x)
```

```
[Out] -(Integral(c**3/(cos(e + f*x) - 1), x) + Integral(d**3*x**3/(cos(e + f*x) -
 1), x) + Integral(3*c*d**2*x**2/(cos(e + f*x) - 1), x) + Integral(3*c**2*d
*x/(cos(e + f*x) - 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^3/(a*cos(f*x + e) - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{a-a\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a - a*cos(e + f*x)),x)
[Out] int((c + d*x)^3/(a - a*cos(e + f*x)), x)
```

3.139 $\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx$

Optimal. Leaf size=102

$$-\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{4id^2\text{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3}$$

```
[Out] -I*(d*x+c)^2/a/f-(d*x+c)^2*cot(1/2*f*x+1/2*e)/a/f+4*d*(d*x+c)*ln(1-exp(I*(f
*x+e)))/a/f^2-4*I*d^2*polylog(2,exp(I*(f*x+e)))/a/f^3
```

Rubi [A]
time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3399, 4269, 3798, 2221, 2317, 2438}

$$\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^2}{af}-\frac{4id^2\text{Li}_2\left(e^{i(e+fx)}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a - a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) - ((c + d*x)^2*Cot[e/2 + (f*x)/2])/(a*f) + (4*d*(c
 + d*x)*Log[1 - E^(I*(e + f*x))])/(a*f^2) - ((4*I)*d^2*PolyLog[2, E^(I*(e +
 f*x))])/(a*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx &= \frac{\int (c+dx)^2\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{2a} \\
&= -\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(2d)\int (c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af} \\
&= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{(4id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af} \\
&= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{(4d^2)\int \mathrm{lo}\cdots}{\cdots} \\
&= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}+\frac{(4id^2)\,\mathrm{Su}\cdots}{\cdots} \\
&= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{4id^2\mathrm{Li}_2\left(e\cdots\right.}{af\cdots}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 292 vs. 2(102) = 204.
time = 3.60, size = 292, normalized size = 2.86

$$
\frac{2\csc\left(\frac{e}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(f^2(c+dx)^2\sin\left(\frac{fx}{2}\right)-2cdf\left(fx\cos\left(\frac{e}{2}\right)-2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{e}{2}\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)+d^2\left(-e^{i\mathrm{ArcTan}\left(\tan\left(\frac{e}{2}\right)\right)}f^2x^2\cos\left(\frac{e}{2}\right)\sqrt{\sec^2\left(\frac{e}{2}\right)}-4\left(-\frac{1}{2}ifx\left(\pi-2\mathrm{ArcTan}\left(\tan\left(\frac{e}{2}\right)\right)\right)-\pi\log\left(1+e^{-ifx}\right)-\left(fx+2\mathrm{ArcTan}\left(\tan\left(\frac{e}{2}\right)\right)\right)\log\left(1-e^{i\left(fx+2\mathrm{ArcTan}\left(\tan\left(\frac{e}{2}\right)\right)\right)}\right)+\pi\log\left(\cos\left(\frac{fx}{2}\right)\right)+2\mathrm{ArcTan}\left(\tan\left(\frac{e}{2}\right)\right)\log\left(\sin\left(\frac{fx}{2}+\mathrm{ArcTan}\left(\tan\left(\frac{e}{2}\right)\right)\right)\right)+i\mathrm{PolyLog}\left(2,e^{i\left(fx+2\mathrm{ArcTan}\left(\tan\left(\frac{e}{2}\right)\right)\right)}\right)\right)\sin\left(\frac{e}{2}\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f^3(a-a\cos(e+fx))}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2/(a - a*Cos[e + f*x]),x]
```

```
[Out] (2*Csc[e/2]*Sin[(e + f*x)/2]*(f^2*(c + d*x)^2*Sin[(f*x)/2] - 2*c*d*f*(f*x*C
os[e/2] - 2*Log[Sin[(e + f*x)/2]]*Sin[e/2])*Sin[(e + f*x)/2] + d^2*(-(E^(I*
ArcTan[Tan[e/2]])*f^2*x^2*Cos[e/2]*Sqrt[Sec[e/2]^2]) - 4*((-1/2*I)*f*x*(Pi
- 2*ArcTan[Tan[e/2]]) - Pi*Log[1 + E^((-I)*f*x)] - (f*x + 2*ArcTan[Tan[e/2]
])*Log[1 - E^(I*(f*x + 2*ArcTan[Tan[e/2]]))] + Pi*Log[Cos[(f*x)/2]] + 2*Arc
Tan[Tan[e/2]]*Log[Sin[(f*x)/2 + ArcTan[Tan[e/2]]]] + I*PolyLog[2, E^(I*(f*x
 + 2*ArcTan[Tan[e/2]]))])*Sin[e/2])*Sin[(e + f*x)/2]))/(f^3*(a - a*Cos[e +
f*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(92) = 184.
time = 0.11, size = 247, normalized size = 2.42

method	result
risch	$-\frac{2i(d^2x^2+2cdx+c^2)}{fa(e^{i(fx+e)}-1)}-\frac{4dc\ln(e^{i(fx+e)})}{a\,f^2}+\frac{4dc\ln(e^{i(fx+e)}-1)}{a\,f^2}-\frac{2id^2x^2}{af}-\frac{4id^2ex}{a\,f^2}-\frac{2id^2e^2}{a\,f^3}+\frac{4d^2\ln(1-e^{i(fx+e)})x}{a\,f^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a-a*cos(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))-1)-4/a/f^2*d*c*ln(exp(I*(f*x
+e)))+4*d/a/f^2*c*ln(exp(I*(f*x+e))-1)-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2*
I/a/f^3*d^2*e^2+4*d^2/a/f^2*ln(1-exp(I*(f*x+e)))*x+4*d^2/a/f^3*ln(1-exp(I*(
f*x+e)))*e-4*I*d^2*polylog(2,exp(I*(f*x+e)))/a/f^3-4*d^2/a/f^3*e*ln(exp(I*(
f*x+e))-1)+4/a/f^3*d^2*e*ln(exp(I*(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(92) = 184.
time = 0.37, size = 330, normalized size = 3.24

$$-\frac{2\left(c^2f^2-2\left(cdf\cos\left(fx+e\right)+i\,cdf\sin\left(fx+e\right)-cdf\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)+2\left(d^2fx\cos\left(fx+e\right)+i\,d^2fx\sin\left(fx+e\right)-d^2fx\right)\arctan\left(\sin\left(fx+e\right),-\cos\left(fx+e\right)+1\right)+\left(d^2f^2x^2+2\,cdf^2x\right)\cos\left(fx+e\right)+2\left(d^2\cos\left(fx+e\right)+i\,d^2\sin\left(fx+e\right)-d^2\right)\mathrm{Li}_2\left(e^{(i\,fx+i\,e)}\right)+\left(-i\,d^2fx-i\,cdf+\left(i\,d^2fx+i\,cdf\right)\cos\left(fx+e\right)-\left(d^2fx+cdf\right)\sin\left(fx+e\right)\right)\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2-2\,\cos\left(fx+e\right)+1\right)+\left(i\,d^2f^2x^2+2i\,cdf^2x\right)\sin\left(fx+e\right)\right)}{-i\,af^3\cos\left(fx+e\right)+af^3\sin\left(fx+e\right)+i\,af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(c^2*f^2 - 2*(c*d*f*cos(f*x + e) + I*c*d*f*sin(f*x + e) - c*d*f)*arctan2
(sin(f*x + e), cos(f*x + e) - 1) + 2*(d^2*f*x*cos(f*x + e) + I*d^2*f*x*sin(
f*x + e) - d^2*f*x)*arctan2(sin(f*x + e), -cos(f*x + e) + 1) + (d^2*f^2*x^2
 + 2*c*d*f^2*x)*cos(f*x + e) + 2*(d^2*cos(f*x + e) + I*d^2*sin(f*x + e) - d
^2)*dilog(e^(I*f*x + I*e)) + (-I*d^2*f*x - I*c*d*f + (I*d^2*f*x + I*c*d*f)*
cos(f*x + e) - (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x
 + e)^2 - 2*cos(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(f*x + e
))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + I*a*f^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(92) = 184$.
time = 0.42, size = 307, normalized size = 3.01

$$-\frac{d^2f^2x^2+2\,cdf^2x+c^2f^2+2i\,d^2\mathrm{Li}_2(\cos(fx+e)+i\,\sin(fx+e))\sin(fx+e)-2i\,d^2\mathrm{Li}_2(\cos(fx+e)-i\,\sin(fx+e))\sin(fx+e)-2\,(cdf-d^2e)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}i\,\sin(fx+e)+\frac{1}{2}\right)\sin(fx+e)-2\,(cdf-d^2e)\log\left(-\frac{1}{2}\cos(fx+e)-\frac{1}{2}i\,\sin(fx+e)+\frac{1}{2}\right)\sin(fx+e)-2\,(d^2fx+d^2e)\log(-\cos(fx+e)+i\,\sin(fx+e)+1)\sin(fx+e)-2\,(d^2fx+d^2e)\log(-\cos(fx+e)-i\,\sin(fx+e)+1)\sin(fx+e)+(d^2f^2x^2+2\,cdf^2x+c^2f^2)\cos(fx+e)}{af^3\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*I*d^2*dilog(cos(f*x + e) + I*sin(
f*x + e))*sin(f*x + e) - 2*I*d^2*dilog(cos(f*x + e) - I*sin(f*x + e))*sin(f
*x + e) - 2*(c*d*f - d^2*e)*log(-1/2*cos(f*x + e) + 1/2*I*sin(f*x + e) + 1/
2)*sin(f*x + e) - 2*(c*d*f - d^2*e)*log(-1/2*cos(f*x + e) - 1/2*I*sin(f*x +
 e) + 1/2)*sin(f*x + e) - 2*(d^2*f*x + d^2*e)*log(-cos(f*x + e) + I*sin(f*x
 + e) + 1)*sin(f*x + e) - 2*(d^2*f*x + d^2*e)*log(-cos(f*x + e) - I*sin(f*x
 + e) + 1)*sin(f*x + e) + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*cos(f*x + e
))/(a*f^3*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{c^2}{\cos(e+fx)-1}\,dx+\int \frac{d^2x^2}{\cos(e+fx)-1}\,dx+\int \frac{2cdx}{\cos(e+fx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a-a*cos(f*x+e)),x)
```

```
[Out] -(Integral(c**2/(cos(e + f*x) - 1), x) + Integral(d**2*x**2/(cos(e + f*x) -
 1), x) + Integral(2*c*d*x/(cos(e + f*x) - 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^2/(a*cos(f*x + e) - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^2}{a-a\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a - a*cos(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a - a*cos(e + f*x)), x)
```

3.140 $\int \frac{c+dx}{a-a\cos(e+fx)}\,dx$

Optimal. Leaf size=50

$$-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

```
[Out] -(d*x+c)*cot(1/2*f*x+1/2*e)/a/f+2*d*ln(sin(1/2*f*x+1/2*e))/a/f^2
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3399, 4269, 3556}

$$\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - a*Cos[e + f*x]),x]
```

```
[Out] -(((c + d*x)*Cot[e/2 + (f*x)/2])/(a*f)) + (2*d*Log[Sin[e/2 + (f*x)/2]])/(a*
f^2)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = \frac{\int (c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{2a}$$
$$= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{d\int \cot\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}$$
$$= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

Mathematica [A]
time = 0.15, size = 57, normalized size = 1.14

$$\frac{-4d\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)+f(c+dx)\sin(e+fx)}{af^2(-1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - a*Cos[e + f*x]),x]
```

```
[Out] (-4*d*Log[Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + f*(c + d*x)*Sin[e + f*x])/
(a*f^2*(-1 + Cos[e + f*x]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.09, size = 72, normalized size = 1.44

method	result	size
risch	$-\frac{2idx}{af}-\frac{2ide}{a f^2}-\frac{2i(dx+c)}{fa\left(e^{i(fx+e)}-1\right)}+\frac{2d\ln\left(e^{i(fx+e)}-1\right)}{a f^2}$	72
norman	$\frac{-\frac{c}{af}-\frac{dx}{af}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a f^2}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a f^2}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a-a*cos(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*d/a/f*x-2*I*d/a/f^2*e-2*I*(d*x+c)/f/a/(exp(I*(f*x+e))-1)+2*d/a/f^2*ln(
exp(I*(f*x+e))-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(44) = 88.
time = 0.28, size = 176, normalized size = 3.52

$$\frac{\frac{\left(\left(\cos(fx+e)^2+\sin(fx+e)^2-2\cos(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2-2\cos(fx+e)+1\right)-2(fx+e)\sin(fx+e)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2-2af\cos(fx+e)+af}-\frac{c(\cos(fx+e)+1)}{a\sin(fx+e)}+\frac{d(\cos(fx+e)+1)e}{af\sin(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] (((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x + e)^2
 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*cos(f*x + e) + a*f) - c*(cos(f*
x + e) + 1)/(a*sin(f*x + e)) + d*(cos(f*x + e) + 1)*e/(a*f*sin(f*x + e)))/f
```

Fricas [A]
time = 0.40, size = 63, normalized size = 1.26

$$-\frac{dfx - d\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+cf+\left(dfx+cf\right)\cos\left(fx+e\right)}{af^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*f*x - d*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + c*f + (d*f*x + c*f)
*cos(f*x + e))/(a*f^2*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.
time = 0.39, size = 90, normalized size = 1.80

$$\begin{cases} -\frac{c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} - \frac{dx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} - \frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2} + \frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} & \text{for } f \neq 0 \\ \frac{cx+\frac{dx^2}{2}}{-a\cos\left(e\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x)
```

```
[Out] Piecewise((-c/(a*f*tan(e/2 + f*x/2)) - d*x/(a*f*tan(e/2 + f*x/2)) - d*log(t
an(e/2 + f*x/2)**2 + 1)/(a*f**2) + 2*d*log(tan(e/2 + f*x/2))/(a*f**2), Ne(f
, 0)), ((c*x + d*x**2/2)/(-a*cos(e) + a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(44) = 88.
time = 0.50, size = 229, normalized size = 4.58

$$\frac{dfx\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+cf\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)-dfx+d\log\left(\frac{4\left(\tan\left(\frac{1}{2}fx\right)^4+2\tan\left(\frac{1}{2}fx\right)^3\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx\right)^2+2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}e\right)^2\right)}{\tan\left(\frac{1}{2}e\right)^2+1}\right)\tan\left(\frac{1}{2}fx\right)+d\log\left(\frac{4\left(\tan\left(\frac{1}{2}fx\right)^4+2\tan\left(\frac{1}{2}fx\right)^3\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx\right)^2+2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}e\right)^2\right)}{\tan\left(\frac{1}{2}e\right)^2+1}\right)\tan\left(\frac{1}{2}e\right)-cf}{af^2\tan\left(\frac{1}{2}fx\right)+af^2\tan\left(\frac{1}{2}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*f*x*tan(1/2*f*x)*tan(1/2*e) + c*f*tan(1/2*f*x)*tan(1/2*e) - d*f*x + d*lo
g(4*(tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*
```

```
e)^2 + tan(1/2*f*x)^2 + 2*tan(1/2*f*x)*tan(1/2*e) + tan(1/2*e)^2)/(tan(1/2*
e)^2 + 1))*tan(1/2*f*x) + d*log(4*(tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^3*tan(1/
2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + 2*tan(1/2*f*x)*tan(1/
2*e) + tan(1/2*e)^2)/(tan(1/2*e)^2 + 1))*tan(1/2*e) - c*f)/(a*f^2*tan(1/2*f
*x) + a*f^2*tan(1/2*e))
```

Mupad [B]
time = 0.52, size = 65, normalized size = 1.30

$$\frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}-1\right)}{a\,f^2}-\frac{\left(c+d\,x\right)\,2\mathrm{i}}{a\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)}-\frac{d\,x\,2\mathrm{i}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - a*cos(e + f*x)),x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) - 1))/(a*f^2) - ((c + d*x)*2i)/(a*f*(exp(e*1
i + f*x*1i) - 1)) - (d*x*2i)/(a*f)
```

3.141 $\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{(c+dx)(a-a\cos(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a - a*Cos[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a - a*Cos[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

Mathematica [A]
time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a - a*Cos[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a - a*Cos[e + f*x])), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a-a\cos(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f*
x + a*c*d*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e))*integra
te(sin(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*a*c
*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*si
n(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)), x) +
sin(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (a*d*f*
x + a*c*f)*sin(f*x + e)^2 - 2*(a*d*f*x + a*c*f)*cos(f*x + e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c - (a*d*x + a*c)*cos(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c\cos{\left(e+fx \right)}-c+dx\cos{\left(e+fx \right)}-dx}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

```
[Out] -Integral(1/(c*cos(e + f*x) - c + d*x*cos(e + f*x) - d*x), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x + c)*(a*cos(f*x + e) - a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a-a\,\cos\left(e+f\,x\right)\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*cos(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a - a*cos(e + f*x))*(c + d*x)), x)
```

3.142 $\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{1}{(c+dx)^2(a-a\cos(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a-a*cos(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a - a*Cos[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)^2*(a - a*Cos[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

Mathematica [A]
time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a - a*Cos[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a - a*Cos[e + f*x])), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a-a\cos(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a-a*cos(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a-a*cos(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*f
*x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*
sin(f*x + e)^2 - 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*cos(f*x + e))*
integrate(sin(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c
^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x +
e)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f*x +
e)^2 - 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x
+ e)), x) + sin(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x
^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a
*c^2*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x +
e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*cos(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c^2\cos{(e+fx)}-c^2+2cdx\cos{(e+fx)}-2cdx+d^2x^2\cos{(e+fx)}-d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a-a*cos(f*x+e)),x)
```

```
[Out] -Integral(1/(c**2*cos(e + f*x) - c**2 + 2*c*d*x*cos(e + f*x) - 2*c*d*x + d*
*2*x**2*cos(e + f*x) - d**2*x**2), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="giac")
[Out] integrate(-1/((d*x + c)^2*(a*cos(f*x + e) - a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a-a\,\cos\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*cos(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a - a*cos(e + f*x))*(c + d*x)^2), x)
```

3.143 $\int x^3 \sqrt{a + a\cos(c + dx)}\, dx$

Optimal. Leaf size=110

$$-\frac{96\sqrt{a + a\cos(c + dx)}}{d^4} + \frac{12x^2\sqrt{a + a\cos(c + dx)}}{d^2} - \frac{48x\sqrt{a + a\cos(c + dx)}\,\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^3} + \frac{2x^3\sqrt{a + a\cos(c + dx)}\,\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

```
[Out] -96*(a+a*cos(d*x+c))^(1/2)/d^4+12*x^2*(a+a*cos(d*x+c))^(1/2)/d^2-48*x*(a+a*
cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)/d^3+2*x^3*(a+a*cos(d*x+c))^(1/2)*tan(1
/2*d*x+1/2*c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3400, 3377, 2718}

$$-\frac{96\sqrt{a\cos(c + dx) + a}}{d^4} - \frac{48x\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cos(c + dx) + a}}{d^3} + \frac{12x^2\sqrt{a\cos(c + dx) + a}}{d^2} + \frac{2x^3\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cos(c + dx) + a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-96*Sqrt[a + a*Cos[c + d*x]])/d^4 + (12*x^2*Sqrt[a + a*Cos[c + d*x]])/d^2
- (48*x*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d^3 + (2*x^3*Sqrt[a +
a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx = \left(\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^3\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\right.$$

$$= \frac{2x^3\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(6\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right.\right.\right.}{d}$$

$$= \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} + \frac{2x^3\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(24\sqrt{a\right.}}{}$$

$$= \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} - \frac{48x\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^3\sqrt{a}}{}$$

$$= -\frac{96\sqrt{a+a\cos(c+dx)}}{d^4} + \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} - \frac{48x\sqrt{a+a\cos(c+}}{d^3}$$

Mathematica [A]
time = 0.15, size = 53, normalized size = 0.48

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,\left(6(-8+d^2x^2)+dx(-24+d^2x^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(6*(-8 + d^2*x^2) + d*x*(-24 + d^2*x^2)*Tan[(
c + d*x)/2]))/d^4
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 132, normalized size = 1.20

method	result
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{i(dx+c)}+1\right)^2\mathrm{e}^{-i(dx+c)}}\,\left(d^3x^3\mathrm{e}^{i(dx+c)}+6id^2x^2\mathrm{e}^{i(dx+c)}-d^3x^3+6id^2x^2-24dx\,\mathrm{e}^{i(dx+c)}-48i\mathrm{e}^{i(dx+c)}+24dx-48\right.}{\left(\mathrm{e}^{i(dx+c)}+1\right)d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d^3*x^3*exp(I*(d*x+c))+6*I*d^2*x^2*exp(I*(d*x+c))-d^3*x^3+6*I*d^2*x^2-24
*d*x*exp(I*(d*x+c))-48*I*exp(I*(d*x+c))+24*d*x-48*I)/d^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(94) = 188.
time = 0.59, size = 206, normalized size = 1.87

$$-\frac{2\left(\sqrt{2}\sqrt{a}c^3\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3\left(\sqrt{2}(dx+c)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}c^2+3\left(\sqrt{2}(dx+c)^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4\sqrt{2}(dx+c)\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}c-\left(\sqrt{2}(dx+c)^3\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6\sqrt{2}(dx+c)^2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-24\sqrt{2}(dx+c)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)-48\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(2)*sqrt(a)*c^3*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*(d*x + c)*sin(1/2
*d*x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a)*c^2 + 3*(sqrt(2)*(d
*x + c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*(d*x + c)*cos(1/2*d*x + 1/2*c) -
 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)*c - (sqrt(2)*(d*x + c)^3*sin(1/2*d
*x + 1/2*c) + 6*sqrt(2)*(d*x + c)^2*cos(1/2*d*x + 1/2*c) - 24*sqrt(2)*(d*x
+ c)*sin(1/2*d*x + 1/2*c) - 48*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a))/d^4
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [**A**]
time = 0.43, size = 98, normalized size = 0.89

$$2\sqrt{2}\sqrt{a}\left(\frac{6\left(d^2x^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-8\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d^4}+\frac{\left(d^3x^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-24\,dx\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*(6*(d^2*x^2*sgn(cos(1/2*d*x + 1/2*c)) - 8*sgn(cos(1/2*d*x
 + 1/2*c)))*cos(1/2*d*x + 1/2*c)/d^4 + (d^3*x^3*sgn(cos(1/2*d*x + 1/2*c)) -
 24*d*x*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d^4)
```

Mupad [**B**]
time = 0.55, size = 83, normalized size = 0.75

$$-\frac{2\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(48\,\cos\left(c+d\,x\right)-6\,d^2\,x^2-6\,d^2\,x^2\,\cos\left(c+d\,x\right)-d^3\,x^3\,\sin\left(c+d\,x\right)+24\,d\,x\,\sin\left(c+d\,x\right)+48\right)}{d^4\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*(a*(cos(c + d*x) + 1))^(1/2)*(48*cos(c + d*x) - 6*d^2*x^2 - 6*d^2*x^2*c
os(c + d*x) - d^3*x^3*sin(c + d*x) + 24*d*x*sin(c + d*x) + 48))/(d^4*(cos(c
 + d*x) + 1))
```

3.144 $\int x^2 \sqrt{a + a\cos(c + dx)}\, dx$

Optimal. Leaf size=88

$$\frac{8x\sqrt{a + a\cos(c + dx)}}{d^2} - \frac{16\sqrt{a + a\cos(c + dx)}\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^3} + \frac{2x^2\sqrt{a + a\cos(c + dx)}\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

```
[Out] 8*x*(a+a*cos(d*x+c))^(1/2)/d^2-16*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)
/d^3+2*x^2*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3400, 3377, 2717}

$$-\frac{16\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cos(c + dx) + a}}{d^3} + \frac{8x\sqrt{a\cos(c + dx) + a}}{d^2} + \frac{2x^2\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cos(c + dx) + a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (8*x*Sqrt[a + a*Cos[c + d*x]])/d^2 - (16*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 +
 (d*x)/2])/d^3 + (2*x^2*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2\sqrt{a+a\cos(c+dx)}\,dx = \left(\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\right.$$

$$= \frac{2x^2\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(4\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right.\right.\right.}{d}$$

$$= \frac{8x\sqrt{a+a\cos(c+dx)}}{d^2} + \frac{2x^2\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(8\sqrt{a+\right.}{}$$

$$= \frac{8x\sqrt{a+a\cos(c+dx)}}{d^2} - \frac{16\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^2\sqrt{a+}}{}$$

Mathematica [A]
time = 0.11, size = 44, normalized size = 0.50

$$\frac{2\sqrt{a(1+\cos(c+dx))}\left(4dx+(-8+d^2x^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(4*d*x + (-8 + d^2*x^2)*Tan[(c + d*x)/2]))/d^
3
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 105, normalized size = 1.19

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{i(dx+c)}+1\right)^2 e^{-i(dx+c)}}\,\left(d^2x^2e^{i(dx+c)}+4idx\,e^{i(dx+c)}-d^2x^2+4idx-8\,e^{i(dx+c)}+8\right)}{\left(e^{i(dx+c)}+1\right)d^3}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d^2*x^2*exp(I*(d*x+c))+4*I*d*x*exp(I*(d*x+c))-d^2*x^2+4*I*d*x-8*exp(I*(d
*x+c))+8)/d^3
```

Maxima [A]
time = 0.55, size = 122, normalized size = 1.39

$$\frac{2\left(\sqrt{2}\,\sqrt{a}\,c^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\left(\sqrt{2}\,(dx+c)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\,c+\left(\sqrt{2}\,(dx+c)^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,\sqrt{2}\,(dx+c)\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,\sqrt{2}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a)*c^2*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*(d*x + c)*sin(1/2*
d*x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a)*c + (sqrt(2)*(d*x +
c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*(d*x + c)*cos(1/2*d*x + 1/2*c) - 8*sq
rt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a))/d^3
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [A]
time = 0.45, size = 77, normalized size = 0.88

$$2\sqrt{2}\sqrt{a}\left(\frac{4\,x\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d^2}+\frac{\left(d^2x^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-8\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*(4*x*cos(1/2*d*x + 1/2*c)*sgn(cos(1/2*d*x + 1/2*c))/d^2 +
 (d^2*x^2*sgn(cos(1/2*d*x + 1/2*c)) - 8*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*
d*x + 1/2*c)/d^3)
```

Mupad [B]
time = 0.43, size = 63, normalized size = 0.72

$$\frac{2\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(4\,d\,x-8\,\sin\left(c+d\,x\right)+d^2\,x^2\,\sin\left(c+d\,x\right)+4\,d\,x\,\cos\left(c+d\,x\right)\right)}{d^3\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*(cos(c + d*x) + 1))^(1/2)*(4*d*x - 8*sin(c + d*x) + d^2*x^2*sin(c + d
*x) + 4*d*x*cos(c + d*x)))/(d^3*(cos(c + d*x) + 1))
```

3.145 $\int x\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{4\sqrt{a+a\cos(c+dx)}}{d^2}+\frac{2x\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

```
[Out] 4*(a+a*cos(d*x+c))^(1/2)/d^2+2*x*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)/
d
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3400, 3377, 2718}

$$\frac{4\sqrt{a\cos(c+dx)+a}}{d^2}+\frac{2x\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + a*Cos[c + d*x]])/d^2 + (2*x*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 +
(d*x)/2])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x\sqrt{a+a\cos(c+dx)}\,dx = \left(\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\right.$$

$$= \frac{2x\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(2\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right.\right.\right.}{d}$$

$$= \frac{4\sqrt{a+a\cos(c+dx)}}{d^2} + \frac{2x\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

Mathematica [A]
time = 0.08, size = 34, normalized size = 0.64

$$\frac{2\sqrt{a(1+\cos(c+dx))}\left(2+dx\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(2 + d*x*Tan[(c + d*x)/2]))/d^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 80, normalized size = 1.51

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{i(dx+c)}+1\right)^2 e^{-i(dx+c)}}\,\left(dx\,e^{i(dx+c)}+2ie^{i(dx+c)}-dx+2i\right)}{\left(e^{i(dx+c)}+1\right)d^2}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d*x*exp(I*(d*x+c))+2*I*exp(I*(d*x+c))-d*x+2*I)/d^2
```

Maxima [A]
time = 0.54, size = 61, normalized size = 1.15

$$-\frac{2\left(\sqrt{2}\,\sqrt{a}\,c\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\left(\sqrt{2}\,(dx+c)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(2)*sqrt(a)*c*sin(1/2*d*x + 1/2*c) - (sqrt(2)*(d*x + c)*sin(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a))/d^2
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(x*sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [A]
time = 0.49, size = 57, normalized size = 1.08

$$2\sqrt{2}\left(\frac{x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}+\frac{2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d^2}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*(x*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d + 2*cos(1/2*d
*x + 1/2*c)*sgn(cos(1/2*d*x + 1/2*c))/d^2)*sqrt(a)
```

Mupad [B]
time = 0.21, size = 46, normalized size = 0.87

$$\frac{2\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(2\,\cos\left(c+d\,x\right)+d\,x\,\sin\left(c+d\,x\right)+2\right)}{d^2\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*cos(c + d*x))^(1/2),x)
[Out] (2*(a*(cos(c + d*x) + 1))^(1/2)*(2*cos(c + d*x) + d*x*sin(c + d*x) + 2))/(d
^2*(cos(c + d*x) + 1))
```

3.146 $\int \sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2725}

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.12

$$\frac{2\sqrt{a(1+\cos(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [A]
time = 0.07, size = 43, normalized size = 1.65

method	result	size
default	$\frac{2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	43
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{i(dx+c)}+1\right)^{2}e^{-i(dx+c)}}\,\left(e^{i(dx+c)}-1\right)}{\left(e^{i(dx+c)}+1\right)d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

Maxima [A]
time = 0.55, size = 20, normalized size = 0.77

$$\frac{2\sqrt{2}\,\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [A]
time = 0.35, size = 32, normalized size = 1.23

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cos\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cos(c + d*x) + a), x)
```

Giac [A]
time = 0.43, size = 30, normalized size = 1.15

$$\frac{2\sqrt{2}\,\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d
```

Mupad [B]
time = 0.33, size = 33, normalized size = 1.27

$$\frac{2\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*(cos(c + d*x) + 1))
```

3.147 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx$

Optimal. Leaf size=84

$$\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)-\sqrt{a+a\cos(c+dx)}\,\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)$$

```
[Out] Ci(1/2*d*x)*cos(1/2*c)*sec(1/2*d*x+1/2*c)*(a+a*cos(d*x+c))^(1/2)-sec(1/2*d*
x+1/2*c)*Si(1/2*d*x)*sin(1/2*c)*(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3400, 3384, 3380, 3383}

$$\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/x,x]
```

```
[Out] Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2] -
 Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
```

```
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \left(\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\right.}{x}$$
$$= \left(\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\cos\left(\frac{dx}{2}\right)}{x}\,dx -$$
$$= \cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{Ci}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)-\sqrt{a+a\cos(c+dx)}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.65

$$\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x,x]
[Out] Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Cos[c/2]*CosIntegral[(d*x)/2]
- Sin[c/2]*SinIntegral[(d*x)/2])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cos(dx+c)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/x,x)
[Out] int((a+a*cos(d*x+c))^(1/2)/x,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 61, normalized size = 0.73

$$-\frac{1}{2}\left(\left(\sqrt{2}\,E_1\left(\frac{1}{2}i\,dx\right)+\sqrt{2}\,E_1\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)-\left(i\sqrt{2}\,E_1\left(\frac{1}{2}i\,dx\right)-i\sqrt{2}\,E_1\left(-\frac{1}{2}i\,dx\right)\right)\sin\left(\frac{1}{2}c\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*exp_integral_e(1, 1/2*I*d*x) + sqrt(2)*exp_integral_e(1, -1/
2*I*d*x))*cos(1/2*c) - (I*sqrt(2)*exp_integral_e(1, 1/2*I*d*x) - I*sqrt(2)*
exp_integral_e(1, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 166, normalized size = 1.98

$$-\frac{\sqrt{2}\left(\Re\left(\mathrm{Ci}\left(\frac{1}{2}\,dx\right)\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{4}\,c\right)^{2}+\Re\left(\mathrm{Ci}\left(-\frac{1}{2}\,dx\right)\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{4}\,c\right)^{2}+2\,\Im\left(\mathrm{Ci}\left(\frac{1}{2}\,dx\right)\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{4}\,c\right)-2\,\Im\left(\mathrm{Ci}\left(-\frac{1}{2}\,dx\right)\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{4}\,c\right)+4\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\mathrm{Si}\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{4}\,c\right)-\Re\left(\mathrm{Ci}\left(\frac{1}{2}\,dx\right)\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\Re\left(\mathrm{Ci}\left(-\frac{1}{2}\,dx\right)\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{a}}{2\left(\tan\left(\frac{1}{4}\,c\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*ta
n(1/4*c)^2 + real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*ta
n(1/4*c)^2 + 2*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/4*c) - 2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/4*c) + 4*sgn(cos(1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*c) -
real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)) - real_part(cos_
integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)))*sqrt(a)/(tan(1/4*c)^2 + 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/x,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/x, x)
```

3.148 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{a+a\cos(c+dx)}}{x}-\frac{1}{2}d\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)-\frac{1}{2}d\cos\left(\frac{c}{2}\right)\sqrt{a-$$

[Out] -(a+a*cos(d*x+c))^(1/2)/x-1/2*d*cos(1/2*c)*sec(1/2*d*x+1/2*c)*Si(1/2*d*x)*(a+a*cos(d*x+c))^(1/2)-1/2*d*Ci(1/2*d*x)*sec(1/2*d*x+1/2*c)*sin(1/2*c)*(a+a*cos(d*x+c))^(1/2)

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}d\sin\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}-\frac{1}{2}d\cos\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}-\frac{\sqrt{a\cos(c+dx)+a}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Cos[c + d*x]]/x^2,x]

[Out] -(Sqrt[a + a*Cos[c + d*x]]/x) - (d*Sqrt[a + a*Cos[c + d*x]]*CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2]*Sin[c/2])/2 - (d*Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*SinIntegral[(d*x)/2])/2

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx &= \left(\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\right.}{x^2}\\ &= -\frac{\sqrt{a+a\cos(c+dx)}}{x}-\frac{1}{2}\left(d\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right.\right.\\ &= -\frac{\sqrt{a+a\cos(c+dx)}}{x}-\frac{1}{2}\left(d\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)\right.\right.\\ &= -\frac{\sqrt{a+a\cos(c+dx)}}{x}-\frac{1}{2}d\sqrt{a+a\cos(c+dx)}\,\text{Ci}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\end{aligned}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.68

$$-\frac{\sqrt{a(1+\cos(c+dx))}\left(2+dx\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}\right)+dx\cos\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\text{Si}\left(\frac{dx}{2}\right)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x^2,x]
```

```
[Out] -1/2*(Sqrt[a*(1 + Cos[c + d*x])]*(2 + d*x*CosIntegral[(d*x)/2]*Sec[(c + d*x
)/2]*Sin[c/2] + d*x*Cos[c/2]*Sec[(c + d*x)/2]*SinIntegral[(d*x)/2]))/x
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cos(dx+c)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/x^2,x)
```

```
[Out] int((a+a*cos(d*x+c))^(1/2)/x^2,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 193, normalized size = 1.75

$$-\frac{\left(\left(E_{2}\left(\frac{1}{2} i\, d x\right)+E_{2}\left(-\frac{1}{2} i\, d x\right)\right) \cos\left(\frac{1}{2} c\right)^{3}+\left(E_{2}\left(\frac{1}{2} i\, d x\right)+E_{2}\left(-\frac{1}{2} i\, d x\right)\right) \cos\left(\frac{1}{2} c\right) \sin\left(\frac{1}{2} c\right)^{2}+\left(-i\, E_{2}\left(\frac{1}{2} i\, d x\right)+i\, E_{2}\left(-\frac{1}{2} i\, d x\right)\right) \sin\left(\frac{1}{2} c\right)^{3}+\left(E_{2}\left(\frac{1}{2} i\, d x\right)+E_{2}\left(-\frac{1}{2} i\, d x\right)\right) \cos\left(\frac{1}{2} c\right)+\left(\left(-i\, E_{2}\left(\frac{1}{2} i\, d x\right)+i\, E_{2}\left(-\frac{1}{2} i\, d x\right)\right) \cos\left(\frac{1}{2} c\right)^{2}-i\, E_{2}\left(\frac{1}{2} i\, d x\right)+i\, E_{2}\left(-\frac{1}{2} i\, d x\right)\right) \sin\left(\frac{1}{2} c\right)\right) \sqrt{a}\, d}{2 \left(\left(\sqrt{2} \cos\left(\frac{1}{2} c\right)^{2}+\sqrt{2} \sin\left(\frac{1}{2} c\right)^{2}\right) (d x+c)-\left(\sqrt{2} \cos\left(\frac{1}{2} c\right)^{2}+\sqrt{2} \sin\left(\frac{1}{2} c\right)^{2}\right) c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*((exp_integral_e(2, 1/2*I*d*x) + exp_integral_e(2, -1/2*I*d*x))*cos(1/
2*c)^3 + (exp_integral_e(2, 1/2*I*d*x) + exp_integral_e(2, -1/2*I*d*x))*cos
(1/2*c)*sin(1/2*c)^2 + (-I*exp_integral_e(2, 1/2*I*d*x) + I*exp_integral_e(
2, -1/2*I*d*x))*sin(1/2*c)^3 + (exp_integral_e(2, 1/2*I*d*x) + exp_integral
_e(2, -1/2*I*d*x))*cos(1/2*c) + ((-I*exp_integral_e(2, 1/2*I*d*x) + I*exp_i
ntegral_e(2, -1/2*I*d*x))*cos(1/2*c)^2 - I*exp_integral_e(2, 1/2*I*d*x) + I
*exp_integral_e(2, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)*d/((sqrt(2)*cos(1/2*c)^
2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c) - (sqrt(2)*cos(1/2*c)^2 + sqrt(2)*sin(1
/2*c)^2)*c)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \left(\cos\left(c+d x\right)+1\right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 560, normalized size = 5.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/4*d*x)^2*tan(1/4*c)^2 - d*x*imag_part(cos_integral(-1/2*d*x))*sgn(co
s(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d*x*sgn(cos(1/2*d*x + 1
/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c)^2 - 2*d*x*real_part(
cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)
- 2*d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4
*d*x)^2*tan(1/4*c) - d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/4*d*x)^2 + d*x*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2
*d*x + 1/2*c))*tan(1/4*d*x)^2 - 2*d*x*sgn(cos(1/2*d*x + 1/2*c))*sin_integra
l(1/2*d*x)*tan(1/4*d*x)^2 + d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*c)^2 - d*x*imag_part(cos_integral(-1/2*d*x))*sgn(co
s(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 2*d*x*sgn(cos(1/2*d*x + 1/2*c))*sin_inte
gral(1/2*d*x)*tan(1/4*c)^2 - 2*d*x*real_part(cos_integral(1/2*d*x))*sgn(cos
(1/2*d*x + 1/2*c))*tan(1/4*c) - 2*d*x*real_part(cos_integral(-1/2*d*x))*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/4*c) - 4*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x
)^2*tan(1/4*c)^2 - d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1
/2*c)) + d*x*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)) -
2*d*x*sgn(cos(1/2*d*x + 1/2*c))*sin_integral(1/2*d*x) + 4*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/4*d*x)^2 + 16*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4
*c) + 4*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 - 4*sgn(cos(1/2*d*x + 1/2*c)
))*sqrt(a)/(x*tan(1/4*d*x)^2*tan(1/4*c)^2 + x*tan(1/4*d*x)^2 + x*tan(1/4*c)
^2 + x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/x^2,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/x^2, x)
```

3.149 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{a+a\cos(c+dx)}}{2x^2}-\frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{1}{8}d^2\sqrt{a+a\cos}$$

```
[Out] -1/2*(a+a*cos(d*x+c))^(1/2)/x^2-1/8*d^2*Ci(1/2*d*x)*cos(1/2*c)*sec(1/2*d*x+
1/2*c)*(a+a*cos(d*x+c))^(1/2)+1/8*d^2*sec(1/2*d*x+1/2*c)*Si(1/2*d*x)*sin(1/
2*c)*(a+a*cos(d*x+c))^(1/2)+1/4*d*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)
/x
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3400, 3378, 3384, 3380, 3383}

$$-\frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}+\frac{1}{8}d^2\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}-\frac{\sqrt{a\cos(c+dx)+a}}{2x^2}+\frac{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + a*Cos[c + d*x]]/x^2 - (d^2*Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*
CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2])/8 + (d^2*Sqrt[a + a*Cos[c + d*x]]*
Sec[c/2 + (d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2])/8 + (d*Sqrt[a + a*Cos[c +
 d*x]]*Tan[c/2 + (d*x)/2])/(4*x)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = \left(\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\right.}{x^3}$$
$$= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} - \frac{1}{4}\left(d\sqrt{a+a\cos(c+dx)}\,\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right.\right.$$
$$= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} + \frac{d\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x} - \frac{1}{8}\left(d^2\sqrt{a+a}\right.$$
$$= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} + \frac{d\sqrt{a+a\cos(c+dx)}\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x} - \frac{1}{8}\left(d^2\cos\left(\frac{c}{2}\right.\right.$$
$$= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} - \frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\mathrm{Ci}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\right.$$

Mathematica [A]
time = 0.17, size = 98, normalized size = 0.65

$$\frac{\sqrt{a(1+\cos(c+dx))}\left(-4-d^2x^2\cos\left(\frac{c}{2}\right)\mathrm{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+d^2x^2\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}\right)\mathrm{Si}\left(\frac{dx}{2}\right)+2dx\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(-4 - d^2*x^2*Cos[c/2]*CosIntegral[(d*x)/2]*Sec
[(c + d*x)/2] + d^2*x^2*Sec[(c + d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2] + 2*
d*x*Tan[(c + d*x)/2]))/(8*x^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a\cos\left(dx + c\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/x^3,x)
```

```
[Out] int((a+a*cos(d*x+c))^(1/2)/x^3,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 227, normalized size = 1.50

$$-\frac{\left(\left(E_3\left(\frac{1}{2} i\, dx\right) + E_3\left(-\frac{1}{2} i\, dx\right)\right)\cos\left(\frac{1}{2}\,c\right)^3 + \left(E_3\left(\frac{1}{2} i\, dx\right) + E_3\left(-\frac{1}{2} i\, dx\right)\right)\cos\left(\frac{1}{2}\,c\right)\sin\left(\frac{1}{2}\,c\right)^2 + \left(-i\,E_3\left(\frac{1}{2} i\, dx\right) + i\,E_3\left(-\frac{1}{2} i\, dx\right)\right)\sin\left(\frac{1}{2}\,c\right)^3 + \left(E_3\left(\frac{1}{2} i\, dx\right) + E_3\left(-\frac{1}{2} i\, dx\right)\right)\cos\left(\frac{1}{2}\,c\right) + \left(\left(-i\,E_3\left(\frac{1}{2} i\, dx\right) + i\,E_3\left(-\frac{1}{2} i\, dx\right)\right)\cos\left(\frac{1}{2}\,c\right)^2 - i\,E_3\left(\frac{1}{2} i\, dx\right) + i\,E_3\left(-\frac{1}{2} i\, dx\right)\right)\sin\left(\frac{1}{2}\,c\right)\right)\sqrt{a}\, d^2}{2\left(\left(\sqrt{2}\cos\left(\frac{1}{2}\,c\right)^2 + \sqrt{2}\sin\left(\frac{1}{2}\,c\right)^2\right)(dx + c)^2 - 2\left(\sqrt{2}\cos\left(\frac{1}{2}\,c\right)^2 + \sqrt{2}\sin\left(\frac{1}{2}\,c\right)^2\right)(dx + c)c + \left(\sqrt{2}\cos\left(\frac{1}{2}\,c\right)^2 + \sqrt{2}\sin\left(\frac{1}{2}\,c\right)^2\right)c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((exp_integral_e(3, 1/2*I*d*x) + exp_integral_e(3, -1/2*I*d*x))*cos(1/
2*c)^3 + (exp_integral_e(3, 1/2*I*d*x) + exp_integral_e(3, -1/2*I*d*x))*cos
(1/2*c)*sin(1/2*c)^2 + (-I*exp_integral_e(3, 1/2*I*d*x) + I*exp_integral_e(
3, -1/2*I*d*x))*sin(1/2*c)^3 + (exp_integral_e(3, 1/2*I*d*x) + exp_integral
_e(3, -1/2*I*d*x))*cos(1/2*c) + ((-I*exp_integral_e(3, 1/2*I*d*x) + I*exp_i
ntegral_e(3, -1/2*I*d*x))*cos(1/2*c)^2 - I*exp_integral_e(3, 1/2*I*d*x) + I
*exp_integral_e(3, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)*d^2/((sqrt(2)*cos(1/2*c
)^2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c)^2 - 2*(sqrt(2)*cos(1/2*c)^2 + sqrt(2)
*sin(1/2*c)^2)*(d*x + c)*c + (sqrt(2)*cos(1/2*c)^2 + sqrt(2)*sin(1/2*c)^2)*
c^2)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c + dx\right) + 1\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 662, normalized size = 4.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/
2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + d^2*x^2*real_part(cos_integral(-1/2*d*x
))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d^2*x^2*imag_p
art(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4
*c) - 2*d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/4*d*x)^2*tan(1/4*c) + 4*d^2*x^2*sgn(cos(1/2*d*x + 1/2*c))*sin_integr
al(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c) - d^2*x^2*real_part(cos_integral(1/2*
d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - d^2*x^2*real_part(cos_inte
gral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 + d^2*x^2*real_par
t(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + d^2*x^2*r
eal_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 2
*d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4
*c) - 2*d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/4*c) + 4*d^2*x^2*sgn(cos(1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan
(1/4*c) - d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)
) - d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)) - 8
*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 8*d*x*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4*c)^2 - 8*sgn(cos(1/2*d*x + 1/2*c))*tan
(1/4*d*x)^2*tan(1/4*c)^2 + 8*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x) + 8
*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c) + 8*sgn(cos(1/2*d*x + 1/2*c))*tan
(1/4*d*x)^2 + 32*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4*c) + 8*sgn(
cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 - 8*sgn(cos(1/2*d*x + 1/2*c)))*sqrt(a)/(
x^2*tan(1/4*d*x)^2*tan(1/4*c)^2 + x^2*tan(1/4*d*x)^2 + x^2*tan(1/4*c)^2 + x
^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/x^3,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/x^3, x)
```

3.150 $\int x^3 \sqrt{a+a\cos(x)}\,dx$

Optimal. Leaf size=68

$$-96\sqrt{a+a\cos(x)}+12x^2\sqrt{a+a\cos(x)}-48x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

```
[Out] -96*(a+a*cos(x))^(1/2)+12*x^2*(a+a*cos(x))^(1/2)-48*x*(a+a*cos(x))^(1/2)*ta
n(1/2*x)+2*x^3*(a+a*cos(x))^(1/2)*tan(1/2*x)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3377, 2718}

$$2x^3\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+12x^2\sqrt{a\cos(x)+a}-96\sqrt{a\cos(x)+a}-48x\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Cos[x]],x]
```

```
[Out] -96*Sqrt[a + a*Cos[x]] + 12*x^2*Sqrt[a + a*Cos[x]] - 48*x*Sqrt[a + a*Cos[x]
]*Tan[x/2] + 2*x^3*Sqrt[a + a*Cos[x]]*Tan[x/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3\sqrt{a+a\cos(x)}\,dx = \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^3\cos\left(\frac{x}{2}\right)dx$$
$$= 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \left(6\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\sin\left(\frac{x}{2}\right)dx$$
$$= 12x^2\sqrt{a+a\cos(x)} + 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \left(24\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right.\right.$$
$$= 12x^2\sqrt{a+a\cos(x)} - 48x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right.$$
$$= -96\sqrt{a+a\cos(x)} + 12x^2\sqrt{a+a\cos(x)} - 48x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + 2x^3$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 0.49

$$2\sqrt{a(1+\cos(x))}\left(6\left(-8+x^2\right)+x\left(-24+x^2\right)\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 2*Sqrt[a*(1 + Cos[x])]*(6*(-8 + x^2) + x*(-24 + x^2)*Tan[x/2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.07, size = 87, normalized size = 1.28

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{ix}+1\right)^2\mathrm{e}^{-ix}}\,\left(6ix^2\mathrm{e}^{ix}+x^3\mathrm{e}^{ix}+6ix^2-x^3-48i\mathrm{e}^{ix}-24x\,\mathrm{e}^{ix}-48i+24x\right)}{\mathrm{e}^{ix}+1}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(6*I*x^2*exp(I*x
)+x^3*exp(I*x)+6*I*x^2-x^3-48*I*exp(I*x)-24*x*exp(I*x)-48*I+24*x)
```

Maxima [A]
time = 0.51, size = 48, normalized size = 0.71

$$2\left(\sqrt{2}\,x^3\sin\left(\frac{1}{2}\,x\right)+6\,\sqrt{2}\,x^2\cos\left(\frac{1}{2}\,x\right)-24\,\sqrt{2}\,x\sin\left(\frac{1}{2}\,x\right)-48\,\sqrt{2}\,\cos\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*x^3*sin(1/2*x) + 6*sqrt(2)*x^2*cos(1/2*x) - 24*sqrt(2)*x*sin(1/2
*x) - 48*sqrt(2)*cos(1/2*x))*sqrt(a)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a (\cos (x) + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cos(x) + 1)), x)
```

Giac [**A**]
time = 0.40, size = 55, normalized size = 0.81

$$2\sqrt{2}\left(6\left(x^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) - 8\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\cos\left(\frac{1}{2}x\right) + \left(x^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) - 24\,x\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*(6*(x^2*sgn(cos(1/2*x)) - 8*sgn(cos(1/2*x)))*cos(1/2*x) + (x^3*sg
n(cos(1/2*x)) - 24*x*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [**B**]
time = 0.43, size = 91, normalized size = 1.34

$$\frac{2\sqrt{a}\ \sqrt{\cos (x) + 1}\ (24\,x - \cos (x)\ 48\mathrm{i} + 48\,\sin (x) + x^2\,\cos (x)\ 6\mathrm{i} + x^3\,\cos (x) - 6\,x^2\,\sin (x) + x^3\,\sin (x)\ 1\mathrm{i} - 24\,x\,\cos (x) - x\,\sin (x)\ 24\mathrm{i} + x^2\,6\mathrm{i} - x^3 - 48\mathrm{i})}{\cos (x)\ 1\mathrm{i} - \sin (x) + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(cos(x) + 1)^(1/2)*(24*x - cos(x)*48i + 48*sin(x) + x^2*cos(x)*6
i + x^3*cos(x) - 6*x^2*sin(x) + x^3*sin(x)*1i - 24*x*cos(x) - x*sin(x)*24i
+ x^2*6i - x^3 - 48i))/(cos(x)*1i - sin(x) + 1i)
```

3.151 $\int x^2 \sqrt{a+a\cos(x)}\,dx$

Optimal. Leaf size=53

$$8x\sqrt{a+a\cos(x)} - 16\sqrt{a+a\cos(x)}\,\tan\left(\frac{x}{2}\right) + 2x^2\sqrt{a+a\cos(x)}\,\tan\left(\frac{x}{2}\right)$$

```
[Out] 8*x*(a+a*cos(x))^(1/2)-16*(a+a*cos(x))^(1/2)*tan(1/2*x)+2*x^2*(a+a*cos(x))^
(1/2)*tan(1/2*x)
```

Rubi [A]

time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3377, 2717}

$$2x^2\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + 8x\sqrt{a\cos(x)+a} - 16\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 8*x*Sqrt[a + a*Cos[x]] - 16*Sqrt[a + a*Cos[x]]*Tan[x/2] + 2*x^2*Sqrt[a + a*
Cos[x]]*Tan[x/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2\sqrt{a+a\cos(x)}\,dx = \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\cos\left(\frac{x}{2}\right)dx$$
$$= 2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \left(4\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)dx$$
$$= 8x\sqrt{a+a\cos(x)} + 2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \left(8\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int$$
$$= 8x\sqrt{a+a\cos(x)} - 16\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + 2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.55

$$8\sqrt{a(1+\cos(x))}\left(x+\frac{1}{4}(-8+x^2)\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 8*Sqrt[a*(1 + Cos[x])]*(x + ((-8 + x^2)*Tan[x/2])/4)
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 70, normalized size = 1.32

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{ix}+1\right)^{2} e^{-ix}}\,\left(4ix\,e^{ix}+x^{2}e^{ix}+4ix-x^{2}-8\,e^{ix}+8\right)}{e^{ix}+1}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(4*I*x*exp(I*x)+
x^2*exp(I*x)+4*I*x-x^2-8*exp(I*x)+8)
```

Maxima [A]
time = 0.52, size = 36, normalized size = 0.68

$$2\left(\sqrt{2}\,x^2\sin\left(\frac{1}{2}\,x\right)+4\,\sqrt{2}\,x\cos\left(\frac{1}{2}\,x\right)-8\,\sqrt{2}\,\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*x^2*sin(1/2*x) + 4*sqrt(2)*x*cos(1/2*x) - 8*sqrt(2)*sin(1/2*x))*
sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cos(x) + 1)), x)
```

Giac [A]
time = 0.43, size = 43, normalized size = 0.81

$$2\sqrt{2}\left(4\,x\cos\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+\left(x^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-8\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*(4*x*cos(1/2*x)*sgn(cos(1/2*x)) + (x^2*sgn(cos(1/2*x)) - 8*sgn(co
s(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [B]
time = 0.34, size = 70, normalized size = 1.32

$$\frac{2\sqrt{a}\,\sqrt{\cos\left(x\right)+1}\,\left(x^2\,\cos\left(x\right)-8\,\cos\left(x\right)-4\,x\,\sin\left(x\right)-x^2+8+x\,4\mathrm{i}-\sin\left(x\right)\,8\mathrm{i}+x^2\,\sin\left(x\right)\,1\mathrm{i}+x\,\cos\left(x\right)\,4\mathrm{i}\right)}{\cos\left(x\right)\,1\mathrm{i}-\sin\left(x\right)+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(cos(x) + 1)^(1/2)*(x*4i - 8*cos(x) - sin(x)*8i + x^2*cos(x) + x
^2*sin(x)*1i + x*cos(x)*4i - 4*x*sin(x) - x^2 + 8))/(cos(x)*1i - sin(x) + 1
i)
```

3.152 $\int x \sqrt{a+a \cos(x)}\, dx$

Optimal. Leaf size=32

$$4 \sqrt{a+a \cos(x)}+2 x \sqrt{a+a \cos(x)} \tan\left(\frac{x}{2}\right)$$

```
[Out] 4*(a+a*cos(x))^(1/2)+2*x*(a+a*cos(x))^(1/2)*tan(1/2*x)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3400, 3377, 2718}

$$4 \sqrt{a \cos(x)+a}+2 x \tan\left(\frac{x}{2}\right) \sqrt{a \cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 4*Sqrt[a + a*Cos[x]] + 2*x*Sqrt[a + a*Cos[x]]*Tan[x/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x \sqrt{a+a \cos(x)}\, dx &= \left(\sqrt{a+a \cos(x)} \sec\left(\frac{x}{2}\right)\right) \int x \cos\left(\frac{x}{2}\right) dx \\
&= 2 x \sqrt{a+a \cos(x)} \tan\left(\frac{x}{2}\right)-\left(2 \sqrt{a+a \cos(x)} \sec\left(\frac{x}{2}\right)\right) \int \sin\left(\frac{x}{2}\right) dx \\
&= 4 \sqrt{a+a \cos(x)}+2 x \sqrt{a+a \cos(x)} \tan\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.69

$$2\sqrt{a(1+\cos(x))}\left(2+x\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 2*Sqrt[a*(1 + Cos[x])]*(2 + x*Tan[x/2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 55, normalized size = 1.72

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{ix}+1\right)^{2} e^{-ix}}\,\left(2ie^{ix}+x\,e^{ix}+2i-x\right)}{e^{ix}+1}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(2*I*exp(I*x)+x*
exp(I*x)+2*I-x)
```

Maxima [A]
time = 0.51, size = 24, normalized size = 0.75

$$2\left(\sqrt{2}\,x\sin\left(\frac{1}{2}\,x\right)+2\,\sqrt{2}\,\cos\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*x*sin(1/2*x) + 2*sqrt(2)*cos(1/2*x))*sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(cos(x) + 1)), x)
```

Giac [A]
time = 0.43, size = 31, normalized size = 0.97

$$2\sqrt{2}\left(x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)+2\,\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*(x*sgn(cos(1/2*x))*sin(1/2*x) + 2*cos(1/2*x)*sgn(cos(1/2*x)))*sqr
t(a)
```

Mupad [B]
time = 0.31, size = 50, normalized size = 1.56

$$\frac{2\sqrt{a}\,\sqrt{\cos\left(x\right)+1}\,\left(x\,\cos\left(x\right)+\cos\left(x\right)\,2\mathrm{i}-2\,\sin\left(x\right)-x+x\,\sin\left(x\right)\,1\mathrm{i}+2\mathrm{i}\right)}{\cos\left(x\right)\,1\mathrm{i}-\sin\left(x\right)+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(cos(x) + 1)^(1/2)*(cos(x)*2i - x - 2*sin(x) + x*cos(x) + x*sin(
x)*1i + 2i))/(cos(x)*1i - sin(x) + 1i)
```

3.153 $\int \sqrt{a+a\cos(x)}\,dx$

Optimal. Leaf size=15

$$\frac{2a\sin(x)}{\sqrt{a+a\cos(x)}}$$

```
[Out] 2*a*sin(x)/(a+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2725}

$$\frac{2a\sin(x)}{\sqrt{a\cos(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]],x]
```

```
[Out] (2*a*Sin[x])/Sqrt[a + a*Cos[x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(x)}\,dx = \frac{2a\sin(x)}{\sqrt{a+a\cos(x)}}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.20

$$2\sqrt{a(1+\cos(x))}\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]],x]
```

```
[Out] 2*Sqrt[a*(1 + Cos[x])]*Tan[x/2]
```

Maple [A]
time = 0.07, size = 25, normalized size = 1.67

method	result	size
default	$\frac{2a\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^{2}\left(\frac{x}{2}\right)\right)}}$	25
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{ix}+1\right)^{2}e^{-ix}}\,\left(e^{ix}-1\right)}{e^{ix}+1}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(a*cos(1/2*x)^2)^(1/2)
```

Maxima [A]
time = 0.53, size = 12, normalized size = 0.80

$$2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*x)
```

Fricas [A]
time = 0.35, size = 18, normalized size = 1.20

$$\frac{2\sqrt{a\cos(x)+a}\,\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*cos(x) + a)*sin(x)/(cos(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cos(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2),x)
[Out] Integral(sqrt(a*cos(x) + a), x)
```

Giac [A]
time = 0.44, size = 17, normalized size = 1.13

$$2\sqrt{2}\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(1/2*x))*sin(1/2*x)
```

Mupad [B]
time = 0.29, size = 34, normalized size = 2.27

$$\frac{2\sqrt{a}\,\sqrt{\cos\left(x\right)+1}\,\left(\cos\left(x\right)-1+\sin\left(x\right)\,1\mathrm{i}\right)}{\cos\left(x\right)\,1\mathrm{i}-\sin\left(x\right)+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(cos(x) + 1)^(1/2)*(cos(x) + sin(x)*1i - 1))/(cos(x)*1i - sin(x)
 + 1i)
```

3.154 $\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx$

Optimal. Leaf size=23

$$\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)$$

```
[Out] Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3400, 3383}

$$\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx &= \left(\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx\\ &= \sqrt{a+a\cos(x)}\,\text{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\sqrt{a(1+\cos(x))}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x,x]
[Out] Sqrt[a*(1 + Cos[x])]*CosIntegral[x/2]*Sec[x/2]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)/x,x)
[Out] int((a+a*cos(x))^(1/2)/x,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{2}\sqrt{a}\left(\text{Ei}\left(\frac{1}{2}i\,x\right)+\text{Ei}\left(-\frac{1}{2}i\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="maxima")
[Out] 1/2*sqrt(2)*sqrt(a)*(Ei(1/2*I*x) + Ei(-1/2*I*x))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos(x)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))/x, x)
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.70

$$\sqrt{2}\,\sqrt{a}\,\operatorname{Ci}\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(2)*sqrt(a)*cos_integral(1/2*x)*sgn(cos(1/2*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+a\,\cos\left(x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(1/2)/x,x)
```

```
[Out] int((a + a*cos(x))^(1/2)/x, x)
```

3.155 $\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{a+a\cos(x)}}{x}-\frac{1}{2}\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\mathrm{Si}\left(\frac{x}{2}\right)$$

```
[Out] -(a+a*cos(x))^(1/2)/x-1/2*sec(1/2*x)*Si(1/2*x)*(a+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3378, 3380}

$$-\frac{1}{2}\mathrm{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{\sqrt{a\cos(x)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cos[x]]/x) - (Sqrt[a + a*Cos[x]]*Sec[x/2]*SinIntegral[x/2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = \left(\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{\sqrt{a+a\cos(x)}}{x} - \frac{1}{2}\left(\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x}\,dx$$
$$= -\frac{\sqrt{a+a\cos(x)}}{x} - \frac{1}{2}\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\mathrm{Si}\left(\frac{x}{2}\right)$$

Mathematica [**A**]
time = 0.04, size = 33, normalized size = 0.79

$$-\frac{\sqrt{a(1+\cos(x))}\left(2+x\sec\left(\frac{x}{2}\right)\mathrm{Si}\left(\frac{x}{2}\right)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x^2,x]
[Out] -1/2*(Sqrt[a*(1 + Cos[x])]*(2 + x*Sec[x/2]*SinIntegral[x/2]))/x
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)/x^2,x)
[Out] int((a+a*cos(x))^(1/2)/x^2,x)
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.52, size = 23, normalized size = 0.55

$$-\frac{1}{4}\sqrt{2}\sqrt{a}\left(i\,\Gamma\left(-1,\frac{1}{2}i\,x\right)-i\,\Gamma\left(-1,-\frac{1}{2}i\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="maxima")
[Out] -1/4*sqrt(2)*sqrt(a)*(I*gamma(-1, 1/2*I*x) - I*gamma(-1, -1/2*I*x))
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cos(x)+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))/x**2, x)
```

Giac [**A**]
time = 0.43, size = 34, normalized size = 0.81

$$-\frac{\sqrt{2}\left(x\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\operatorname{Si}\left(\frac{1}{2}\,x\right)+2\,\cos\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(x*sgn(cos(1/2*x))*sin_integral(1/2*x) + 2*cos(1/2*x)*sgn(cos(
1/2*x)))*sqrt(a)/x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\cos(x)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(1/2)/x^2,x)
```

```
[Out] int((a + a*cos(x))^(1/2)/x^2, x)
```

3.156 $\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{a+a\cos(x)}}{2x^2}-\frac{1}{8}\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)+\frac{\sqrt{a+a\cos(x)}\,\tan\left(\frac{x}{2}\right)}{4x}$$

```
[Out] -1/2*(a+a*cos(x))^(1/2)/x^2-1/8*Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+1/4
*(a+a*cos(x))^(1/2)*tan(1/2*x)/x
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3378, 3383}

$$-\frac{1}{8}\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{\sqrt{a\cos(x)+a}}{2x^2}+\frac{\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + a*Cos[x]]/x^2 - (Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
)/8 + (Sqrt[a + a*Cos[x]]*Tan[x/2])/(4*x)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x^3}\,dx \\
&= -\frac{\sqrt{a+a\cos(x)}}{2x^2} - \frac{1}{4}\left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}\,dx \\
&= -\frac{\sqrt{a+a\cos(x)}}{2x^2} + \frac{\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \cos \\
&= -\frac{\sqrt{a+a\cos(x)}}{2x^2} - \frac{1}{8}\sqrt{a+a\cos(x)}\,\mathrm{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)}{4x}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.66

$$
-\frac{\sqrt{a(1+\cos(x))}\left(4+x^2\mathrm{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-2x\tan\left(\frac{x}{2}\right)\right)}{8x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x^3,x]
```

```
[Out] -1/8*(Sqrt[a*(1 + Cos[x])]*(4 + x^2*CosIntegral[x/2]*Sec[x/2] - 2*x*Tan[x/2
]))/x^2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)/x^3,x)
```

```
[Out] int((a+a*cos(x))^(1/2)/x^3,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 19, normalized size = 0.28

$$
\frac{1}{8}\sqrt{2}\sqrt{a}\left(\Gamma\left(-2,\frac{1}{2}i\,x\right)+\Gamma\left(-2,-\frac{1}{2}i\,x\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*sqrt(a)*(gamma(-2, 1/2*I*x) + gamma(-2, -1/2*I*x))
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(x\right)+1\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))/x**3, x)
```

Giac [**A**]
time = 0.45, size = 48, normalized size = 0.72

$$-\frac{\sqrt{2}\left(x^{2}\operatorname{Ci}\left(\frac{1}{2}x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)-2\,x\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)+4\,\cos\left(\frac{1}{2}x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\sqrt{a}}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(x^2*cos_integral(1/2*x)*sgn(cos(1/2*x)) - 2*x*sgn(cos(1/2*x))
*sin(1/2*x) + 4*cos(1/2*x)*sgn(cos(1/2*x)))*sqrt(a)/x^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\cos\left(x\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(1/2)/x^3,x)
```

```
[Out] int((a + a*cos(x))^(1/2)/x^3, x)
```

3.157 $\int x^3 \sqrt{a-a\cos(x)}\,dx$

Optimal. Leaf size=72

$$-96\sqrt{a-a\cos(x)}+12x^2\sqrt{a-a\cos(x)}+48x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

```
[Out] -96*(a-a*cos(x))^(1/2)+12*x^2*(a-a*cos(x))^(1/2)+48*x*cot(1/2*x)*(a-a*cos(x
))^(1/2)-2*x^3*cot(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3400, 3377, 2717}

$$-2x^3\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}+12x^2\sqrt{a-a\cos(x)}-96\sqrt{a-a\cos(x)}+48x\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Cos[x]],x]
```

```
[Out] -96*Sqrt[a - a*Cos[x]] + 12*x^2*Sqrt[a - a*Cos[x]] + 48*x*Sqrt[a - a*Cos[x]
]*Cot[x/2] - 2*x^3*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^3\sqrt{a-a\cos(x)}\,dx &= \left(\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\right)\int x^3\sin\left(\frac{x}{2}\right)dx\\
&= -2x^3\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)+\left(6\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\right)\int x^2\cos\left(\frac{x}{2}\right)dx\\
&= 12x^2\sqrt{a-a\cos(x)}-2x^3\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)-\left(24\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right.\right.\\
&= 12x^2\sqrt{a-a\cos(x)}+48x\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)-2x^3\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)\\
&= -96\sqrt{a-a\cos(x)}+12x^2\sqrt{a-a\cos(x)}+48x\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)-2x^3\sqrt{}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.47

$$
-2\sqrt{a-a\cos(x)}\left(-6(-8+x^2)+x(-24+x^2)\cot\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a - a*Cos[x]],x]
[Out] -2*Sqrt[a - a*Cos[x]]*(-6*(-8 + x^2) + x*(-24 + x^2)*Cot[x/2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 86, normalized size = 1.19

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a\left(e^{ix}-1\right)^{2}e^{-ix}}\,\left(6ix^2e^{ix}+x^3e^{ix}-6ix^2+x^3-48ie^{ix}-24x\,e^{ix}+48i-24x\right)}{e^{ix}-1}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a-a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(6*I*x^2*exp(I*
x)+x^3*exp(I*x)-6*I*x^2+x^3-48*I*exp(I*x)-24*x*exp(I*x)+48*I-24*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(60) = 120.
time = 0.53, size = 129, normalized size = 1.79

$$
-\left(\left(6\sqrt{2}x^2-6\left(\sqrt{2}x^2-8\sqrt{2}\right)\cos(x)-\left(\sqrt{2}x^3-24\sqrt{2}x\right)\sin(x)-48\sqrt{2}\right)\cos\left(\frac{1}{2}\pi+\frac{1}{2}\arctan(\sin(x),\cos(x))\right)+\left(\sqrt{2}x^3+\left(\sqrt{2}x^3-24\sqrt{2}x\right)\cos(x)-6\left(\sqrt{2}x^2-8\sqrt{2}\right)\sin(x)-24\sqrt{2}x\right)\sin\left(\frac{1}{2}\pi+\frac{1}{2}\arctan(\sin(x),\cos(x))\right)\right)\sqrt{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -((6*sqrt(2)*x^2 - 6*(sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - (sqrt(2)*x^3 - 24*s
qrt(2)*x)*sin(x) - 48*sqrt(2))*cos(1/2*pi + 1/2*arctan2(sin(x), cos(x))) +
(sqrt(2)*x^3 + (sqrt(2)*x^3 - 24*sqrt(2)*x)*cos(x) - 6*(sqrt(2)*x^2 - 8*sqr
t(2))*sin(x) - 24*sqrt(2)*x)*sin(1/2*pi + 1/2*arctan2(sin(x), cos(x))))*sqr
t(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{-a (\cos(x) - 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-a*(cos(x) - 1)), x)
```

Giac [A]
time = 0.42, size = 55, normalized size = 0.76

$$-2\sqrt{2}\left(\left(x^3\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-24\,x\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\right)\cos\left(\frac{1}{2}x\right)-6\left(x^2\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-8\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*((x^3*sgn(sin(1/2*x)) - 24*x*sgn(sin(1/2*x)))*cos(1/2*x) - 6*(x^
2*sgn(sin(1/2*x)) - 8*sgn(sin(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [B]
time = 0.43, size = 92, normalized size = 1.28

$$\frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,\left(24\,x+\cos(x)\,48\mathrm{i}-48\,\sin(x)-x^2\,\cos(x)\,6\mathrm{i}-x^3\,\cos(x)+6\,x^2\,\sin(x)-x^3\,\sin(x)\,1\mathrm{i}+24\,x\,\cos(x)+x\,\sin(x)\,24\mathrm{i}+x^2\,6\mathrm{i}-x^3-48\mathrm{i}\right)}{\sin(x)-\cos(x)\,1\mathrm{i}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a - a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(1 - cos(x))^(1/2)*(24*x + cos(x)*48i - 48*sin(x) - x^2*cos(x)*6
i - x^3*cos(x) + 6*x^2*sin(x) - x^3*sin(x)*1i + 24*x*cos(x) + x*sin(x)*24i
+ x^2*6i - x^3 - 48i))/(sin(x) - cos(x)*1i + 1i)
```

3.158 $\int x^2 \sqrt{a - a\cos(x)}\, dx$

Optimal. Leaf size=56

$$8x\sqrt{a-a\cos(x)}+16\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

```
[Out] 8*x*(a-a*cos(x))^(1/2)+16*cot(1/2*x)*(a-a*cos(x))^(1/2)-2*x^2*cot(1/2*x)*(a
-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3400, 3377, 2718}

$$-2x^2\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}+8x\sqrt{a-a\cos(x)}+16\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a - a*Cos[x]],x]
```

```
[Out] 8*x*Sqrt[a - a*Cos[x]] + 16*Sqrt[a - a*Cos[x]]*Cot[x/2] - 2*x^2*Sqrt[a - a*
Cos[x]]*Cot[x/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a-a\cos(x)}\,dx &= \left(\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\right)\int x^2\sin\left(\frac{x}{2}\right)\,dx \\ &= -2x^2\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)+\left(4\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx \\ &= 8x\sqrt{a-a\cos(x)}-2x^2\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)-\left(8\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\right) \\ &= 8x\sqrt{a-a\cos(x)}+16\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)-2x^2\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.54

$$8\sqrt{a-a\cos(x)}\left(x-\frac{1}{4}(-8+x^2)\cot\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a - a*Cos[x]],x]
[Out] 8*Sqrt[a - a*Cos[x]]*(x - ((-8 + x^2)*Cot[x/2])/4)
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 69, normalized size = 1.23

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a\left(e^{ix}-1\right)^2 e^{-ix}}\,\left(4ix\,e^{ix}+x^2e^{ix}-4ix+x^2-8\,e^{ix}-8\right)}{e^{ix}-1}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a-a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(4*I*x*exp(I*x)
+x^2*exp(I*x)-4*I*x+x^2-8*exp(I*x)-8)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(46) = 92.
time = 0.52, size = 100, normalized size = 1.79

$$\left(\left(4\sqrt{2}\,x\cos\left(x\right)+\left(\sqrt{2}\,x^2-8\sqrt{2}\right)\sin\left(x\right)-4\sqrt{2}\,x\right)\cos\left(\frac{1}{2}\pi+\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)-\left(\sqrt{2}\,x^2-4\sqrt{2}\,x\sin\left(x\right)+\left(\sqrt{2}\,x^2-8\sqrt{2}\right)\cos\left(x\right)-8\sqrt{2}\right)\sin\left(\frac{1}{2}\pi+\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] ((4*sqrt(2)*x*cos(x) + (sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 4*sqrt(2)*x)*cos(
1/2*pi + 1/2*arctan2(sin(x), cos(x))) - (sqrt(2)*x^2 - 4*sqrt(2)*x*sin(x) +
 (sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - 8*sqrt(2))*sin(1/2*pi + 1/2*arctan2(sin
(x), cos(x))))*sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{-a (\cos (x) - 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a-a*cos(x))**(1/2),x)
[Out] Integral(x**2*sqrt(-a*(cos(x) - 1)), x)
```

Giac [A]
time = 0.44, size = 51, normalized size = 0.91

$$2 \sqrt{2} \left(4 x \mathrm{sgn}\left(\sin \left(\frac{1}{2} x\right)\right) \sin \left(\frac{1}{2} x\right) - \left(x^2 \mathrm{sgn}\left(\sin \left(\frac{1}{2} x\right)\right) - 8 \,\mathrm{sgn}\left(\sin \left(\frac{1}{2} x\right)\right)\right) \cos \left(\frac{1}{2} x\right) - 8 \,\mathrm{sgn}\left(\sin \left(\frac{1}{2} x\right)\right)\right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*(4*x*sgn(sin(1/2*x))*sin(1/2*x) - (x^2*sgn(sin(1/2*x)) - 8*sgn(si
n(1/2*x)))*cos(1/2*x) - 8*sgn(sin(1/2*x)))*sqrt(a)
```

Mupad [B]
time = 0.34, size = 71, normalized size = 1.27

$$\frac{2 \sqrt{a}\, \sqrt{1 - \cos (x)}\, (8 \cos (x) - x^2 \cos (x) + 4 x \sin (x) - x^2 + 8 + x\, 4\mathrm{i} + \sin (x)\, 8\mathrm{i} - x^2 \sin (x)\, 1\mathrm{i} - x \cos (x)\, 4\mathrm{i})}{\sin (x) - \cos (x)\, 1\mathrm{i} + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - a*cos(x))^(1/2),x)
[Out] (2*a^(1/2)*(1 - cos(x))^(1/2)*(x*4i + 8*cos(x) + sin(x)*8i - x^2*cos(x) - x
^2*sin(x)*1i - x*cos(x)*4i + 4*x*sin(x) - x^2 + 8))/(sin(x) - cos(x)*1i + 1
i)
```

3.159 $\int x \sqrt{a - a\cos(x)}\, dx$

Optimal. Leaf size=34

$$4\sqrt{a - a\cos(x)} - 2x\sqrt{a - a\cos(x)}\cot\left(\frac{x}{2}\right)$$

```
[Out] 4*(a-a*cos(x))^(1/2)-2*x*cot(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3400, 3377, 2717}

$$4\sqrt{a - a\cos(x)} - 2x\cot\left(\frac{x}{2}\right)\sqrt{a - a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a - a*Cos[x]],x]
```

```
[Out] 4*Sqrt[a - a*Cos[x]] - 2*x*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{a - a\cos(x)}\, dx &= \left(\sqrt{a - a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)\, dx \\
&= -2x\sqrt{a - a\cos(x)}\cot\left(\frac{x}{2}\right) + \left(2\sqrt{a - a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \cos\left(\frac{x}{2}\right)\, dx \\
&= 4\sqrt{a - a\cos(x)} - 2x\sqrt{a - a\cos(x)}\cot\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.68

$$-2\sqrt{a-a\cos(x)}\left(-2+x\cot\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Cos[x]],x]
```

```
[Out] -2*Sqrt[a - a*Cos[x]]*(-2 + x*Cot[x/2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 54, normalized size = 1.59

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a\left(e^{ix}-1\right)^{2}e^{-ix}}\,\left(2ie^{ix}+x\,e^{ix}-2i+x\right)}{e^{ix}-1}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(2*I*exp(I*x)+x
*exp(I*x)-2*I+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(28) = 56.
time = 0.50, size = 72, normalized size = 2.12

$$\left(\left(\sqrt{2}\,x\sin\left(x\right)+2\,\sqrt{2}\,\cos\left(x\right)-2\,\sqrt{2}\right)\cos\left(\frac{1}{2}\,\pi+\frac{1}{2}\,\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)-\left(\sqrt{2}\,x\cos\left(x\right)+\sqrt{2}\,x-2\,\sqrt{2}\,\sin\left(x\right)\right)\sin\left(\frac{1}{2}\,\pi+\frac{1}{2}\,\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] ((sqrt(2)*x*sin(x) + 2*sqrt(2)*cos(x) - 2*sqrt(2))*cos(1/2*pi + 1/2*arctan2
(sin(x), cos(x))) - (sqrt(2)*x*cos(x) + sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(1
/2*pi + 1/2*arctan2(sin(x), cos(x))))*sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-a(\cos(x)-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a*(cos(x) - 1)), x)
```

Giac [**A**]
time = 0.46, size = 31, normalized size = 0.91

$$-2\sqrt{2}\left(x\cos\left(\frac{1}{2}x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-2\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(x*cos(1/2*x)*sgn(sin(1/2*x)) - 2*sgn(sin(1/2*x))*sin(1/2*x))*sq
rt(a)
```

Mupad [**B**]
time = 0.32, size = 48, normalized size = 1.41

$$-\frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,\left(x+\cos(x)\,2\mathrm{i}-2\sin(x)+x\,\cos(x)+x\,\sin(x)\,1\mathrm{i}-2\mathrm{i}\right)}{\sin(x)-\cos(x)\,1\mathrm{i}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a - a*cos(x))^(1/2),x)
```

```
[Out] -(2*a^(1/2)*(1 - cos(x))^(1/2)*(x + cos(x)*2i - 2*sin(x) + x*cos(x) + x*sin
(x)*1i - 2i))/(sin(x) - cos(x)*1i + 1i)
```

3.160 $\int \sqrt{a-a\cos(x)}\,dx$

Optimal. Leaf size=16

$$-\frac{2a\sin(x)}{\sqrt{a-a\cos(x)}}$$

```
[Out] -2*a*sin(x)/(a-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2725}

$$-\frac{2a\sin(x)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-2*a*Sin[x])/Sqrt[a - a*Cos[x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a-a\cos(x)}\,dx = -\frac{2a\sin(x)}{\sqrt{a-a\cos(x)}}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.19

$$-2\sqrt{a-a\cos(x)}\,\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[x]],x]
```

```
[Out] -2*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Maple [A]
time = 0.08, size = 25, normalized size = 1.56

method	result	size
default	$-\frac{2 \sin \left(\frac{x}{2}\right) a \cos \left(\frac{x}{2}\right) \sqrt{2}}{\sqrt{a \left(\sin^{2}\left(\frac{x}{2}\right)\right)}}$	25
risch	$-\frac{i \sqrt{2}\, \sqrt{-a \left({e}^{i x}-1\right)^{2} {e}^{-i x}}\, \left({e}^{i x}+1\right)}{{e}^{i x}-1}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*sin(1/2*x)*a*cos(1/2*x)*2^(1/2)/(a*sin(1/2*x)^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 23, normalized size = 1.44

$$-\frac{2 \sqrt{2} \sqrt{a}}{\sqrt{\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right) + 1\right)^{2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)*sqrt(a)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 0.37, size = 19, normalized size = 1.19

$$-\frac{2 \sqrt{-a \cos\left(x\right) + a} \left(\cos\left(x\right) + 1\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*cos(x) + a)*(cos(x) + 1)/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \cos\left(x\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*cos(x) + a), x)
```

Giac [A]
time = 0.42, size = 26, normalized size = 1.62

$$-2\sqrt{2}\left(\cos\left(\frac{1}{2}x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(cos(1/2*x)*sgn(sin(1/2*x)) - sgn(sin(1/2*x)))*sqrt(a)
```

Mupad [B]
time = 0.29, size = 34, normalized size = 2.12

$$-\frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,\left(\cos(x)+1+\sin(x)\,1\mathrm{i}\right)}{\sin(x)-\cos(x)\,1\mathrm{i}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(x))^(1/2),x)
```

```
[Out] -(2*a^(1/2)*(1 - cos(x))^(1/2)*(cos(x) + sin(x)*1i + 1))/(sin(x) - cos(x)*1
i + 1i)
```

3.161 $\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx$

Optimal. Leaf size=24

$$\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)$$

```
[Out] csc(1/2*x)*Si(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3400, 3380}

$$\text{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx &= \left(\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x}\,dx\\ &= \sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\sqrt{a-a\cos(x)}\,\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - a\cos(x)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(x))^(1/2)/x,x)
```

```
[Out] int((a-a*cos(x))^(1/2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\cos(x) - 1)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-a*(cos(x) - 1))/x, x)
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.67

$$\sqrt{2}\,\sqrt{a}\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\,\mathrm{Si}\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="giac")
[Out] sqrt(2)*sqrt(a)*sgn(sin(1/2*x))*sin_integral(1/2*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a-a\,\cos\left(x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(x))^(1/2)/x,x)
[Out] int((a - a*cos(x))^(1/2)/x, x)
```

3.162 $\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx$

Optimal. Leaf size=44

$$-\frac{\sqrt{a-a\cos(x)}}{x}+\frac{1}{2}\sqrt{a-a\cos(x)}\,\mathrm{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

```
[Out] -(a-a*cos(x))^(1/2)/x+1/2*Ci(1/2*x)*csc(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3400, 3378, 3383}

$$\frac{1}{2}\mathrm{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}-\frac{\sqrt{a-a\cos(x)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[x]]/x^2,x]
```

```
[Out] -(Sqrt[a - a*Cos[x]]/x) + (Sqrt[a - a*Cos[x]]*CosIntegral[x/2]*Csc[x/2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{\sqrt{a-a\cos(x)}}{x} + \frac{1}{2}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx$$
$$= -\frac{\sqrt{a-a\cos(x)}}{x} + \frac{1}{2}\sqrt{a-a\cos(x)}\,\mathrm{Ci}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.77

$$\frac{\sqrt{a-a\cos(x)}\left(-2+x\mathrm{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x^2,x]
[Out] (Sqrt[a - a*Cos[x]]*(-2 + x*CosIntegral[x/2]*Csc[x/2]))/(2*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\cos\left(x\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(x))^(1/2)/x^2,x)
[Out] int((a-a*cos(x))^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(-a*cos(x) + a)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\cos\left(x\right)-1\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-a*(cos(x) - 1))/x**2, x)
```

Giac [A]
time = 0.60, size = 34, normalized size = 0.77

$$\frac{\sqrt{2}\left(x\operatorname{Ci}\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)-2\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(x*cos_integral(1/2*x)*sgn(sin(1/2*x)) - 2*sgn(sin(1/2*x))*sin(
1/2*x))*sqrt(a)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a-a\,\cos\left(x\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(x))^(1/2)/x^2,x)
```

```
[Out] int((a - a*cos(x))^(1/2)/x^2, x)
```

3.163 $\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{a-a\cos(x)}}{2x^2}-\frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x}-\frac{1}{8}\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)$$

```
[Out] -1/2*(a-a*cos(x))^(1/2)/x^2-1/4*cot(1/2*x)*(a-a*cos(x))^(1/2)/x-1/8*csc(1/2
*x)*Si(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3400, 3378, 3380}

$$-\frac{1}{8}\text{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}-\frac{\sqrt{a-a\cos(x)}}{2x^2}-\frac{\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a - a*Cos[x]]/x^2 - (Sqrt[a - a*Cos[x]]*Cot[x/2])/(4*x) - (Sqrt[a
 - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x^3}\,dx$$
$$= -\frac{\sqrt{a-a\cos(x)}}{2x^2} + \frac{1}{4}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{\sqrt{a-a\cos(x)}}{2x^2} - \frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin(}{x}$$
$$= -\frac{\sqrt{a-a\cos(x)}}{2x^2} - \frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)$$

Mathematica [**A**]
time = 0.04, size = 45, normalized size = 0.64

$$-\frac{\sqrt{a-a\cos(x)}\left(4+2x\cot\left(\frac{x}{2}\right)+x^2\csc\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x^3,x]
[Out] -1/8*(Sqrt[a - a*Cos[x]]*(4 + 2*x*Cot[x/2] + x^2*Csc[x/2]*SinIntegral[x/2])
)/x^2
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(x))^(1/2)/x^3,x)
[Out] int((a-a*cos(x))^(1/2)/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(-a*cos(x) + a)/x^3, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\cos\left(x\right)-1\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-a*(cos(x) - 1))/x**3, x)
```

Giac [**A**]
time = 0.48, size = 48, normalized size = 0.69

$$-\frac{\sqrt{2}\left(x^{2}\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{1}{2}\,x\right)+2\,x\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)+4\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(x^2*sgn(sin(1/2*x))*sin_integral(1/2*x) + 2*x*cos(1/2*x)*sgn(
sin(1/2*x)) + 4*sgn(sin(1/2*x))*sin(1/2*x))*sqrt(a)/x^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\cos\left(x\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(x))^(1/2)/x^3,x)
```

```
[Out] int((a - a*cos(x))^(1/2)/x^3, x)
```

3.164 $\int x^3(a+a\cos(x))^{3/2}\,dx$

Optimal. Leaf size=185

$$-\frac{1280}{9}a\sqrt{a+a\cos(x)}+16ax^2\sqrt{a+a\cos(x)}-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x}$$

```
[Out] -1280/9*a*(a+a*cos(x))^(1/2)+16*a*x^2*(a+a*cos(x))^(1/2)-64/27*a*cos(1/2*x)
^2*(a+a*cos(x))^(1/2)+8/3*a*x^2*cos(1/2*x)^2*(a+a*cos(x))^(1/2)-32/9*a*x*co
s(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)+4/3*a*x^3*cos(1/2*x)*sin(1/2*x)*(a+a
*cos(x))^(1/2)-640/9*a*x*(a+a*cos(x))^(1/2)*tan(1/2*x)+8/3*a*x^3*(a+a*cos(x
))^(1/2)*tan(1/2*x)
```

Rubi [A]
time = 0.13, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3400, 3392, 3377, 2718, 3391}

$$\frac{4}{3}ax^3\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax^3\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+16ax^2\sqrt{a\cos(x)+a}-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{1280}{9}a\sqrt{a\cos(x)+a}-\frac{32}{9}ax\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{640}{9}ax\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (-1280*a*Sqrt[a + a*Cos[x]])/9 + 16*a*x^2*Sqrt[a + a*Cos[x]] - (64*a*Cos[x/
2]^2*Sqrt[a + a*Cos[x]])/27 + (8*a*x^2*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/3 - (
32*a*x*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/9 + (4*a*x^3*Cos[x/2]*Sqrt[a +
 a*Cos[x]]*Sin[x/2])/3 - (640*a*x*Sqrt[a + a*Cos[x]]*Tan[x/2])/9 + (8*a*x^3
*Sqrt[a + a*Cos[x]]*Tan[x/2])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3(a+a\cos(x))^{3/2}\,dx &= \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^3\cos^3\left(\frac{x}{2}\right)\,dx \\
&= \frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)+\frac{1}{3}\Big(4a \\
&= -\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}-\frac{32}{9}ax\cos\Big(\\
&= 16ax^2\sqrt{a+a\cos(x)}-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a} \\
&= -\frac{128}{9}a\sqrt{a+a\cos(x)}+16ax^2\sqrt{a+a\cos(x)}-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\
&= -\frac{1280}{9}a\sqrt{a+a\cos(x)}+16ax^2\sqrt{a+a\cos(x)}-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 67, normalized size = 0.36

$$\frac{2}{27}a\sqrt{a(1+\cos(x))}\left(-1936+234x^2+3x(-328+15x^2)\tan\left(\frac{x}{2}\right)+\cos(x)\left(2(-8+9x^2)+3x(-8+3x^2)\tan\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[x])]*(-1936 + 234*x^2 + 3*x*(-328 + 15*x^2)*Tan[x/2] +
 Cos[x]*(2*(-8 + 9*x^2) + 3*x*(-8 + 3*x^2)*Tan[x/2])))/27
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^3(a+a\cos(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x^3*(a+a*cos(x))^(3/2),x)
```

Maxima [A]
time = 0.50, size = 98, normalized size = 0.53

$$\frac{1}{27}\left(81\sqrt{2}\,ax^3\sin\left(\frac{1}{2}x\right)+486\sqrt{2}\,ax^2\cos\left(\frac{1}{2}x\right)-1944\sqrt{2}\,ax\sin\left(\frac{1}{2}x\right)-3888\sqrt{2}\,a\cos\left(\frac{1}{2}x\right)+2\left(9\sqrt{2}\,ax^2-8\sqrt{2}\,a\right)\cos\left(\frac{3}{2}x\right)+3\left(3\sqrt{2}\,ax^3-8\sqrt{2}\,ax\right)\sin\left(\frac{3}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/27*(81*sqrt(2)*a*x^3*sin(1/2*x) + 486*sqrt(2)*a*x^2*cos(1/2*x) - 1944*sqr
t(2)*a*x*sin(1/2*x) - 3888*sqrt(2)*a*cos(1/2*x) + 2*(9*sqrt(2)*a*x^2 - 8*sq
rt(2)*a)*cos(3/2*x) + 3*(3*sqrt(2)*a*x^3 - 8*sqrt(2)*a*x)*sin(3/2*x))*sqrt(
a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x**3*(a*(cos(x) + 1))**(3/2), x)
```

Giac [A]
time = 0.43, size = 113, normalized size = 0.61

$$\frac{1}{27}\sqrt{2}\left(2\left(9\,ax^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-8\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\cos\left(\frac{3}{2}\,x\right)+486\left(ax^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-8\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\cos\left(\frac{1}{2}\,x\right)+3\left(3\,ax^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-8\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sin\left(\frac{3}{2}\,x\right)+81\left(ax^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-24\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/27*sqrt(2)*(2*(9*a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*cos(3/2*x)
+ 486*(a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*cos(1/2*x) + 3*(3*a*x^3
*sgn(cos(1/2*x)) - 8*a*x*sgn(cos(1/2*x)))*sin(3/2*x) + 81*(a*x^3*sgn(cos(1/
2*x)) - 24*a*x*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(a+a\,\cos\left(x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*cos(x))^(3/2),x)
```

```
[Out] int(x^3*(a + a*cos(x))^(3/2), x)
```

3.165 $\int x^2(a+a\cos(x))^{3/2}\,dx$

Optimal. Leaf size=145

$$\frac{32}{3}ax\sqrt{a+a\cos(x)}+\frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)-\frac{224}{9}a\sqrt{a+a$$

```
[Out] 32/3*a*x*(a+a*cos(x))^(1/2)+16/9*a*x*cos(1/2*x)^2*(a+a*cos(x))^(1/2)+4/3*a*
x^2*cos(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)-224/9*a*(a+a*cos(x))^(1/2)*tan
(1/2*x)+8/3*a*x^2*(a+a*cos(x))^(1/2)*tan(1/2*x)+32/27*a*sin(1/2*x)^2*(a+a*c
os(x))^(1/2)*tan(1/2*x)
```

Rubi [A]

time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3400, 3392, 3377, 2717, 2713}

$$\frac{4}{3}ax^2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax^2\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{32}{3}ax\sqrt{a\cos(x)+a}-\frac{224}{9}a\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{32}{27}a\sin^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (32*a*x*Sqrt[a + a*Cos[x]])/3 + (16*a*x*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/9 +
(4*a*x^2*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/3 - (224*a*Sqrt[a + a*Cos[x]
]*Tan[x/2])/9 + (8*a*x^2*Sqrt[a + a*Cos[x]]*Tan[x/2])/3 + (32*a*Sqrt[a + a*
Cos[x]]*Sin[x/2]^2*Tan[x/2])/27
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \left(2a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int x^2\cos^3\left(\frac{x}{2}\right)\,dx$$
$$= \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\,\sin\left(\frac{x}{2}\right) + \frac{1}{3}(4$$
$$= \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\,\sin\left(\frac{x}{2}\right) + \frac{8}{3}ax$$
$$= \frac{32}{3}ax\sqrt{a+a\cos(x)} + \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a}$$
$$= \frac{32}{3}ax\sqrt{a+a\cos(x)} + \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a}$$

Mathematica [A]
time = 0.15, size = 54, normalized size = 0.37

$$\frac{2}{27}a\sqrt{a(1+\cos(x))}\left(156x+\left(-328+45x^2\right)\tan\left(\frac{x}{2}\right)+\cos(x)\left(12x+\left(-8+9x^2\right)\tan\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[x])]*(156*x + (-328 + 45*x^2)*Tan[x/2] + Cos[x]*(12*x
+ (-8 + 9*x^2)*Tan[x/2])))/27
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2(a+a\cos(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x^2*(a+a*cos(x))^(3/2),x)
```

Maxima [A]
time = 0.50, size = 72, normalized size = 0.50

$$\frac{1}{27}\left(81\sqrt{2}\,ax^2\sin\left(\frac{1}{2}x\right)+12\sqrt{2}\,ax\cos\left(\frac{3}{2}x\right)+324\sqrt{2}\,ax\cos\left(\frac{1}{2}x\right)-648\sqrt{2}\,a\sin\left(\frac{1}{2}x\right)+\left(9\sqrt{2}\,ax^2-8\sqrt{2}\,a\right)\sin\left(\frac{3}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/27*(81*sqrt(2)*a*x^2*sin(1/2*x) + 12*sqrt(2)*a*x*cos(3/2*x) + 324*sqrt(2)
*a*x*cos(1/2*x) - 648*sqrt(2)*a*sin(1/2*x) + (9*sqrt(2)*a*x^2 - 8*sqrt(2)*a
)*sin(3/2*x))*sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x**2*(a*(cos(x) + 1))**(3/2), x)
```

Giac [A]
time = 0.42, size = 85, normalized size = 0.59

$$\frac{1}{27}\sqrt{2}\left(12\,ax\cos\left(\frac{3}{2}x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)+324\,ax\cos\left(\frac{1}{2}x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)+\left(9\,ax^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)-8\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\sin\left(\frac{3}{2}x\right)+81\left(ax^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)-8\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/27*sqrt(2)*(12*a*x*cos(3/2*x)*sgn(cos(1/2*x)) + 324*a*x*cos(1/2*x)*sgn(co
s(1/2*x)) + (9*a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*sin(3/2*x) + 81
*(a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + a \cos\left(x\right)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*cos(x))^(3/2),x)
[Out] int(x^2*(a + a*cos(x))^(3/2), x)
```

3.166 $\int x(a+a\cos(x))^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{16}{3}a\sqrt{a+a\cos(x)}+\frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)+\frac{8}{3}ax\sqrt{a+a\cos(x$$

```
[Out] 16/3*a*(a+a*cos(x))^(1/2)+8/9*a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)+4/3*a*x*cos
(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)+8/3*a*x*(a+a*cos(x))^(1/2)*tan(1/2*x)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3400, 3391, 3377, 2718}

$$\frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{16}{3}a\sqrt{a\cos(x)+a}+\frac{4}{3}ax\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (16*a*Sqrt[a + a*Cos[x]])/3 + (8*a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/9 + (4*a*
x*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/3 + (8*a*x*Sqrt[a + a*Cos[x]]*Tan[x
/2])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
```

```
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x(a+a\cos(x))^{3/2}\,dx = \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos^3\left(\frac{x}{2}\right)\,dx$$
$$= \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{1}{3}\left(4a\sqrt{a\right.$$
$$= \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{8}{3}ax\sqrt{a+}$$
$$= \frac{16}{3}a\sqrt{a+a\cos(x)} + \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x}$$

Mathematica [**A**]
time = 0.05, size = 45, normalized size = 0.51

$$\frac{1}{9}a\sqrt{a(1+\cos(x))}\left(52+4\cos(x)+3x\sec\left(\frac{x}{2}\right)\sin\left(\frac{3x}{2}\right)+27x\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[x])]*(52 + 4*Cos[x] + 3*x*Sec[x/2]*Sin[(3*x)/2] + 27*x*T
an[x/2]))/9
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int x(a+a\cos(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x*(a+a*cos(x))^(3/2),x)
```

Maxima [**A**]
time = 0.52, size = 48, normalized size = 0.54

$$\frac{1}{9}\left(3\sqrt{2}\,ax\sin\left(\frac{3}{2}x\right)+27\sqrt{2}\,ax\sin\left(\frac{1}{2}x\right)+2\sqrt{2}\,a\cos\left(\frac{3}{2}x\right)+54\sqrt{2}\,a\cos\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*(3*sqrt(2)*a*x*sin(3/2*x) + 27*sqrt(2)*a*x*sin(1/2*x) + 2*sqrt(2)*a*cos
(3/2*x) + 54*sqrt(2)*a*cos(1/2*x))*sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x*(a*(cos(x) + 1))**(3/2), x)
```

Giac [A]
time = 0.44, size = 59, normalized size = 0.66

$$\frac{1}{9}\sqrt{2}\left(3\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{3}{2}\,x\right)+27\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)+2\,a\cos\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+54\,a\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(2)*(3*a*x*sgn(cos(1/2*x))*sin(3/2*x) + 27*a*x*sgn(cos(1/2*x))*sin(
1/2*x) + 2*a*cos(3/2*x)*sgn(cos(1/2*x)) + 54*a*cos(1/2*x)*sgn(cos(1/2*x)))*
sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+a\,\cos\left(x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*cos(x))^(3/2),x)
```

```
[Out] int(x*(a + a*cos(x))^(3/2), x)
```

3.167 $\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{3}{2}a\sqrt{a+a\cos(x)}\ \text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)+\frac{1}{2}a\sqrt{a+a\cos(x)}\ \text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)$$

```
[Out] 3/2*a*Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+1/2*a*Ci(3/2*x)*sec(1/2*x)*(a
+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3400, 3393, 3383}

$$\frac{3}{2}a\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{1}{2}a\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)/x,x]
```

```
[Out] (3*a*Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2])/2 + (a*Sqrt[a + a*Cos[x]
]*CosIntegral[(3*x)/2]*Sec[x/2])/2
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx &= \left(2a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x}\,dx \\
&= \left(2a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \left(\frac{3\cos\left(\frac{x}{2}\right)}{4x}+\frac{\cos\left(\frac{3x}{2}\right)}{4x}\right)dx \\
&= \frac{1}{2}\left(a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{3x}{2}\right)}{x}\,dx+\frac{1}{2}\left(3a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \\
&= \frac{3}{2}a\sqrt{a+a\cos(x)}\,\mathrm{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)+\frac{1}{2}a\sqrt{a+a\cos(x)}\,\mathrm{Ci}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.65

$$\frac{1}{2}a\sqrt{a(1+\cos(x))}\left(3\mathrm{CosIntegral}\left(\frac{x}{2}\right)+\mathrm{CosIntegral}\left(\frac{3x}{2}\right)\right)\sec\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x,x]
[Out] (a*Sqrt[a*(1 + Cos[x])]*(3*CosIntegral[x/2] + CosIntegral[(3*x)/2])*Sec[x/2
])/2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cos(x))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(3/2)/x,x)
[Out] int((a+a*cos(x))^(3/2)/x,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 29, normalized size = 0.53

$$\frac{1}{4}\sqrt{2}\,a^{\frac{3}{2}}\left(\mathrm{Ei}\left(\frac{3}{2}i\,x\right)+3\,\mathrm{Ei}\left(\frac{1}{2}i\,x\right)+3\,\mathrm{Ei}\left(-\frac{1}{2}i\,x\right)+\mathrm{Ei}\left(-\frac{3}{2}i\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*a^(3/2)*(Ei(3/2*I*x) + 3*Ei(1/2*I*x) + 3*Ei(-1/2*I*x) + Ei(-3/2
*I*x))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)/x,x)
```

```
[Out] Integral((a*(cos(x) + 1))**(3/2)/x, x)
```

Giac [A]
time = 0.44, size = 32, normalized size = 0.58

$$\frac{1}{2}\sqrt{2}\left(a\,\mathrm{Ci}\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+3\,a\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*cos_integral(3/2*x)*sgn(cos(1/2*x)) + 3*a*cos_integral(1/2*x
)*sgn(cos(1/2*x)))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\,\cos(x))^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(3/2)/x,x)
```

```
[Out] int((a + a*cos(x))^(3/2)/x, x)
```

3.168 $\int \frac{(a+a\cos(x))^{3/2}}{x^2} dx$

Optimal. Leaf size=79

$$-\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x}-\frac{3}{4}a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)-\frac{3}{4}a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\text{Si}\left(\frac{3x}{2}\right)$$

```
[Out] -2*a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)/x-3/4*a*sec(1/2*x)*Si(1/2*x)*(a+a*cos(
x))^(1/2)-3/4*a*sec(1/2*x)*Si(3/2*x)*(a+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3394, 3380}

$$-\frac{3}{4}a\text{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{3}{4}a\text{Si}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)/x^2,x]
```

```
[Out] (-2*a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/x - (3*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*S
inIntegral[x/2])/4 - (3*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*SinIntegral[(3*x)/2])
/4
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = \left(2a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x} + \left(3a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int\left(-\frac{\sin\left(\frac{x}{2}\right)}{4x} - \text{si}\right.$$
$$= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x} - \frac{1}{4}\left(3a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int\frac{\sin\left(\frac{x}{2}\right)}{x}\,dx - \frac{1}{4}$$
$$= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x} - \frac{3}{4}a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right) - \frac{3}{4}a\sqrt{a+a\,\text{c}}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.67

$$-\frac{a\sqrt{a(1+\cos(x))}\,\sec\left(\frac{x}{2}\right)\left(8\cos^3\left(\frac{x}{2}\right)+3x\text{Si}\left(\frac{x}{2}\right)+3x\text{Si}\left(\frac{3x}{2}\right)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x^2,x]
```

```
[Out] -1/4*(a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(8*Cos[x/2]^3 + 3*x*SinIntegral[x/2]
+ 3*x*SinIntegral[(3*x)/2]))/x
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cos(x))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(3/2)/x^2,x)
```

```
[Out] int((a+a*cos(x))^(3/2)/x^2,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.59, size = 37, normalized size = 0.47

$$\frac{3}{8}\sqrt{2}\,a^{\frac{3}{2}}\left(-i\,\Gamma\left(-1,\frac{3}{2}i\,x\right)-i\,\Gamma\left(-1,\frac{1}{2}i\,x\right)+i\,\Gamma\left(-1,-\frac{1}{2}i\,x\right)+i\,\Gamma\left(-1,-\frac{3}{2}i\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*a^(3/2)*(-I*gamma(-1, 3/2*I*x) - I*gamma(-1, 1/2*I*x) + I*gamma
(-1, -1/2*I*x) + I*gamma(-1, -3/2*I*x))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)/x**2,x)
```

```
[Out] Integral((a*(cos(x) + 1))**(3/2)/x**2, x)
```

Giac [A]
time = 0.48, size = 62, normalized size = 0.78

$$-\frac{\sqrt{2}\left(3\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{3}{2}\,x\right)+3\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{1}{2}\,x\right)+2\,a\cos\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+6\,a\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(3*a*x*sgn(cos(1/2*x))*sin_integral(3/2*x) + 3*a*x*sgn(cos(1/2
*x))*sin_integral(1/2*x) + 2*a*cos(3/2*x)*sgn(cos(1/2*x)) + 6*a*cos(1/2*x)*
sgn(cos(1/2*x)))*sqrt(a)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\cos(x))^{3/2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(3/2)/x^2,x)
```

```
[Out] int((a + a*cos(x))^(3/2)/x^2, x)
```

3.169 $\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx$

Optimal. Leaf size=109

$$-\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2}-\frac{3}{16}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-\frac{9}{16}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}$$

```
[Out] -a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)/x^2-3/16*a*Ci(1/2*x)*sec(1/2*x)*(a+a*cos
(x))^(1/2)-9/16*a*Ci(3/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+3/2*a*cos(1/2*x)*
sin(1/2*x)*(a+a*cos(x))^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3400, 3395, 3383, 3393}

$$-\frac{3}{16}a\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{9}{16}a\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{x^2}+\frac{3a\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)/x^3,x]
```

```
[Out] -((a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/x^2) - (3*a*Sqrt[a + a*Cos[x]]*CosInteg
ral[x/2]*Sec[x/2])/16 - (9*a*Sqrt[a + a*Cos[x]]*CosIntegral[(3*x)/2]*Sec[x/
2])/16 + (3*a*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/(2*x)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
```

```
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \left(2a\sqrt{a+a\cos(x)}\,\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x^3}\,dx$$
$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\,\sin\left(\frac{x}{2}\right)}{2x} + \frac{1}{2}\Big(3a\sqrt{a+a\,\mathrm{c}}$$
$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3}{2}a\sqrt{a+a\cos(x)}\,\mathrm{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a}}{}$$
$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3}{2}a\sqrt{a+a\cos(x)}\,\mathrm{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a}}{}$$
$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} - \frac{3}{16}a\sqrt{a+a\cos(x)}\,\mathrm{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) - \frac{9}{16}a\sqrt{a+a\,\mathrm{c}}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.61

$$-\frac{(a(1+\cos(x)))^{3/2}\left(16+3x^2\mathrm{CosIntegral}\left(\frac{x}{2}\right)\sec^3\left(\frac{x}{2}\right)+9x^2\mathrm{CosIntegral}\left(\frac{3x}{2}\right)\sec^3\left(\frac{x}{2}\right)-24x\tan\left(\frac{x}{2}\right)\right)}{32x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x^3,x]
[Out] -1/32*((a*(1 + Cos[x]))^(3/2)*(16 + 3*x^2*CosIntegral[x/2]*Sec[x/2]^3 + 9*x
^2*CosIntegral[(3*x)/2]*Sec[x/2]^3 - 24*x*Tan[x/2]))/x^2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cos(x))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(3/2)/x^3,x)
```

```
[Out] int((a+a*cos(x))^(3/2)/x^3,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 33, normalized size = 0.30

$$\frac{3}{16}\sqrt{2}\,a^{\frac{3}{2}}\left(3\,\Gamma\left(-2,\frac{3}{2}i\,x\right)+\Gamma\left(-2,\frac{1}{2}i\,x\right)+\Gamma\left(-2,-\frac{1}{2}i\,x\right)+3\,\Gamma\left(-2,-\frac{3}{2}i\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*a^(3/2)*(3*gamma(-2, 3/2*I*x) + gamma(-2, 1/2*I*x) + gamma(-2,
 -1/2*I*x) + 3*gamma(-2, -3/2*I*x))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)/x**3,x)
```

```
[Out] Integral((a*(cos(x) + 1))**(3/2)/x**3, x)
```

Giac [A]
time = 0.43, size = 92, normalized size = 0.84

$$-\frac{\sqrt{2}\,\left(9\,ax^2\,\mathrm{Ci}\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+3\,ax^2\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-6\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{3}{2}\,x\right)-6\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)+4\,a\cos\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+12\,a\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(9*a*x^2*cos_integral(3/2*x)*sgn(cos(1/2*x)) + 3*a*x^2*cos_in
tegral(1/2*x)*sgn(cos(1/2*x)) - 6*a*x*sgn(cos(1/2*x))*sin(3/2*x) - 6*a*x*sg
```

```
n(cos(1/2*x))*sin(1/2*x) + 4*a*cos(3/2*x)*sgn(cos(1/2*x)) + 12*a*cos(1/2*x)
*sgn(cos(1/2*x)))*sqrt(a)/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\cos(x))^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(3/2)/x^3,x)
[Out] int((a + a*cos(x))^(3/2)/x^3, x)
```

3.170 $\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=374

$$-\frac{4ix^3\text{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}}+\frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}-\frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Pol}}{d^2\sqrt{a+a\,\text{co}}}$$

```
[Out] -4*I*x^3*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^(
1/2)+12*I*x^2*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+a*
cos(d*x+c))^(1/2)-12*I*x^2*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x+c)
))/d^2/(a+a*cos(d*x+c))^(1/2)-48*x*cos(1/2*d*x+1/2*c)*polylog(3,-I*exp(1/2*
I*(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)+48*x*cos(1/2*d*x+1/2*c)*polylog(3,I*
exp(1/2*I*(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)-96*I*cos(1/2*d*x+1/2*c)*poly
log(4,-I*exp(1/2*I*(d*x+c)))/d^4/(a+a*cos(d*x+c))^(1/2)+96*I*cos(1/2*d*x+1/
2*c)*polylog(4,I*exp(1/2*I*(d*x+c)))/d^4/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 4266, 2611, 6744, 2320, 6724}

$$-\frac{4ix^3\text{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cos(c+dx)+a}}-\frac{96i\text{Li}_4\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^4\sqrt{a\cos(c+dx)+a}}+\frac{96i\text{Li}_4\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^4\sqrt{a\cos(c+dx)+a}}-\frac{48x\text{Li}_3\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}+\frac{48x\text{Li}_3\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}+\frac{12ix^2\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{12ix^2\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x^3*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*C
os[c + d*x]]) + ((12*I)*x^2*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c
+ d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((12*I)*x^2*Cos[c/2 + (d*x)/2]*P
olyLog[2, I*E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - (48*x*Co
s[c/2 + (d*x)/2]*PolyLog[3, (-I)*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a + a*Cos[
c + d*x]]) + (48*x*Cos[c/2 + (d*x)/2]*PolyLog[3, I*E^((I/2)*(c + d*x))])/(d
^3*Sqrt[a + a*Cos[c + d*x]]) - ((96*I)*Cos[c/2 + (d*x)/2]*PolyLog[4, (-I)*E
^((I/2)*(c + d*x))])/(d^4*Sqrt[a + a*Cos[c + d*x]]) + ((96*I)*Cos[c/2 + (d*
x)/2]*PolyLog[4, I*E^((I/2)*(c + d*x))])/(d^4*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^3\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(6\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2\log\left(\right.}{d\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} -$$

Mathematica [A]
time = 0.09, size = 199, normalized size = 0.53

$$-\frac{4i\cos\left(\frac{1}{2}(c+dx)\right)\left(d^3x^3\text{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)-3d^2x^2\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)+3d^2x^2\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)-12idx\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)+12idx\text{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)+24\text{PolyLog}\left(4,-ie^{\frac{1}{2}i(c+dx)}\right)-24\text{PolyLog}\left(4,ie^{\frac{1}{2}i(c+dx)}\right)\right)}{d^4\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]*(d^3*x^3*ArcTan[E^((I/2)*(c + d*x))] - 3*d^2*x^2*P
olyLog[2, (-I)*E^((I/2)*(c + d*x))] + 3*d^2*x^2*PolyLog[2, I*E^((I/2)*(c +
d*x))] - (12*I)*d*x*PolyLog[3, (-I)*E^((I/2)*(c + d*x))] + (12*I)*d*x*PolyL
og[3, I*E^((I/2)*(c + d*x))] + 24*PolyLog[4, (-I)*E^((I/2)*(c + d*x))] - 24
*PolyLog[4, I*E^((I/2)*(c + d*x))]))/(d^4*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+a\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] int(x^3/(a+a*cos(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(6*sqrt(2)*d^2*x^2*cos(1/2*d*x + 1/2*c) + 24*(sqrt(2)*cos(d*x + c)^2 + sq
rt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(cos(1/2*d*
x + 1/2*c), sin(1/2*d*x + 1/2*c) + 1) + 24*(sqrt(2)*cos(d*x + c)^2 + sqrt(2
)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(cos(1/2*d*x +
1/2*c), -sin(1/2*d*x + 1/2*c) + 1) + (6*sqrt(2)*d^2*x^2*cos(1/2*d*x + 1/2*c
) - (sqrt(2)*d^3*x^3 - 24*sqrt(2)*d*x)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) +
 (sqrt(2)*a*d^7*cos(d*x + c)^2 + sqrt(2)*a*d^7*sin(d*x + c)^2 + 2*sqrt(2)*a
*d^7*cos(d*x + c) + sqrt(2)*a*d^7)*integrate((x^3*cos(2*d*x + 2*c)*cos(1/2*
d*x + 1/2*c) + 2*x^3*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + x^3*sin(2*d*x + 2*
c)*sin(1/2*d*x + 1/2*c) + 2*x^3*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + x^3*cos
(1/2*d*x + 1/2*c))/(a*d^3*cos(2*d*x + 2*c)^2 + 4*a*d^3*cos(d*x + c)^2 + a*d
^3*sin(2*d*x + 2*c)^2 + 4*a*d^3*sin(2*d*x + 2*c)*sin(d*x + c) + 4*a*d^3*sin
(d*x + c)^2 + 4*a*d^3*cos(d*x + c) + a*d^3 + 2*(2*a*d^3*cos(d*x + c) + a*d^
3)*cos(2*d*x + 2*c)), x) - 6*(sqrt(2)*a*d^6*cos(d*x + c)^2 + sqrt(2)*a*d^6*
sin(d*x + c)^2 + 2*sqrt(2)*a*d^6*cos(d*x + c) + sqrt(2)*a*d^6)*integrate((x
^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + 2*x^2*cos(1/2*d*x + 1/2*c)*sin(d
*x + c) - x^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 2*x^2*cos(d*x + c)*si
n(1/2*d*x + 1/2*c) - x^2*sin(1/2*d*x + 1/2*c))/(a*d^3*cos(2*d*x + 2*c)^2 +
4*a*d^3*cos(d*x + c)^2 + a*d^3*sin(2*d*x + 2*c)^2 + 4*a*d^3*sin(2*d*x + 2*c
)*sin(d*x + c) + 4*a*d^3*sin(d*x + c)^2 + 4*a*d^3*cos(d*x + c) + a*d^3 + 2*
(2*a*d^3*cos(d*x + c) + a*d^3)*cos(2*d*x + 2*c)), x) - 24*(sqrt(2)*a*d^5*co
s(d*x + c)^2 + sqrt(2)*a*d^5*sin(d*x + c)^2 + 2*sqrt(2)*a*d^5*cos(d*x + c)
+ sqrt(2)*a*d^5)*integrate((x*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + 2*x*c
os(d*x + c)*cos(1/2*d*x + 1/2*c) + x*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c)
+ 2*x*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + x*cos(1/2*d*x + 1/2*c))/(a*d^3*co
s(2*d*x + 2*c)^2 + 4*a*d^3*cos(d*x + c)^2 + a*d^3*sin(2*d*x + 2*c)^2 + 4*a*
d^3*sin(2*d*x + 2*c)*sin(d*x + c) + 4*a*d^3*sin(d*x + c)^2 + 4*a*d^3*cos(d*
x + c) + a*d^3 + 2*(2*a*d^3*cos(d*x + c) + a*d^3)*cos(2*d*x + 2*c)), x) + (
6*sqrt(2)*d^2*x^2*sin(1/2*d*x + 1/2*c) + (sqrt(2)*d^3*x^3 - 24*sqrt(2)*d*x)
*cos(1/2*d*x + 1/2*c))*sin(d*x + c) - (sqrt(2)*d^3*x^3 - 24*sqrt(2)*d*x)*si
n(1/2*d*x + 1/2*c))/((d^4*cos(d*x + c)^2 + d^4*sin(d*x + c)^2 + 2*d^4*cos(d
*x + c) + d^4)*sqrt(a))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(x^3/(a + a*cos(c + d*x))^(1/2), x)
```

3.171 $\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=262

$$-\frac{4ix^2\operatorname{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}}+\frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}-\frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}}{d^2\sqrt{a+a\cos(c+}}$$

```
[Out] -4*I*x^2*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^(
1/2)+8*I*x*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+a*cos
(d*x+c))^(1/2)-8*I*x*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x+c)))/d^2
/(a+a*cos(d*x+c))^(1/2)-16*cos(1/2*d*x+1/2*c)*polylog(3,-I*exp(1/2*I*(d*x+c
)))/d^3/(a+a*cos(d*x+c))^(1/2)+16*cos(1/2*d*x+1/2*c)*polylog(3,I*exp(1/2*I*
(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 4266, 2611, 2320, 6724}

$$-\frac{4ix^2\operatorname{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16\operatorname{Li}_3\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}+\frac{16\operatorname{Li}_3\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}+\frac{8ix\operatorname{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{8ix\operatorname{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x^2*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*C
os[c + d*x]]) + ((8*I)*x*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c + d
*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((8*I)*x*Cos[c/2 + (d*x)/2]*PolyLog
[2, I*E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - (16*Cos[c/2 +
(d*x)/2]*PolyLog[3, (-I)*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a + a*Cos[c + d*x]
]) + (16*Cos[c/2 + (d*x)/2]*PolyLog[3, I*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a
+ a*Cos[c + d*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^2\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(4\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\log\left(\right.}{d\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{4ix^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{4ix^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{4ix^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} -$$

Mathematica [A]
time = 0.05, size = 146, normalized size = 0.56

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(-id^2x^2\text{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)+2idx\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)-2idx\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)-4\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)+4\text{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)\right)}{d^3\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]*((-I)*d^2*x^2*ArcTan[E^((I/2)*(c + d*x))] + (2*I)*d*x*P
olyLog[2, (-I)*E^((I/2)*(c + d*x))] - (2*I)*d*x*PolyLog[2, I*E^((I/2)*(c +
d*x))] - 4*PolyLog[3, (-I)*E^((I/2)*(c + d*x))] + 4*PolyLog[3, I*E^((I/2)*(
c + d*x))]))/(d^3*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+a\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] int(x^2/(a+a*cos(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(2)*d^2*x^2*sin(1/2*d*x + 1/2*c) - 4*sqrt(2)*d*x*cos(1/2*d*x + 1/2*
c) + (sqrt(2)*d^2*x^2*sin(1/2*d*x + 1/2*c) - 4*sqrt(2)*d*x*cos(1/2*d*x + 1/
2*c))*cos(d*x + c) - (sqrt(2)*a*d^5*cos(d*x + c)^2 + sqrt(2)*a*d^5*sin(d*x
+ c)^2 + 2*sqrt(2)*a*d^5*cos(d*x + c) + sqrt(2)*a*d^5)*integrate((x^2*cos(2
*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + 2*x^2*cos(d*x + c)*cos(1/2*d*x + 1/2*c)
+ x^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + 2*x^2*sin(d*x + c)*sin(1/2*d*
x + 1/2*c) + x^2*cos(1/2*d*x + 1/2*c))/(a*d^2*cos(2*d*x + 2*c)^2 + 4*a*d^2*
cos(d*x + c)^2 + a*d^2*sin(2*d*x + 2*c)^2 + 4*a*d^2*sin(2*d*x + 2*c)*sin(d*
x + c) + 4*a*d^2*sin(d*x + c)^2 + 4*a*d^2*cos(d*x + c) + a*d^2 + 2*(2*a*d^2
*cos(d*x + c) + a*d^2)*cos(2*d*x + 2*c)), x) + 4*(sqrt(2)*a*d^4*cos(d*x + c
)^2 + sqrt(2)*a*d^4*sin(d*x + c)^2 + 2*sqrt(2)*a*d^4*cos(d*x + c) + sqrt(2)
*a*d^4)*integrate((x*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + 2*x*cos(1/2*d*
x + 1/2*c)*sin(d*x + c) - x*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 2*x*cos
```

```
(d*x + c)*sin(1/2*d*x + 1/2*c) - x*sin(1/2*d*x + 1/2*c))/(a*d^2*cos(2*d*x +
 2*c)^2 + 4*a*d^2*cos(d*x + c)^2 + a*d^2*sin(2*d*x + 2*c)^2 + 4*a*d^2*sin(2
*d*x + 2*c)*sin(d*x + c) + 4*a*d^2*sin(d*x + c)^2 + 4*a*d^2*cos(d*x + c) +
a*d^2 + 2*(2*a*d^2*cos(d*x + c) + a*d^2)*cos(2*d*x + 2*c)), x) + 2*(sqrt(2)
*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2)
)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 2*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*c
os(d*x + c) + sqrt(2))*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - (sqrt(2)*d^2*x^2*cos(1/2*d*x + 1/2*c) + 4*s
qrt(2)*d*x*sin(1/2*d*x + 1/2*c))*sin(d*x + c))/((d^3*cos(d*x + c)^2 + d^3*s
in(d*x + c)^2 + 2*d^3*cos(d*x + c) + d^3)*sqrt(a))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(x**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(a*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*cos(c + d*x))^(1/2),x)
[Out] int(x^2/(a + a*cos(c + d*x))^(1/2), x)
```

3.172 $\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=156

$$-\frac{4ix\text{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}}+\frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}-\frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2\right.}{d^2\sqrt{a+a\cos(c+}}$$

```
[Out] -4*I*x*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^(1/
2)+4*I*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+a*cos(d*x
+c))^(1/2)-4*I*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x+c)))/d^2/(a+a*
cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3400, 4266, 2317, 2438}

$$-\frac{4ix\text{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cos(c+dx)+a}}+\frac{4i\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{4i\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cos
[c + d*x]]) + ((4*I)*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c + d*x))
])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((4*I)*Cos[c/2 + (d*x)/2]*PolyLog[2, I*
E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
```

```
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\log\left(1-\right.}{d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\mathrm{Subst}\left(\int\right.}{d^2\sqrt{a+a\cos(c+}}$$

$$= -\frac{4ix\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{4i\,\mathrm{co}}{d}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 0.57

$$-\frac{4i\cos\left(\frac{1}{2}(c+dx)\right)\left(dx\,\mathrm{ArcTan}\left(e^{\frac{1}{2}i(c+dx)}\right)-\mathrm{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)+\mathrm{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)\right)}{d^2\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + a*Cos[c + d*x]],x]
[Out] ((-4*I)*Cos[(c + d*x)/2]*(d*x*ArcTan[E^((I/2)*(c + d*x))] - PolyLog[2, (-I)
*E^((I/2)*(c + d*x))] + PolyLog[2, I*E^((I/2)*(c + d*x))]))/(d^2*Sqrt[a*(1
+ Cos[c + d*x])])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+a\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] int(x/(a+a*cos(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*d*x*cos(1/2*d*x + 1/2*c)*sin(d*x + c) - sqrt(2)*d*x*cos(d*x + c)
*sin(1/2*d*x + 1/2*c) - sqrt(2)*d*x*sin(1/2*d*x + 1/2*c) - (sqrt(2)*cos(d*x
 + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan
2(cos(1/2*d*x + 1/2*c), sin(1/2*d*x + 1/2*c) + 1) - (sqrt(2)*cos(d*x + c)^2
 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(cos(1
/2*d*x + 1/2*c), -sin(1/2*d*x + 1/2*c) + 1) + (sqrt(2)*a*d^3*cos(d*x + c)^2
 + sqrt(2)*a*d^3*sin(d*x + c)^2 + 2*sqrt(2)*a*d^3*cos(d*x + c) + sqrt(2)*a*
d^3)*integrate((x*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + 2*x*cos(d*x + c)*
cos(1/2*d*x + 1/2*c) + x*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + 2*x*sin(d*
x + c)*sin(1/2*d*x + 1/2*c) + x*cos(1/2*d*x + 1/2*c))/(a*d*cos(2*d*x + 2*c)
^2 + 4*a*d*cos(d*x + c)^2 + a*d*sin(2*d*x + 2*c)^2 + 4*a*d*sin(2*d*x + 2*c)
*sin(d*x + c) + 4*a*d*sin(d*x + c)^2 + 4*a*d*cos(d*x + c) + a*d + 2*(2*a*d*
cos(d*x + c) + a*d)*cos(2*d*x + 2*c)), x))/((d^2*cos(d*x + c)^2 + d^2*sin(d
*x + c)^2 + 2*d^2*cos(d*x + c) + d^2)*sqrt(a))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(x/(a + a*cos(c + d*x))^(1/2), x)
```

3.173 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2728, 212\}$

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.87

$$\frac{2\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])
])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 54, normalized size = 1.17

method	result	size
default	$\frac{\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert 1\right)}{d\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\mathrm{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/csgn(cos(1/2*d*x+1/2*c))*cos(1/2
*d*x+1/2*c)*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.
time = 0.55, size = 90, normalized size = 1.96

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{2\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Fricas [A]
time = 0.38, size = 126, normalized size = 2.74

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}\,d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}}{\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sqrt(-1/a)/sin(d*x + c))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cos(c + d*x) + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(37) = 74.
time = 0.46, size = 93, normalized size = 2.02

$$\frac{\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right|\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\right|\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/(s
qrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c)
+ sin(1/2*d*x + 1/2*c) - 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 0.33, size = 45, normalized size = 0.98

$$\frac{\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\cos\left(c+d\,x\right)\right)}{a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*
cos(c + d*x))^(1/2))
```

3.174 $\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\sqrt{a+a\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/x/(a+a*cos(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + a*Cos[c + d*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[a + a*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + a*Cos[c + d*x]]),x]
[Out] Integrate[1/(x*Sqrt[a + a*Cos[c + d*x]]), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+a\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/x/(a+a*cos(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + a)/(a*x*cos(d*x + c) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*(cos(c + d*x) + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*cos(c + d*x))^(1/2)),x)
[Out] int(1/(x*(a + a*cos(c + d*x))^(1/2)), x)
```

3.175 $\int \frac{x^3}{\sqrt{a - a\cos(x)}}\, dx$

Optimal. Leaf size=235

$$-\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{12ix^2\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{12ix^2\text{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{48x\text{PolyLog}\left(3\right.}{\sqrt{a-a}}$$

```
[Out] -4*x^3*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+12*I*x^2*polylog
(2,-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-12*I*x^2*polylog(2,exp(1/2*
I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-48*x*polylog(3,-exp(1/2*I*x))*sin(1/2*x
)/(a-a*cos(x))^(1/2)+48*x*polylog(3,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(
1/2)-96*I*polylog(4,-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+96*I*polyl
og(4,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3400, 4268, 2611, 6744, 2320, 6724}

$$\frac{12ix^2\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{12ix^2\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{48x\text{Li}_3\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{48x\text{Li}_3\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{96i\text{Li}_4\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{96i\text{Li}_4\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4x^3\sin\left(\frac{x}{2}\right)\tanh^{-1}\left(e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-4*x^3*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((12*I)*x^2*Pol
yLog[2, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((12*I)*x^2*PolyLog[2,
 E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - (48*x*PolyLog[3, -E^((I/2)*x)]
*Sin[x/2])/Sqrt[a - a*Cos[x]] + (48*x*PolyLog[3, E^((I/2)*x)]*Sin[x/2])/Sqr
t[a - a*Cos[x]] - ((96*I)*PolyLog[4, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos
[x]] + ((96*I)*PolyLog[4, E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx = \frac{\sin\left(\frac{x}{2}\right)\int x^3\csc\left(\frac{x}{2}\right)\,dx}{\sqrt{a-a\cos(x)}}$$
$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(6\sin\left(\frac{x}{2}\right)\right)\int x^2\log\left(1-e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}} + \frac{\left(6\sin\left(\frac{x}{2}\right)\right)\int x^2\,\mathrm{lo}}{\sqrt{a-a\,\mathrm{c}}}$$
$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{(}{}$$
$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{4}{}$$
$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{4}{}$$
$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{4}{}$$

Mathematica [A]
time = 0.07, size = 170, normalized size = 0.72

$$-\frac{i\left(8\pi^4 - x^4 + 8ix^3\log\left(1-e^{-\frac{ix}{2}}\right) - 8ix^3\log\left(1+e^{\frac{ix}{2}}\right) - 48x^2\mathrm{PolyLog}\left(2,e^{-\frac{ix}{2}}\right) - 48x^2\mathrm{PolyLog}\left(2,-e^{\frac{ix}{2}}\right) + 192ix\mathrm{PolyLog}\left(3,e^{-\frac{ix}{2}}\right) - 192ix\mathrm{PolyLog}\left(3,-e^{\frac{ix}{2}}\right) + 384\mathrm{PolyLog}\left(4,e^{-\frac{ix}{2}}\right) + 384\mathrm{PolyLog}\left(4,-e^{\frac{ix}{2}}\right)\right)\sin\left(\frac{x}{2}\right)}{4\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a - a*Cos[x]],x]
```

```
[Out] ((-1/4*I)*(8*Pi^4 - x^4 + (8*I)*x^3*Log[1 - E^((-1/2*I)*x)] - (8*I)*x^3*Log
[1 + E^((I/2)*x)] - 48*x^2*PolyLog[2, E^((-1/2*I)*x)] - 48*x^2*PolyLog[2, -
E^((I/2)*x)] + (192*I)*x*PolyLog[3, E^((-1/2*I)*x)] - (192*I)*x*PolyLog[3,
-E^((I/2)*x)] + 384*PolyLog[4, E^((-1/2*I)*x)] + 384*PolyLog[4, -E^((I/2)*x
)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a-a*cos(x))^(1/2),x)
```

```
[Out] int(x^3/(a-a*cos(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(-a*cos(x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*cos(x) + a)*x^3/(a*cos(x) - a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-a\left(\cos\left(x\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a-a*cos(x))**(1/2),x)
[Out] Integral(x**3/sqrt(-a*(cos(x) - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(-a*cos(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a-a\,\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - a*cos(x))^(1/2),x)
[Out] int(x^3/(a - a*cos(x))^(1/2), x)
```

3.176 $\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=163

$$-\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{8ix\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{8ix\text{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{16\text{PolyLog}\left(3,-e^{\frac{ix}{2}}\right.}{\sqrt{a-a\cos(}}$$

```
[Out] -4*x^2*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+8*I*x*polylog(2,
-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-8*I*x*polylog(2,exp(1/2*I*x))*
sin(1/2*x)/(a-a*cos(x))^(1/2)-16*polylog(3,-exp(1/2*I*x))*sin(1/2*x)/(a-a*c
os(x))^(1/2)+16*polylog(3,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 4268, 2611, 2320, 6724}

$$\frac{8ix\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{8ix\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{16\text{Li}_3\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{16\text{Li}_3\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4x^2\sin\left(\frac{x}{2}\right)\tanh^{-1}\left(e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-4*x^2*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((8*I)*x*PolyLo
g[2, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((8*I)*x*PolyLog[2, E^((I
/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - (16*PolyLog[3, -E^((I/2)*x)]*Sin[x/2
])/Sqrt[a - a*Cos[x]] + (16*PolyLog[3, E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Co
s[x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx &= \frac{\sin\left(\frac{x}{2}\right)\int x^2\csc\left(\frac{x}{2}\right)\,dx}{\sqrt{a-a\cos(x)}}\\ &= -\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{\left(4\sin\left(\frac{x}{2}\right)\right)\int x\log\left(1-e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}}+\frac{\left(4\sin\left(\frac{x}{2}\right)\right)\int x\log \cdots}{\sqrt{a-a\cos \cdots}}\\ &= -\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{8ix\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{8ix\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{(8i\,\text{s}\cdots}{\cdots}\\ &= -\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{8ix\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{8ix\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{(16\,\text{s}\cdots}{\cdots}\\ &= -\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{8ix\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{8ix\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{16\text{Li}\cdots}{\sqrt{\cdots}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 117, normalized size = 0.72

$$\frac{2\left(x^2\log\left(1-e^{\frac{ix}{2}}\right)-x^2\log\left(1+e^{\frac{ix}{2}}\right)+4ix\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4ix\text{PolyLog}\left(2,e^{\frac{ix}{2}}\right)-8\text{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)+8\text{PolyLog}\left(3,e^{\frac{ix}{2}}\right)\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (2*(x^2*Log[1 - E^((I/2)*x)] - x^2*Log[1 + E^((I/2)*x)] + (4*I)*x*PolyLog[2
, -E^((I/2)*x)] - (4*I)*x*PolyLog[2, E^((I/2)*x)] - 8*PolyLog[3, -E^((I/2)*
x)] + 8*PolyLog[3, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a-a*cos(x))^(1/2),x)
```

```
[Out] int(x^2/(a-a*cos(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-a*cos(x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*cos(x) + a)*x^2/(a*cos(x) - a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-a(\cos(x) - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-a*(cos(x) - 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-a*cos(x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a-a\,\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - a*cos(x))^(1/2),x)
```

```
[Out] int(x^2/(a - a*cos(x))^(1/2), x)
```

3.177 $\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=97

$$-\frac{4x\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{4i\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4i\text{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

```
[Out] -4*x*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+4*I*polylog(2,-exp
(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-4*I*polylog(2,exp(1/2*I*x))*sin(1/
2*x)/(a-a*cos(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3400, 4268, 2317, 2438}

$$\frac{4i\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4i\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4x\sin\left(\frac{x}{2}\right)\tanh^{-1}\left(e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-4*x*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((4*I)*PolyLog[2,
 -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((4*I)*PolyLog[2, E^((I/2)*x)
]*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx &= \frac{\sin\left(\frac{x}{2}\right)\int x\csc\left(\frac{x}{2}\right)\,dx}{\sqrt{a-a\cos(x)}}\\ &= -\frac{4x\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{\left(2\sin\left(\frac{x}{2}\right)\right)\int\log\left(1-e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}}+\frac{\left(2\sin\left(\frac{x}{2}\right)\right)\int\log\left(1\right.}{\sqrt{a-a\cos(}}\\ &= -\frac{4x\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{\left(4i\sin\left(\frac{x}{2}\right)\right)\mathrm{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}-\frac{\left(4i\sin\left(\frac{x}{2}\right)\right.}{}\\ &= -\frac{4x\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{4i\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4i\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.86

$$\frac{2\left(x\left(\log\left(1-e^{\frac{ix}{2}}\right)-\log\left(1+e^{\frac{ix}{2}}\right)\right)+2i\mathrm{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-2i\mathrm{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a - a*Cos[x]],x]
[Out] (2*(x*(Log[1 - E^((I/2)*x)] - Log[1 + E^((I/2)*x)]) + (2*I)*PolyLog[2, -E^(
(I/2)*x)] - (2*I)*PolyLog[2, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a-a\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a-a*cos(x))^(1/2),x)
```

```
[Out] int(x/(a-a*cos(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(-a*cos(x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*cos(x) + a)*x/(a*cos(x) - a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a\left(\cos\left(x\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a-a*cos(x))**(1/2),x)
[Out] Integral(x/sqrt(-a*(cos(x) - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(-a*cos(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a-a\,\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - a*cos(x))^(1/2),x)
[Out] int(x/(a - a*cos(x))^(1/2), x)
```

3.178 $\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a-a*cos(x))^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2728, 212}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Cos[x]],x]
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a - a*Cos[x]])])/Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\sin(x)}{\sqrt{a-a\cos(x)}}\right)\right)\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.97

$$\frac{2\left(-\log\left(\cos\left(\frac{x}{4}\right)\right)+\log\left(\sin\left(\frac{x}{4}\right)\right)\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (2*(-Log[Cos[x/4]] + Log[Sin[x/4]])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.68

method	result	size
default	$-\frac{\sin\left(\frac{x}{2}\right)\operatorname{arctanh}\left(\cos\left(\frac{x}{2}\right)\right)\sqrt{2}}{\sqrt{a\left(\sin^{2}\left(\frac{x}{2}\right)\right)}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -sin(1/2*x)*arctanh(cos(1/2*x))*2^(1/2)/(a*sin(1/2*x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(28) = 56.
time = 0.56, size = 81, normalized size = 2.19

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)^{2}+2\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)^{2}-2\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)+1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(x), cos(x)))^2 + sin(1/2*arctan2(sin(
x), cos(x)))^2 + 2*cos(1/2*arctan2(sin(x), cos(x))) + 1) - sqrt(2)*log(cos(
1/2*arctan2(sin(x), cos(x)))^2 + sin(1/2*arctan2(sin(x), cos(x)))^2 - 2*cos
(1/2*arctan2(sin(x), cos(x))) + 1))/sqrt(a)
```

Fricas [A]
time = 0.38, size = 87, normalized size = 2.35

$$\left[\frac{\sqrt{2}\log\left(-\frac{(\cos(x)+3)\sin(x)-\frac{2\sqrt{2}\sqrt{-a\cos\left(x\right)+a}\,(\cos(x)+1)}{\sqrt{a}}}{(\cos(x)-1)\sin(x)}\right)}{2\sqrt{a}},\ \sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-a\cos\left(x\right)+a}\sqrt{-\frac{1}{a}}}{\sin\left(x\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-((cos(x) + 3)*sin(x) - 2*sqrt(2)*sqrt(-a*cos(x) + a)*(cos
(x) + 1)/sqrt(a))/((cos(x) - 1)*sin(x)))/sqrt(a), sqrt(2)*sqrt(-1/a)*arctan
(sqrt(2)*sqrt(-a*cos(x) + a)*sqrt(-1/a)/sin(x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\cos{(x)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*cos(x) + a), x)
```

Giac [**A**]
time = 0.46, size = 20, normalized size = 0.54

$$\frac{\sqrt{2}\,\log\left(\left|\tan\left(\frac{1}{4}\,x\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(abs(tan(1/4*x)))/(sqrt(a)*sgn(sin(1/2*x)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a-a\,\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*cos(x))^(1/2),x)
```

```
[Out] int(1/(a - a*cos(x))^(1/2), x)
```

3.179 $\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{1}{x\sqrt{a-a\cos(x)}},x\right)$$

```
[Out] Unintegrable(1/x/(a-a*cos(x))^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[a - a*Cos[x]]),x]
[Out] Defer[Int][1/(x*Sqrt[a - a*Cos[x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

Mathematica [A]
time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a - a*Cos[x]]),x]
[Out] Integrate[1/(x*Sqrt[a - a*Cos[x]]), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a-a\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a-a*cos(x))^(1/2),x)
```

```
[Out] int(1/x/(a-a*cos(x))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*cos(x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*cos(x) + a)/(a*x*cos(x) - a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-a\left(\cos\left(x\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a*(cos(x) - 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a*cos(x) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\sqrt{a-a\cos{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - a*cos(x))^(1/2)),x)
```

```
[Out] int(1/(x*(a - a*cos(x))^(1/2)), x)
```

3.180 $\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{3x^2}{a\sqrt{a+a\cos(x)}}-\frac{24ix\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}-\frac{ix^3\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{24i\cos\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$

```
[Out] -3*x^2/a/(a+a*cos(x))^(1/2)-24*I*x*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*c
os(x))^(1/2)-I*x^3*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos(x))^(1/2)+24*
I*cos(1/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+3*I*x^2*cos(1/
2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-24*I*cos(1/2*x)*polylo
g(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-3*I*x^2*cos(1/2*x)*polylog(2,I*exp
(1/2*I*x))/a/(a+a*cos(x))^(1/2)-12*x*cos(1/2*x)*polylog(3,-I*exp(1/2*I*x))/
a/(a+a*cos(x))^(1/2)+12*x*cos(1/2*x)*polylog(3,I*exp(1/2*I*x))/a/(a+a*cos(x
))^(1/2)-24*I*cos(1/2*x)*polylog(4,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+24
*I*cos(1/2*x)*polylog(4,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+1/2*x^3*tan(1/
2*x)/a/(a+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3400, 4271, 4266, 2317, 2438, 2611, 6744, 2320, 6724}

$$-\frac{ix^3\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{24ix\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{3ix^2\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{3ix^2\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{12x\text{Li}_3\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{12x\text{Li}_3\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{24i\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{24i\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{24i\text{Li}_4\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{24i\text{Li}_4\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a\cos(x)+a}}-\frac{3x^2}{a\sqrt{a\cos(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (-3*x^2)/(a*Sqrt[a + a*Cos[x]]) - ((24*I)*x*ArcTan[E^((I/2)*x)]*Cos[x/2])/(
a*Sqrt[a + a*Cos[x]]) - (I*x^3*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqrt[a + a*
Cos[x]]) + ((24*I)*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos
[x]]) + ((3*I)*x^2*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos
[x]]) - ((24*I)*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]])
- ((3*I)*x^2*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - (
12*x*Cos[x/2]*PolyLog[3, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (12*x*
Cos[x/2]*PolyLog[3, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - ((24*I)*Cos[x/
2]*PolyLog[4, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + ((24*I)*Cos[x/2]*
PolyLog[4, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x^3*Tan[x/2])/(2*a*Sqr
t[a + a*Cos[x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
```

```
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx &= \frac{\cos\left(\frac{x}{2}\right)\int x^3\sec^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cos(x)}} \\
&= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x^3\sec\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cos(x)}} + \cdots \\
&= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \cdots \\
&= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \cdots \\
&= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \cdots \\
&= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \cdots \\
&= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \cdots
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 257, normalized size = 0.61

$$
-\frac{i\cos\left(\frac{x}{2}\right)\left(-6ix^2\cos\left(\frac{x}{2}\right)+48x\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)+2x^3\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)-6(8+x^2)\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)+6(8+x^2)\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)-24ix\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)+24ix\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)+48\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(4,-ie^{\frac{ix}{2}}\right)-48\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(4,ie^{\frac{ix}{2}}\right)+ix^3\sin\left(\frac{x}{2}\right)\right)}{(a(1+\cos(x)))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + a*Cos[x])^(3/2),x]
```

```
[Out] ((-I)*Cos[x/2]*((-6*I)*x^2*Cos[x/2] + 48*x*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 +
 2*x^3*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 - 6*(8 + x^2)*Cos[x/2]^2*PolyLog[2, (
-I)*E^((I/2)*x)] + 6*(8 + x^2)*Cos[x/2]^2*PolyLog[2, I*E^((I/2)*x)] - (24*I
)*x*Cos[x/2]^2*PolyLog[3, (-I)*E^((I/2)*x)] + (24*I)*x*Cos[x/2]^2*PolyLog[3
, I*E^((I/2)*x)] + 48*Cos[x/2]^2*PolyLog[4, (-I)*E^((I/2)*x)] - 48*Cos[x/2]
^2*PolyLog[4, I*E^((I/2)*x)] + I*x^3*Sin[x/2]))/(a*(1 + Cos[x]))^(3/2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a + a\cos\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x^3/(a+a*cos(x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 4/9*((6*sqrt(2)*x^2*cos(3/2*x) - (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*
cos(3*x)^3 + (6*sqrt(2)*x^2*sin(3/2*x) + (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(
3/2*x))*sin(3*x)^3 + 48*sqrt(2)*cos(2*x)^2*cos(3/2*x) + 48*sqrt(2)*cos(3/2*
x)*sin(2*x)^2 + ((2*(9*sqrt(2)*x^2 + 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x
^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)*x^2 + 2*(9*sqrt(2)*x^2
 + 8*sqrt(2))*cos(x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x))*cos(3/2*x) +
 (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2)
)*sin(3/2*x))*sin(2*x) - (9*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*c
os(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 24*sqrt(2)*x)*sin(3/2*x))*co
s(3*x)^2 + 3*((2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3
- 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) + 3*(2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(
3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)
*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(x) + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*
x)*sin(x) - 16*sqrt(2))*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2
)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt
(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^
```

```
2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt
(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*
a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*int
egrate(1/9*(x^3*cos(4*x)*cos(3/2*x) + 4*x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos
(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3/2*x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*
x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3/2*x)*sin(x) + (4*x^3*cos(x) + x^3)*co
s(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*
cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*
sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x)
+ 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2
*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3
*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*
sin(x))*sin(3*x)), x) - 486*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x
)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2
*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)
*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x
) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x)
 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x
^2*cos(3/2*x)*sin(4*x) + 4*x^2*cos(3/2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2
*x) - x^2*cos(4*x)*sin(3/2*x) - 4*x^2*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*
sin(3/2*x) + 4*x^2*cos(3/2*x)*sin(x) - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^
2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^
2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(
x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2
*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2
)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*s
in(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3
*x)), x) - 648*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(
2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sq
rt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) +
 sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a
^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)
*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x*cos(4*x)*cos
(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*si
n(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*
x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^
2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^
2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*co
s(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4
*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) +
 a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x
) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (3*(3*sqrt(2)*x^3 -
8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(3*x
) + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sq
rt(2))*sin(3/2*x))*sin(2*x) - 3*(3*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(
```

```
2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 8*sqrt(2)*x)*sin(3/2*
x))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 3*((2*(9*sqrt(2)*x^2 - 8*s
qrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) +
 3*(2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)
*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*
cos(x) + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x)...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cos(x) + a)*x^3/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a(\cos(x)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*cos(x))**(3/2),x)
[Out] Integral(x**3/(a*(cos(x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(a*cos(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(a+a\,\cos\left(x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*cos(x))^(3/2),x)
[Out] int(x^3/(a + a*cos(x))^(3/2), x)
```

3.181 $\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{2x}{a\sqrt{a+a\cos(x)}}-\frac{ix^2\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{2ix\cos\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$

```
[Out] -2*x/a/(a+a*cos(x))^(1/2)-I*x^2*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos(
x))^(1/2)+4*arctanh(sin(1/2*x))*cos(1/2*x)/a/(a+a*cos(x))^(1/2)+2*I*x*cos(1
/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-2*I*x*cos(1/2*x)*poly
log(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-4*cos(1/2*x)*polylog(3,-I*exp(1/
2*I*x))/a/(a+a*cos(x))^(1/2)+4*cos(1/2*x)*polylog(3,I*exp(1/2*I*x))/a/(a+a*
cos(x))^(1/2)+1/2*x^2*tan(1/2*x)/a/(a+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3400, 4271, 3855, 4266, 2611, 2320, 6724}

$$-\frac{ix^2\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{2ix\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{2ix\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{4\text{Li}_3\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{4\text{Li}_3\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a\cos(x)+a}}-\frac{2x}{a\sqrt{a\cos(x)+a}}+\frac{4\cos\left(\frac{x}{2}\right)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)}{a\sqrt{a\cos(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (-2*x)/(a*Sqrt[a + a*Cos[x]]) - (I*x^2*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqr
t[a + a*Cos[x]]) + (4*ArcTanh[Sin[x/2]]*Cos[x/2])/(a*Sqrt[a + a*Cos[x]]) +
((2*I)*x*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - ((
2*I)*x*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - (4*Cos[
x/2]*PolyLog[3, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (4*Cos[x/2]*Pol
yLog[3, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x^2*Tan[x/2])/(2*a*Sqrt[a
 + a*Cos[x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \frac{\cos\left(\frac{x}{2}\right)\int x^2\sec^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} + \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x^2\sec\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cos(x)}} + \frac{\left(2\cos\left(\frac{x}{2}\right)\right)}{a\sqrt{a+}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{}{2a}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{2ix}{}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{2ix}{}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{2ix}{}$$

Mathematica [A]
time = 0.08, size = 185, normalized size = 0.72

$$\frac{\cos\left(\frac{x}{2}\right)\left(-4x\cos\left(\frac{x}{2}\right) - 2ix^2\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right) + 8\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos^2\left(\frac{x}{2}\right) + 4ix\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right) - 4ix\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right) - 8\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right) + 8\cos^2\left(\frac{x}{2}\right)\text{PolyLog}\left(3,ie^{\frac{ix}{2}}\right) + x^2\sin\left(\frac{x}{2}\right)\right)}{(a(1+\cos(x)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (Cos[x/2]*(-4*x*Cos[x/2] - (2*I)*x^2*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 + 8*Arc
Tanh[Sin[x/2]]*Cos[x/2]^2 + (4*I)*x*Cos[x/2]^2*PolyLog[2, (-I)*E^((I/2)*x)]
 - (4*I)*x*Cos[x/2]^2*PolyLog[2, I*E^((I/2)*x)] - 8*Cos[x/2]^2*PolyLog[3, (
-I)*E^((I/2)*x)] + 8*Cos[x/2]^2*PolyLog[3, I*E^((I/2)*x)] + x^2*Sin[x/2]))/
(a*(1 + Cos[x]))^(3/2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+a\cos\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x^2/(a+a*cos(x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*(4*(3*sqrt(2)*x^2*sin(3/2*x) - 4*sqrt(2)*x*cos(3/2*x))*cos(3*x)^3 - 4*
(3*sqrt(2)*x^2*cos(3/2*x) + 4*sqrt(2)*x*sin(3/2*x))*sin(3*x)^3 + 96*sqrt(2)
*cos(2*x)^2*sin(3/2*x) + 96*sqrt(2)*sin(2*x)^2*sin(3/2*x) - 4*((12*sqrt(2)*
x*cos(3/2*x) - (9*sqrt(2)*x^2 + 8*sqrt(2))*sin(3/2*x))*cos(2*x) + (12*sqrt(
2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 12*sqrt(2)*x)*cos(3/2*x)
 + (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin(2
*x) - (9*sqrt(2)*x^2 - 12*sqrt(2)*x*sin(x) + (9*sqrt(2)*x^2 + 8*sqrt(2))*co
s(x))*sin(3/2*x))*cos(3*x)^2 - 12*((12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^
2 - 8*sqrt(2))*sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(
2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x) + (9*sqrt
(2)*x^2 - 8*sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqrt(2)*a^2*co
s(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*
sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*
sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^
2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^
2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*si
n(x))*sin(3*x))*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos
(3/2*x) + 6*x^2*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3
*x)*sin(3/2*x) + 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x
^2*cos(x) + x^2)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*c
os(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*s
in(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 +
 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a
^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*
x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^
2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 324*(sqrt(2)*a^2*cos(3*x)^2 + 9*
sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 +
9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*s
in(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) +
3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sq
rt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x
))*integrate(1/9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos
(3/2*x)*sin(2*x) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*co
s(2*x)*sin(3/2*x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a
^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a
^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin
(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(
```

```
2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^
2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*
sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(
3*x)), x) + (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*
x))*sin(3*x) + 3*(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos
(3/2*x))*sin(2*x) - (9*sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*sqrt(2)*x^2
 - 8*sqrt(2))*cos(x) - 8*sqrt(2))*sin(3/2*x))*cos(4/3*arctan2(sin(3/2*x), c
os(3/2*x)))^2 - 12*((12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*
sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt
(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqr
t(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x)^2 + 9*sq
rt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*
sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin
(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*
sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt
(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))
*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*x^2
*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*x)
+ 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) + x^2
)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*
a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*
a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3
*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4
*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*s
in(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*
a^2*sin(x))*sin(3*x)), x) - 324*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos
(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*s
in(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqr
t(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*c
os(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x) + a)*x^2/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\left(\cos\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x**2/(a*(cos(x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*cos(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a+a\,\cos(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*cos(x))^(3/2),x)
```

```
[Out] int(x^2/(a + a*cos(x))^(3/2), x)
```

3.182 $\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{1}{a\sqrt{a+a\cos(x)}}-\frac{ix\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{i\cos\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}-\frac{i\cos\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}+\frac{}{2a}$$

```
[Out] -1/a/(a+a*cos(x))^(1/2)-I*x*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos(x))^
(1/2)+I*cos(1/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-I*cos(1/
2*x)*polylog(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+1/2*x*tan(1/2*x)/a/(a+a
*cos(x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3400, 4270, 4266, 2317, 2438}

$$-\frac{ix\text{ArcTan}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{i\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{i\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{1}{a\sqrt{a\cos(x)+a}}+\frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a\cos(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + a*Cos[x])^(3/2),x]
```

```
[Out] -(1/(a*Sqrt[a + a*Cos[x]])) - (I*x*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqrt[a
+ a*Cos[x]]) + (I*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[
x]]) - (I*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x*T
an[x/2])/(2*a*Sqrt[a + a*Cos[x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
```

```
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx &= \frac{\cos\left(\frac{x}{2}\right)\int x\sec^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cos(x)}}\\
&= -\frac{1}{a\sqrt{a+a\cos(x)}}+\frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}+\frac{\cos\left(\frac{x}{2}\right)\int x\sec\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cos(x)}}\\
&= -\frac{1}{a\sqrt{a+a\cos(x)}}-\frac{ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}-\frac{\cos\left(\frac{x}{2}\right)\int\log\left(\right.}{2a\sqrt{a+\right.}}\\
&= -\frac{1}{a\sqrt{a+a\cos(x)}}-\frac{ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}+\frac{\left(i\cos\left(\frac{x}{2}\right)\right)\mathrm{Su}}{a\sqrt{}}\\
&= -\frac{1}{a\sqrt{a+a\cos(x)}}-\frac{ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{i\cos\left(\frac{x}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}-\frac{i\cos\left(\frac{x}{2}\right)\mathrm{Li}}{a\sqrt{a+a\,c}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 165, normalized size = 1.10

$$\frac{\sec\left(\frac{x}{2}\right)\left(-4\cos\left(\frac{x}{2}\right)+x\log\left(1-ie^{\frac{ix}{2}}\right)+x\cos(x)\log\left(1-ie^{\frac{ix}{2}}\right)-x\log\left(1+ie^{\frac{ix}{2}}\right)-x\cos(x)\log\left(1+ie^{\frac{ix}{2}}\right)+2i(1+\cos(x))\mathrm{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-2i(1+\cos(x))\mathrm{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)+2x\sin\left(\frac{x}{2}\right)\right)}{4a\sqrt{a(1+\cos(x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (Sec[x/2]*(-4*Cos[x/2] + x*Log[1 - I*E^((I/2)*x)] + x*Cos[x]*Log[1 - I*E^((
I/2)*x)] - x*Log[1 + I*E^((I/2)*x)] - x*Cos[x]*Log[1 + I*E^((I/2)*x)] + (2*
I)*(1 + Cos[x])*PolyLog[2, (-I)*E^((I/2)*x)] - (2*I)*(1 + Cos[x])*PolyLog[2
, I*E^((I/2)*x)] + 2*x*Sin[x/2]))/(4*a*Sqrt[a*(1 + Cos[x])])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+a\cos\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x/(a+a*cos(x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(8*x*cos(3/2*x)*sin(3*x)^3 - 8*x*cos(3*x)^3*sin(3/2*x) - 8*((3*x*sin(3/
2*x) + 2*cos(3/2*x))*cos(2*x) - (3*x*sin(x) - 2*cos(x))*cos(3/2*x) - (3*x*c
os(3/2*x) + 2*sin(3/2*x))*sin(2*x) + (3*x*cos(x) + 3*x - 2*sin(x))*sin(3/2*
x))*cos(3*x)^2 - 48*cos(2*x)^2*cos(3/2*x) - 24*((3*x*sin(3/2*x) - 2*cos(3/2
*x))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) +
6*cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*co
s(x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2
*sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*co
s(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*si
n(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*
x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6
*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x
))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^
2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x
)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2
*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x)
 + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3
*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))
*sin(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x) - 3*(3*x*cos(3/2*
x) + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*cos
(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 - 24*((3*x*sin(3/2*x) - 2*cos(3/2*x
))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) + 6*
```

```
cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*cos(
x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2*s
in(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*cos(
3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*sin(
3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*
cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x
*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))
/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2
+ a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*
sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*c
os(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) +
 a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a
^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*s
in(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x) - 3*(3*x*cos(3/2*x)
 + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*cos(2
/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 - 8*(x*cos(3*x)*sin(3/2*x) + (3*x*sin
(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (3*x*sin(x) + 2*cos(x))*cos(3/2*x) - (3*
x*cos(3/2*x) - 2*sin(3/2*x))*sin(2*x) + (3*x*cos(x) + x + 2*sin(x))*sin(3/2
*x))*sin(3*x)^2 - 48*cos(3/2*x)*sin(2*x)^2 - 24*((3*x*sin(3/2*x) - 2*cos(3/
2*x))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) +
 6*cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*c
os(x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^
2*sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*c
os(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*s
in(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6
*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) +
6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*
x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)
^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*
x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^
2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x
) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) +
3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x)
)*sin(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x) - 3*(3*x*cos(3/2
*x) + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*si
n(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 - 24*((3*x*sin(3/2*x) - 2*cos(3/2*
x))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) + 6
*cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*cos
(x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2*
sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*cos
(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*sin
(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x
*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*
x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x)
)/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cos(x) + a)*x/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a\left(\cos\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(x))**(3/2),x)
[Out] Integral(x/(a*(cos(x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="giac")
[Out] integrate(x/(a*cos(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(a+a\,\cos\left(x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*cos(x))^(3/2),x)
[Out] int(x/(a + a*cos(x))^(3/2), x)
```

3.183 $\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{1}{x(a+a\cos(x))^{3/2}},x\right)$$

`[Out] Unintegrable(1/x/(a+a*cos(x))^(3/2),x)`

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/(x*(a + a*Cos[x])^(3/2)),x]`

`[Out] Defer[Int][1/(x*(a + a*Cos[x])^(3/2)), x]`

Rubi steps

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

Mathematica [A]
time = 6.79, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/(x*(a + a*Cos[x])^(3/2)),x]`

`[Out] Integrate[1/(x*(a + a*Cos[x])^(3/2)), x]`

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+a\cos\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*cos(x))^(3/2),x)
[Out] int(1/x/(a+a*cos(x))^(3/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(x) + a)^(3/2)*x), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cos(x) + a)/(a^2*x*cos(x)^2 + 2*a^2*x*cos(x) + a^2*x), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\left(\cos\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(x))**(3/2),x)
[Out] Integral(1/(x*(a*(cos(x) + 1))**(3/2)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*cos(x) + a)^(3/2)*x), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\,{\left(a+a\,\cos\left(x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*cos(x))^(3/2)),x)
[Out] int(1/(x*(a + a*cos(x))^(3/2)), x)
```

3.184 $\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\sqrt[3]{a+a\cos(c+dx)}}{x},x\right)$$

```
[Out] Unintegrable((a+a*cos(d*x+c))^(1/3)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)/x,x]
```

```
[Out] Defer[Int][(a + a*Cos[c + d*x])^(1/3)/x, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

Mathematica [A]
time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)/x,x]
```

```
[Out] Integrate[(a + a*Cos[c + d*x])^(1/3)/x, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cos(dx+c))^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/3)/x,x)
```

```
[Out] int((a+a*cos(d*x+c))^(1/3)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(1/3)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/3)/x,x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(1/3)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + a \cos(c + d x))^{1/3}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/3)/x,x)
[Out] int((a + a*cos(c + d*x))^(1/3)/x, x)
```

3.185 $\int \frac{x^3}{a+b\cos(x)}\,dx$

Optimal. Leaf size=383

$$-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{3x^2\text{PolyLog}\left(\right.}{\sqrt{}}$$

```
[Out] -I*x^3*ln(1+b*exp(I*x)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)+I*x^3*ln(1+b*ex
p(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)-3*x^2*polylog(2,-b*exp(I*x)/(a-
(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)+3*x^2*polylog(2,-b*exp(I*x)/(a+(a^2-b^2)^
(1/2)))/(a^2-b^2)^(1/2)-6*I*x*polylog(3,-b*exp(I*x)/(a-(a^2-b^2)^(1/2)))/(a
^2-b^2)^(1/2)+6*I*x*polylog(3,-b*exp(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1
/2)+6*polylog(4,-b*exp(I*x)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)-6*polylog(
4,-b*exp(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3402, 2296, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6ix\text{Li}_3\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6ix\text{Li}_3\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6\text{Li}_4\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6\text{Li}_4\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1+\frac{be^{ix}}{\sqrt{a^2-b^2}+a}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cos[x]),x]
```

```
[Out] ((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] + (I*
x^3*Log[1 + (b*E^(I*x))/(a + Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] - (3*x^2*Po
lyLog[2, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + (3*x^2*Po
lyLog[2, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] - ((6*I)*x*
PolyLog[3, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + ((6*I)*
x*PolyLog[3, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + (6*Po
lyLog[4, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] - (6*PolyLo
g[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+b\cos(x)}\,dx &= 2\int \frac{e^{ix}x^3}{b+2ae^{ix}+be^{2ix}}\,dx \\
&= \frac{(2b)\int \frac{e^{ix}x^3}{2a-2\sqrt{a^2-b^2}+2be^{ix}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{ix}x^3}{2a+2\sqrt{a^2-b^2}+2be^{ix}}\,dx}{\sqrt{a^2-b^2}} \\
&= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{(3i)\int x^2\log\left(1+\frac{2}{2a-2\sqrt{}}\right.}{\sqrt{a^2-b^2}} \\
&= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}
\end{aligned}$$

Mathematica [A]
time = 0.56, size = 290, normalized size = 0.76

$$\frac{-ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)+ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)-3x^2\text{PolyLog}\left(2,\frac{be^{ix}}{-a+\sqrt{a^2-b^2}}\right)+3x^2\text{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)-6ix\text{PolyLog}\left(3,\frac{be^{ix}}{-a+\sqrt{a^2-b^2}}\right)+6ix\text{PolyLog}\left(3,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)+6\text{PolyLog}\left(4,\frac{be^{ix}}{-a+\sqrt{a^2-b^2}}\right)-6\text{PolyLog}\left(4,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cos[x]),x]
```

```
[Out] ((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])] + I*x^3*Log[1 + (b*E^(
I*x))/(a + Sqrt[a^2 - b^2])] - 3*x^2*PolyLog[2, (b*E^(I*x))/(-a + Sqrt[a^2
- b^2])] + 3*x^2*PolyLog[2, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))] - (6*I)*x
*PolyLog[3, (b*E^(I*x))/(-a + Sqrt[a^2 - b^2])] + (6*I)*x*PolyLog[3, -((b*E
^(I*x))/(a + Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(I*x))/(-a + Sqrt[a^2 -
 b^2])] - 6*PolyLog[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^
2]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cos(x)),x)
```

```
[Out] int(x^3/(a+b*cos(x)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(317) = 634$.
time = 0.48, size = 1030, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b*x^3*sqrt((a^2 - b^2)/b^2)*log((a*cos(x) + I*a*sin(x) + (b*cos(x)
+ I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b) + I*b*x^3*sqrt((a^2 - b^2)/b^2)
*log((a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2)
 + b)/b) + I*b*x^3*sqrt((a^2 - b^2)/b^2)*log((a*cos(x) - I*a*sin(x) + (b*co
s(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b) - I*b*x^3*sqrt((a^2 - b^2)
/b^2)*log((a*cos(x) - I*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)
/b^2) + b)/b) - 3*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(x) + I*a*sin(x)
 + (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 3*b*x^2*sqrt
((a^2 - b^2)/b^2)*dilog(-(a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*s
qrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*
cos(x) - I*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b
+ 1) + 3*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(x) - I*a*sin(x) - (b*cos
(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 6*I*b*x*sqrt((a^2 - b
^2)/b^2)*polylog(3, -(a*cos(x) + I*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(
(a^2 - b^2)/b^2))/b) + 6*I*b*x*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x)
+ I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) + 6*I*b*x*
sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x) - I*a*sin(x) + (b*cos(x) - I*b*
sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 6*I*b*x*sqrt((a^2 - b^2)/b^2)*polylog(3
```

```
, -(a*cos(x) - I*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/
b) + 6*b*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cos(x) + I*a*sin(x) + (b*cos(
x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 6*b*sqrt((a^2 - b^2)/b^2)*poly
log(4, -(a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b
^2))/b) + 6*b*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cos(x) - I*a*sin(x) + (b
*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 6*b*sqrt((a^2 - b^2)/b^2)
*polylog(4, -(a*cos(x) - I*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b
^2)/b^2))/b))/(a^2 - b^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\cos{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cos(x)),x)
```

```
[Out] Integral(x**3/(a + b*cos(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cos(x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + b\,\cos{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*cos(x)),x)
```

```
[Out] int(x^3/(a + b*cos(x)), x)
```

3.186 $\int \frac{x^2}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=329

$$-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}+\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}-\frac{2x\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^2}+\frac{2x\text{PolyLog}\left(\right.}{\sqrt{a}}$$

```
[Out] -I*x^2*ln(1+b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)+I*x^2*l
n(1+b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)-2*x*polylog(2,-
b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)+2*x*polylog(2,-b*
exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)-2*I*polylog(3,-b*ex
p(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2)+2*I*polylog(3,-b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3402, 2296, 2221, 2611, 2320, 6724}

$$-\frac{2i\text{Li}_3\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}+\frac{2i\text{Li}_3\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}-\frac{2x\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{2x\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-I)*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b
^2]*d) + (I*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) - (2*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))
])/(Sqrt[a^2 - b^2]*d^2) + (2*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^2) - ((2*I)*PolyLog[3, -((b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^3) + ((2*I)*PolyLog[3, -((b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx = 2\int \frac{e^{i(c+dx)}x^2}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{i(c+dx)}x^2}{2a-2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{i(c+dx)}x^2}{2a+2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{(2i)\int x\log\left(1+ \right.}{\sqrt{a^2}}$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{2x\text{Li}_2\left(-\frac{be^{i(c+d}}{a-\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,d^2}$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{2x\text{Li}_2\left(-\frac{be^{i(c+d}}{a-\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,d^2}$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{2x\text{Li}_2\left(-\frac{be^{i(c+d}}{a-\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,d^2}$$

Mathematica [A]
time = 0.46, size = 379, normalized size = 1.15

$$\frac{e^{ic}\left(-2dx\text{PolyLog}\left(2,-\frac{be^{i(2c+dx)}}{ae^{ic}-\sqrt{(a^2-b^2)\,e^{2ic}}}\right)-i\left(d^2x^2\log\left(1+\frac{be^{i(2c+dx)}}{ae^{ic}-\sqrt{(a^2-b^2)\,e^{2ic}}}\right)-d^2x^2\log\left(1+\frac{be^{i(2c+dx)}}{ae^{ic}+\sqrt{(a^2-b^2)\,e^{2ic}}}\right)+2idx\text{PolyLog}\left(2,-\frac{be^{i(2c+dx)}}{ae^{ic}+\sqrt{(a^2-b^2)\,e^{2ic}}}\right)+2\text{PolyLog}\left(3,-\frac{be^{i(2c+dx)}}{ae^{ic}-\sqrt{(a^2-b^2)\,e^{2ic}}}\right)-2\text{PolyLog}\left(3,-\frac{be^{i(2c+dx)}}{ae^{ic}+\sqrt{(a^2-b^2)\,e^{2ic}}}\right)\right)\right)}{d^3\sqrt{(a^2-b^2)\,e^{2ic}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] (E^(I*c)*(-2*d*x*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2
- b^2)*E^((2*I)*c)]))] - I*(d^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(a*E^(I*c
) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - d^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))
/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (2*I)*d*x*PolyLog[2, -((b*E
^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 2*PolyLog
[3, -((b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] -
 2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I
)*c)]))])))/(d^3*Sqrt[(a^2 - b^2)*E^((2*I)*c)])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cos(d*x+c)),x)
```

```
[Out] int(x^2/(a+b*cos(d*x+c)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1263 vs. 2(283) = 566.
time = 0.53, size = 1263, normalized size = 3.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1)
- 2*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) -
 (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2*
b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c) + (b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 2*b*d*
x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c) - (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - I*b*c^2*sq
rt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a
^2 - b^2)/b^2) + 2*a) + I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c)
- 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - I*b*c^2*sqrt((a^2
 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b
^2)/b^2) - 2*a) + I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I
*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) + (I*b*d^2*x^2 - I*b*c^2
)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (-I*b*d^2*x^2 + I
*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x + c) - (b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (-I*b*d^2*x
```

```
^2 + I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) - I*a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (I*b*
d^2*x^2 - I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) - I*a*sin(d*x
+ c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) +
2*I*b*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(d*x + c) + I*a*sin(d*x + c)
+ (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 2*I*b*sqr
t((a^2 - b^2)/b^2)*polylog(3, -(a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(
d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 2*I*b*sqrt((a^2 -
b^2)/b^2)*polylog(3, -(a*cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c)
- I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 2*I*b*sqrt((a^2 - b^2)/b^2)
*polylog(3, -(a*cos(d*x + c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin
(d*x + c))*sqrt((a^2 - b^2)/b^2))/b))/((a^2 - b^2)*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(x**2/(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{a + b\,\cos(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*cos(c + d*x)),x)
```

```
[Out] int(x^2/(a + b*cos(c + d*x)), x)
```

3.187 $\int \frac{x}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=214

$$-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}+\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}-\frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^2}$$

```
[Out] -I*x*ln(1+b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)+I*x*ln(1+
b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)-polylog(2,-b*exp(I*
(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)+polylog(2,-b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3402, 2296, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-I)*x*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2
]*d) + (I*x*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 -
 b^2]*d) - PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))]/(Sqrt[a
^2 - b^2]*d^2) + PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))]/(
Sqrt[a^2 - b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = 2\int \frac{e^{i(c+dx)}x}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{i\int\log\left(1+\frac{2be^{i(}}{2a-2\sqrt{a}}\right.}{\sqrt{a^2-b^2}}$$

$$= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{\text{Subst}\left(\int\frac{\log\left(1+\right.}{2a-2}\right.}{\sqrt{}}$$

$$= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,d^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 756 vs. 2(214) = 428.
time = 0.56, size = 756, normalized size = 3.53

$$\frac{2(c+dx)\tanh^{-1}\left(\frac{(a+b)\cot\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right) - 2\left(c+\text{ArcCos}\left(-\frac{a}{b}\right)\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right) + \left(\text{ArcCos}\left(-\frac{a}{b}\right) - 2i\tanh^{-1}\left(\frac{(a+b)\cot\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right) + 2i\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)\right)\log\left(\frac{\sqrt{-a^2+b^2}\,e^{-\frac{1}{2}i(c+dx)}}{\sqrt{2}\sqrt{b}\sqrt{a+b\cos(c+dx)}}\right) + \left(\text{ArcCos}\left(-\frac{a}{b}\right) + 2i\left(\tanh^{-1}\left(\frac{(a+b)\cot\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right) - \tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)\right)\right)\log\left(\frac{\sqrt{-a^2+b^2}\,e^{\frac{1}{2}i(c+dx)}}{\sqrt{2}\sqrt{b}\sqrt{a+b\cos(c+dx)}}\right) - \left(\text{ArcCos}\left(-\frac{a}{b}\right) - 2i\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)\right)\log\left(\frac{(a+b)\left(-a+b-i\sqrt{-a^2+b^2}\right)\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b\left(a+b+\sqrt{-a^2+b^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right) - \left(\text{ArcCos}\left(-\frac{a}{b}\right) + 2i\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)\right)\log\left(\frac{(a+b)\left(ia-ib+\sqrt{-a^2+b^2}\right)\left(i+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b\left(a+b+\sqrt{-a^2+b^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right) + i\left(\text{PolyLog}\left(2,\frac{\left(a-i\sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b\left(a+b+\sqrt{-a^2+b^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right) - \text{PolyLog}\left(2,\frac{\left(a+i\sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b\left(a+b+\sqrt{-a^2+b^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)}{\sqrt{-a^2+b^2}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(c + d*x)*ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]] - 2*(c +
ArcCos[-(a/b)])*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]] + (Ar
cCos[-(a/b)] - (2*I)*ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]] +
 (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*Log[Sqrt[-a^2
 + b^2]/(Sqrt[2]*Sqrt[b]*E^((I/2)*(c + d*x))*Sqrt[a + b*Cos[c + d*x]])] + (
ArcCos[-(a/b)] + (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]
] - ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[(Sqrt[-a^2
+ b^2]*E^((I/2)*(c + d*x)))/(Sqrt[2]*Sqrt[b]*Sqrt[a + b*Cos[c + d*x]])] - (
ArcCos[-(a/b)] - (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])*Log[((a + b)*(-a + b - I*Sqrt[-a^2 + b^2])*(1 + I*Tan[(c + d*x)/2]))/(b*
(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2]))] - (ArcCos[-(a/b)] + (2*I)*Arc
Tanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*Log[((a + b)*(I*a - I*b
 + Sqrt[-a^2 + b^2])*(I + Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*T
an[(c + d*x)/2]))] + I*(PolyLog[2, ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[
-a^2 + b^2]*Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2
]))] - PolyLog[2, ((a + I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(
c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2]))]))/(Sqrt[-a^2
 + b^2]*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(188) = 376$.
time = 0.07, size = 414, normalized size = 1.93

method	result
risch	$-\frac{i\ln\left(\frac{-e^{i(dx+c)}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{d\sqrt{a^2-b^2}}+\frac{i\ln\left(\frac{e^{i(dx+c)}b+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{d\sqrt{a^2-b^2}}-\frac{i\ln\left(\frac{-e^{i(dx+c)}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)c}{d^2\sqrt{a^2-b^2}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d/(a^2-b^2)^(1/2)*ln((-exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)
^(1/2)))*x+I/d/(a^2-b^2)^(1/2)*ln((exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)+a)/(a+(
a^2-b^2)^(1/2)))*x-I/d^2/(a^2-b^2)^(1/2)*ln((-exp(I*(d*x+c))*b+(a^2-b^2)^(1
/2)-a)/(-a+(a^2-b^2)^(1/2)))*c+I/d^2/(a^2-b^2)^(1/2)*ln((exp(I*(d*x+c))*b+(
a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c-1/d^2/(a^2-b^2)^(1/2)*dilog((-exp(
I*(d*x+c))*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/d^2/(a^2-b^2)^(1/2)
*dilog((exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))+2*I/d^2*c/
(-a^2+b^2)^(1/2)*arctan(1/2*(2*exp(I*(d*x+c))*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(184) = 368$.
time = 0.50, size = 915, normalized size = 4.28

$$\frac{-ibc\sqrt{\frac{a^2-b^2}{b^2}}\log\left(2b\cos(dx+c)+2ib\sin(dx+c)+2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right)+ibc\sqrt{\frac{a^2-b^2}{b^2}}\log\left(2b\cos(dx+c)-2ib\sin(dx+c)+2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right)-ibc\sqrt{\frac{a^2-b^2}{b^2}}\log\left(-2b\cos(dx+c)+2ib\sin(dx+c)+2b\sqrt{\frac{a^2-b^2}{b^2}}-2a\right)+ibc\sqrt{\frac{a^2-b^2}{b^2}}\log\left(-2b\cos(dx+c)-2ib\sin(dx+c)+2b\sqrt{\frac{a^2-b^2}{b^2}}-2a\right)-b\sqrt{\frac{a^2-b^2}{b^2}}\,\mathrm{Li}_2\left(-\frac{a\cos(dx+c)+ia\sin(dx+c)+(b\cos(dx+c)+ib\sin(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)+b\sqrt{\frac{a^2-b^2}{b^2}}\,\mathrm{Li}_2\left(-\frac{a\cos(dx+c)+ia\sin(dx+c)-(b\cos(dx+c)+ib\sin(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)-b\sqrt{\frac{a^2-b^2}{b^2}}\,\mathrm{Li}_2\left(-\frac{a\cos(dx+c)-ia\sin(dx+c)+(b\cos(dx+c)-ib\sin(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)+\cdots}{2(a^2-b^2)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b*c*sqrt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
 + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + I*b*c*sqrt((a^2 - b^2)/b^2)*log(2*b*c
os(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - I*b*c
*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr
t((a^2 - b^2)/b^2) - 2*a) + I*b*c*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x +
c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) - b*sqrt((a^2 -
b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + b*sqrt((a^2 - b^2)/b^2)*
dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - b*sqrt((a^2 - b^2)/b^2)*dilog(-(a*
cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(
(a^2 - b^2)/b^2) + b)/b + 1) + b*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x +
c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2
)/b^2) + b)/b + 1) - (I*b*d*x + I*b*c)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x
 + c) + I*a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b) - (-I*b*d*x - I*b*c)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x
 + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b) - (-I*b*d*x - I*b*c)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x
 + c) - I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b) - (I*b*d*x + I*b*c)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x
+ c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b
^2)/b^2) + b)/b))/((a^2 - b^2)*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\cos(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(x/(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*cos(c + d*x)),x)
```

```
[Out] int(x/(a + b*cos(c + d*x)), x)
```

3.188 $\int \frac{1}{x(a+b\cos(x))}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x(a+b\cos(x))},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*cos(x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+b\cos(x))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + b*Cos[x])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Cos[x])), x]
```

Rubi steps

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{x(a+b\cos(x))}\,dx$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\cos(x))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Cos[x])),x]
```

```
[Out] Integrate[1/(x*(a + b*Cos[x])), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\cos\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*cos(x)),x)
[Out] int(1/x/(a+b*cos(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="maxima")
[Out] integrate(1/((b*cos(x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="fricas")
[Out] integral(1/(b*x*cos(x) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(a+b\cos{(x)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)),x)
[Out] Integral(1/(x*(a + b*cos(x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="giac")
[Out] integrate(1/((b*cos(x) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\,\left(a+b\,\cos\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*cos(x))),x)
[Out] int(1/(x*(a + b*cos(x))), x)
```

3.189 $\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=296

$$-\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}+\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{f\log(a+b\cos(c+dx))}{(a^2-b^2)\,d^2}-\frac{af\mathrm{Poly}}{}$$

```
[Out] -f*ln(a+b*cos(d*x+c))/(a^2-b^2)/d^2-I*a*(f*x+e)*ln(1+b*exp(I*(d*x+c))/(a-(a
^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+I*a*(f*x+e)*ln(1+b*exp(I*(d*x+c))/(a+(a^2
-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-a*f*polylog(2,-b*exp(I*(d*x+c))/(a-(a^2-b^2
)^(1/2)))/(a^2-b^2)^(3/2)/d^2+a*f*polylog(2,-b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/(a^2-b^2)^(3/2)/d^2-b*(f*x+e)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c
))
```

Rubi [A]
time = 0.35, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3405, 3402, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{af\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{af\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-I)*a*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*d) + (I*a*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/((a^2 - b^2)^(3/2)*d) - (f*Log[a + b*Cos[c + d*x]])/((a^2 - b^2)
*d^2) - (a*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))])/((a^
2 - b^2)^(3/2)*d^2) + (a*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2]))])/((a^2 - b^2)^(3/2)*d^2) - (b*(e + f*x)*Sin[c + d*x])/((a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = -\frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{a\int \frac{e+fx}{a+b\cos(c+dx)}\,dx}{a^2-b^2} + \frac{(bf)\int \frac{\sin(c+dx)}{a+b\cos(c+dx)}\,dx}{(a^2-b^2)\,d}$$

$$= -\frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx}{a^2-b^2} - \frac{f\text{Subst}(\int \frac{1}{a+ \cdots}}{(\cdots}$$

$$= -\frac{f\log(a+b\cos(c+dx))}{(a^2-b^2)\,d^2} - \frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{(2ab)\int \frac{e^{i(c+\cdots}}{2a-2\sqrt{a^2-\cdots}}}{(a^2-\cdots}$$

$$= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{f\cdots}{\cdots}$$

$$= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{f\cdots}{\cdots}$$

$$= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{f\cdots}{\cdots}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 933 vs. $2(296) = 592$.
time = 8.42, size = 933, normalized size = 3.15

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-(b*d*e*Sin[c + d*x]) + b*c*f*Sin[c + d*x] - b*f*(c + d*x)*Sin[c + d*x])/(
(a - b)*(a + b)*d^2*(a + b*Cos[c + d*x])) + (Cos[(c + d*x)/2]^2*((2*a*(d*e
- c*f)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]) + f*Log[Sec[(c + d*x)/2]^2] - f*Log[(a + b*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2] - (I*a*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(Sqrt[a + b] - Sqrt[-
a + b]*Tan[(c + d*x)/2])/(I*Sqrt[-a + b] + Sqrt[a + b])] + PolyLog[2, (Sqrt
[-a + b]*(1 - I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] - I*Sqrt[a + b])]))/(Sqrt[
-a + b]*Sqrt[a + b]) + (I*a*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(I*(Sqrt[a +
 b] + Sqrt[-a + b]*Tan[(c + d*x)/2]))/(Sqrt[-a + b] + I*Sqrt[a + b])] + Pol
yLog[2, (Sqrt[-a + b]*(1 - I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] + I*Sqrt[a +
b])]))/(Sqrt[-a + b]*Sqrt[a + b]) - (I*a*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log
```

```
[(Sqrt[a + b] + Sqrt[-a + b]*Tan[(c + d*x)/2])/(I*Sqrt[-a + b] + Sqrt[a + b
])] + PolyLog[2, (Sqrt[-a + b]*(1 + I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] - I*
Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]) + (I*a*f*(Log[1 + I*Tan[(c + d*x
)/2]]*Log[(I*(Sqrt[a + b] - Sqrt[-a + b]*Tan[(c + d*x)/2]))/(Sqrt[-a + b] +
 I*Sqrt[a + b])] + PolyLog[2, (Sqrt[-a + b]*(1 + I*Tan[(c + d*x)/2]))/(Sqrt
[-a + b] + I*Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]))*(a*d*e + a*d*f*x +
 b*f*Sin[c + d*x])*(Sqrt[a + b] - Sqrt[-a + b]*Tan[(c + d*x)/2])*(Sqrt[a +
b] + Sqrt[-a + b]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d^2*(a + b*Cos[c + d*x])*
(a*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*
x)/2]]) + b*f*Sin[c + d*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(270) = 540$.
time = 0.90, size = 674, normalized size = 2.28

method	result
risch	$\frac{2i(fx+e)\left(a\,e^{i(dx+c)}+b\right)}{d(-a^2+b^2)\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{2f\ln\left(e^{i(dx+c)}\right)}{d^2(-a^2+b^2)}+\frac{f\ln\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}{d^2(-a^2+b^2)}+\frac{2iae\arctan\left(\frac{2\,e^{i(dx+c)}b+2a}{2\sqrt{-a^2+b^2}}\right.}{d(-a^2+b^2)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(f*x+e)*(a*exp(I*(d*x+c))+b)/d/(-a^2+b^2)/(b*exp(2*I*(d*x+c))+2*a*exp(I
*(d*x+c))+b)-2/d^2/(-a^2+b^2)*f*ln(exp(I*(d*x+c)))+1/d^2/(-a^2+b^2)*f*ln(b*
exp(2*I*(d*x+c))+2*a*exp(I*(d*x+c))+b)+2*I/d/(-a^2+b^2)^(3/2)*a*e*arctan(1/
2*(2*exp(I*(d*x+c))*b+2*a)/(-a^2+b^2)^(1/2))+I/d/(-a^2+b^2)*a*f/(a^2-b^2)^(
1/2)*ln((-exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x+I/d^2
/(-a^2+b^2)*a*f/(a^2-b^2)^(1/2)*ln((-exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)-a)/(-
a+(a^2-b^2)^(1/2)))*c-I/d/(-a^2+b^2)*a*f/(a^2-b^2)^(1/2)*ln((exp(I*(d*x+c))
*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x-I/d^2/(-a^2+b^2)*a*f/(a^2-b^2)
^(1/2)*ln((exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/d^2
/(-a^2+b^2)*a*f/(a^2-b^2)^(1/2)*dilog((-exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)-a)
/(-a+(a^2-b^2)^(1/2)))-1/d^2/(-a^2+b^2)*a*f/(a^2-b^2)^(1/2)*dilog((exp(I*(d
*x+c))*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2*I/d^2/(-a^2+b^2)^(3/2)*a
*f*c*arctan(1/2*(2*exp(I*(d*x+c))*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1491 vs. 2(269) = 538.
time = 0.57, size = 1491, normalized size = 5.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(
d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2
 - b^2)/b^2) + b)/b + 1) - (a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2
)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (a*b^2*f*cos(d*x + c) + a^2
*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c) + (b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - (a*b^2
*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I
*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2)
 + b)/b + 1) - (-I*a^2*b*d*f*x - I*a^2*b*c*f + (-I*a*b^2*d*f*x - I*a*b^2*c*
f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - (I
*a^2*b*d*f*x + I*a^2*b*c*f + (I*a*b^2*d*f*x + I*a*b^2*c*f)*cos(d*x + c))*sq
rt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - (I*a^2*b*d*f*x + I*a^
2*b*c*f + (I*a*b^2*d*f*x + I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)
*log((a*cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c
))*sqrt((a^2 - b^2)/b^2) + b)/b) - (-I*a^2*b*d*f*x - I*a^2*b*c*f + (-I*a*b^
2*d*f*x - I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x +
 c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^
2)/b^2) + b)/b) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 - a*b^2)*f - (-I*a^2
*b*c*f + I*a^2*b*d*e + (-I*a*b^2*c*f + I*a*b^2*d*e)*cos(d*x + c))*sqrt((a^2
 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b
^2)/b^2) + 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 - a*b^2)*f - (I*a^2*
b*c*f - I*a^2*b*d*e + (I*a*b^2*c*f - I*a*b^2*d*e)*cos(d*x + c))*sqrt((a^2 -
 b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2
)/b^2) + 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 - a*b^2)*f - (-I*a^2*b
*c*f + I*a^2*b*d*e + (-I*a*b^2*c*f + I*a*b^2*d*e)*cos(d*x + c))*sqrt((a^2 -
 b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^
2)/b^2) - 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 - a*b^2)*f - (I*a^2*b
*c*f - I*a^2*b*d*e + (I*a*b^2*c*f - I*a*b^2*d*e)*cos(d*x + c))*sqrt((a^2 -
```

```
b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2
)/b^2) - 2*a) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*d*e)*sin(d*x + c))/(
(a^4*b - 2*a^2*b^3 + b^5)*d^2*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*cos(d*x + c) + a)^2, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] \text{Hanged}
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```